

# Comparison of particle swarm optimization and firefly algorithm in parameter estimation of Lotka-Volterra

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# Comparison of Particle Swarm Optimization and Firefly Algorithm in Parameter Estimation of Lotka-Volterra

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**Abstract.** Lotka-Volterra competition model has been applied in life sciences including competition between species, predicting the *Aeromonas hydrophila* growth on fish surface. The competition model also has been used in social sciences, including competition in the Korean stock market and competition between two types of bank, namely commercial bank and rural bank in Indonesia. It is well-known that the analytical solution of the Lotka-Volterra is unknown. Here gradient-based methods such as Newton method and Levenberg-Marquardt are difficult to be implemented to estimate parameters of the model. In order to estimate parameters in the model, one need to use heuristic method such as genetic algorithm, particle swarm optimization, firefly algorithm or other heuristic methods. In this paper, we compared performance of particle swarm optimization and firefly algorithm in parameter estimation of Lotka-Volterra type competition model. Here we used the profit data of commercial bank and rural bank, where the data cited from literature. We found the mean absolute percentage error (MAPE) of firefly algorithm is a little bit smaller than the error of particle swarm optimization method. We also found variance of the error of firefly algorithm is lower than the particle swarm optimization method. Hence, for parameter estimation of Lotka-Volterra competition model, firefly algorithm is more beneficial than the particle swarm optimization method.

**Keywords:** particle swarm optimization, firefly algorithm, parameter estimation, competition model

## INTRODUCTION

In general, a mathematical model contains one or more parameters. Many mathematical models are represented either in the form of a non-linear differential equation or in the form of systems of non-linear differential equations. Hence the exact solutions of the mathematical models could not be determined. As a result, parameter values in the model cannot or are difficult to determine using standard optimization methods such as the Newton method, the Levenberg-Marquardt method and the steepest descent method. In addition, the gradient-based methods typically converge to a local optimum solution [1]. In this condition, one can use heuristic methods such as genetic algorithms, particle swarm optimization, or firefly algorithms to obtain parameter values from a system of nonlinear differential equations [2, 3].

The Lotka-Volterra competition mathematical model is widely applied in the field of biological sciences, including inter-species competition and prediction of *Aeromonas hydrophila* growth on surface of fish during the refrigerated storage [4]. The competition model has also been utilized in social sciences, including competition of the Korean stock market [5], retail industry competition in Taiwan [6], retailing formats competition [7] and competition between commercial banks and rural banks in Indonesia [8]. The competition model is a system of nonlinear differential equations. Hence the analytic solution of the model is unknown.

Like genetic algorithms, particle swarm optimization and firefly algorithms are bio-inspired optimization methods. Basically, the particle swarm optimization method is a continuous optimization method. Moreover, the particle Swarm Optimization method has been applied to combinatorial optimization problems including scheduling problems [9, 10], travelling salesman problems [11], flow shop [12], and vehicle routing problems [13]. Firefly

algorithm also has been applied to many areas including design optimization of a robotic gripper [14], modelling and study of SSSC compensator [15] and optimization design of FIR digital filter [16].

Some researchers performed a performance comparison of particle swarm optimization and firefly algorithm. Palmieri et al. analyzed performance of particle swarm optimization and firefly algorithm in terms of total energy used by the robots to finished an assignment [17]. Hussain and Jenkins compared the methods for system identification with various types of nonlinear systems [18]. Farzana and Mahadevan compared performance of the methods on congestion management of deregulated power market [19]. In this article, we compare performance of particle swarm optimization and firefly algorithm in parameter estimation of Lotka-Volterra type competition model.

We organized this paper as follows. In section 2 we briefly presents particle swarm optimization and firefly algorithm. We present performance comparison of the methods in parameter estimation of Lotka-Volterra competition model in section 3. Finally, we draw some conclusions in the last section.

## 1.4 PARTICLE SWARM OPTIMIZATION AND FIREFLY ALGORITHM

Particle swarm optimization (PSO) is an optimization method inspired from the behavior of birds in a swarm. In PSO method, value of a solution is represented by position of a particle. PSO procedure consists of following main steps:

- (1) Generate initial solutions (initial positions) and initial velocities.
- (2) Fitness evaluation of every particle.
- (3) Renew the individual best solution (pbest particle) and the global best solution (gbest particle).
- (4) Renew velocity (v) and position (x) of every particle.

The PSO procedure is repeated until a termination condition is achieved.

In PSO method, particle velocity update is performed by using the following equation

$$v_j(k+1) = wv_j(k) + c_1r_1(p_j(k) - x_j(k)) + c_2r_2(g(k) - x_j(k)), j = 1, 2, \dots, np. \quad (1)$$

Here  $v_j(k)$  and  $x_j(k)$  are the j-th particle velocity and particle position respectively, while  $p_j(k)$  and  $g(k)$  are individual best particle and global best particle at k iteration. In eq. (1),  $w$ ,  $c_1$  and  $c_2$  are initial weight parameter, individual cognitive parameter and social cognitive parameter respectively. Here  $r_1$  and  $r_2$  are random numbers between zero and one with uniform distribution. The typical values of  $w$ ,  $c_1$  and  $c_2$  are  $w = 1$ ,  $c_1 = c_2 = 2.2$ . Position update is performed by using the following equation

$$x_j(k+1) = x_j(k) + v_j(k+1). \quad (2)$$

Firefly algorithm is an optimization method inspired from fireflies flashing behaviour, where a firefly moves to another brighter firefly. In the algorithm, a solution is represented by a firefly position. Firefly algorithm consists of three main steps namely generate initial solutions, evaluate fitness of every solution and update the solutions until some termination condition is reached. In a minimization problem of a function  $f$ , the main update of firefly position of  $x_i$  and  $x_j$  where  $f(x_j) < f(x_i)$ , is given by the following equation

$$x_j(k+1) = x_i(k) + \beta \exp(-\gamma r_{ij}^2) (x_i(k) - x_j(k)) + \alpha(k)\epsilon(k). \quad (3)$$

Here  $r_{ij}$  is distance between position  $x_i$  and  $x_j$ , while  $\alpha(k)$  is a parameter controlling the step size at the k iteration and  $\epsilon(k)$  is a random vector with Gaussian or other distribution at the k iteration. In eq. (3)  $\beta$  and  $\gamma$  are scalar parameters.

Premature convergence is commonly problem in implementation of a heuristic method including in the PSO method and the firefly algorithm. In order to obtain the best solution from PSO and FA, both algorithm should be applied many times. In the next section, we compared performance comparison of the methods in parameter estimation of a dynamical system model, namely the Lotka-Volterra competition model.

## A COMPARISON OF PARTICLE SWARM OPTIMIZATION AND FIREFLY ALGORITHM

In this section, we study performance of PSO and FA in estimating the parameters of the Lotka-Volterra competition model. The Lotka-Volterra competition model could be represented as

$$\frac{dy_1}{dt} = r_1y_1 \left(1 - \frac{y_1}{K_1}\right) - b_1y_1y_2, \quad (4)$$

$$\frac{dy_2}{dt} = r_2y_2 \left(1 - \frac{y_2}{K_2}\right) - b_2y_1y_2. \quad (5)$$

Fatmawati et al. applied the model to describe the competition between commercial banks and rural banks in Indonesia. Here  $y_1(t)$  and  $y_2(t)$  are annual profit of commercial bank and rural bank in Indonesia at t year. Parameters  $r_1$  and  $r_2$  are annual profit growth rate, while  $K_1$  and  $K_2$  are maximum profit of commercial bank and rural bank respectively. Parameters  $b_1$  and  $b_2$  are reduction rate of the annual profit of the banks due to competition.

We apply PSO and FA to estimate parameters of the competition model by using annual profit of commercial bank and rural bank in Indonesia from 2004 ( $t = 0$ ) until 2018 ( $t = 14$ ). Here the profit unit is in billion rupiah. The annual profit data is cited from the Indonesia Banking Statistic data [20]. The annual profit data dynamics of commercial bank and rural bank are presented in the Table 1.

TABLE 1. Annual profit (in billion IDR) of commercial bank and rural bank.

t	Year	y1	y2
0	2004	29463	539
1	2005	24899	604
2	2006	28334	509
3	2007	35015	663
4	2008	30606	849
5	2009	45215	1158
6	2010	57309	1447
7	2011	75077	1853
8	2012	92830	2328
9	2013	106707	2661
10	2014	112160	2682
11	2015	104628	2755
12	2016	106544	2936
13	2017	131156	3210
14	2018	150013	3371

We estimated parameters in the Lotka-Volterra competition model such that the mean absolute percentage error (MAPE)

$$MAPE = \frac{1}{2n} \sum_{i=1}^n \left( \left| \frac{y_{1i} - \hat{y}_{1i}}{y_{1i}} \right| + \left| \frac{y_{2i} - \hat{y}_{2i}}{y_{2i}} \right| \right) \quad (6)$$

is minimum. Here n is number of observation data.

Table 2 presents performance comparison of Firefly Algorithm and the Particle Swarm Optimization (PSO) from 20 trials. Both algorithms are stopped after 200 iterations.

TABLE 2. Statistics of MAPE from 20 trials.

Statistics	FA	PSO
Minimum (the best results)	0.1429895	0.144149
Mean	0.1429920 <sup>a</sup>	0.148925 <sup>b</sup>
Standard deviation	0.0000049 <sup>a</sup>	0.001895 <sup>b</sup>
Maximum	0.1430123	0.151836

a,b different superscripts showed significant difference between group at the level 0.05.

From the Table 2, we found that the MAPE average of firefly algorithm was little bit smaller than the MAPE average of particle swarm optimization. We also found that MAPE variance of firefly algorithm was much smaller than MAPE variance of particle swarm optimization. Hence the firefly algorithm is more robust than the particle swarm optimization method. This results indicate that for parameter estimation of Lotka-Volterra competition model, firefly algorithm is more beneficial than the particle swarm optimization method.

## CONCLUSIONS

We have compared performance of particle swarm optimization and firefly algorithm to estimate parameters from Lotka-Volterra competition model. We found that both algorithms could be applied to estimate parameters of the



Lotka-Volterra competition mathematical model. We found that the firefly algorithm is more beneficial than the particle swarm optimization, since the firefly algorithm gives a smaller error and smaller variance than the particle swarm optimization method.

## ACKNOWLEDGMENTS

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