THIRD-ORDER SEMIDISCRETE CENTRAL-UPWIND SCHEME FOR THE SAINT VENANT EQUATIONS

Noor Hidayat [1)], Suhariningsih [2)], Agus Suryanto [1)]

[1)] Jurusan Matematika FMIPA Universitas Brawijaya Malang
[2)] Departemen Fisika FST Universitas Airlangga Surabaya

ABSTRACT

Saint Venant equations are a system of nonhomogeneous partial differential equations, which model the fluid flow in an open channel. These equations are hyperbolic equation and have form of balance law which consist of continuity equation and momentum equation. Saint Venant equations do not always have analytical solution, so that a numerical approximation becomes important when solving these equations. There are two important solutions of the Saint Venant equations, namely the solution of the stationary steady state and discontinuity solution. Both solutions are not always easy to obtain, because we need a numerical scheme that can accurately capture the discontinuous solution and preserve a stationary steady state solution. A scheme which can preserve a stationary steady state solution is called a well balanced scheme.

In this dissertation we discuss a finite volume method to solve the Saint Venant equations that can accurately capture the discontinuous solution and satisfy the well balanced properties. Such scheme is called the third order semi discrete central upwind method which contains the correction term for Saint Venant equations. Basically, this new scheme is a generalization of the second order semi discrete central upwind scheme for the conservation law.

The scheme is constructed in three steps, namely reconstruction, evolution and projection. In the first stage of the reconstruction step, we determine the cell average which is used to reconstruct the piecewise polynomial. This polynomial is a convex combinations of linear and basic quadratic polynomials. The quadratic polynomials satisfy conservation, third order accurate and shape preserving properties. Based on the quadratic polynomials, we obtain the left and right values of the total height (stage), discharge, velocity of fluid, one side local speed propagation, respectively on the cell interface.

In the evolution step, the spatial domain is partitioned into the narrower cell by using the one side local speed propagation to get smooth and nonsmooth regions. Based on the narrower cell we obtain an intermediate average. Furthermore, we reconstruct the piecewise linear polynomial using this intermediate average. In this step we obtain a discretization of the slope which is referred to as a less dissipation way.

In the projection step, this reconstruction is projected into the original cell to obtain the fully discrete scheme. By using the concept of a derivative, we obtain a third order semi discrete central upwind scheme which contains a correction term for the conservation laws. This scheme is a system of ordinary differential equations. Furthermore, this system is solved by using the first order Runge-Kutta solver. Then we complete this scheme by discretizing the source term to obtain the third order semi discrete central upwind scheme for Saint Venant equations.

The performance of the proposed scheme is verified by considering some initial and/or boundary value problems. The problems are lake at rest, perturbation, dam break and obstruction parabolic. Lake at rest problem is used to check the well balanced properties. The result of numerical test shows that the schemes is well balanced. Perturbation problem test shows that the scheme can accurately capture the discontinuous solution. This test also shows that the dissipation of the third order scheme is lower than the second order one. Dam break problem test shows that the scheme can accurately capture the discontinuous solution and the increasing of the order of the scheme can increase its order of accuracy. Parabolic obstruction problem test shows that in the case of a steady flow, convergence of the third order scheme better than the second order one.

Keywords: central upwind semi discrete, Saint Venant, quadratic polynomial.