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A comparison study of bank data in fractional calculus

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ABSTRACT

The present paper investigate the dynamics of the bank data through a competition model with real field data for the year 2004–2014. Initially, we formulate a competition model for the bank data and then use different fractional approaches to simulate the model with the real data for many fractional order parameters α. Then, we present a novel approach for each fractional model and provide a graphical illustration with real data. We show that all these fractional approaches have good resemblance to each other and can be used to model such real data case. We prove in general that the results of the fractional approaches utilized here are good for modeling purpose but also we prove the results of the fractional Atangana–Baleanu operator is more accurate and flexible and can be used confidently to modeled such real case problem.

1. Introduction

Mathematical models are not only used to describe the dynamics of the physical and biological sciences but also used to modeled the phenomenon of other sciences areas. The uses of mathematical models in bank and finance is also a strong growing area of research nowadays for researchers and scientists. Bank are the business places that collect funds from the individuals of a particle area or region and then utilize these funds on public in different activities in order to facilitate the people and improve their life style [1]. The banks are only used to store the funds and lend to the people but also used a useful tool to stabilized the host country and encourage the national economic growth rate. There are some type of banks in particularly the rural and commercial banks are used for such activities of economic growth. The commercial bank is such type of bank that carries the business activities thats based on Syariah Principles which their activities provide services in payment traffic. The providing banking services its activities can be conventional but it can also be islamic based principles, some of them uses both the principles [1]. The aim of the commercial bank is to help the implementation of national development and provide stability to the economic growth and to the public [2].

According to the Act No. 10 of 1998 the rural bank is defined to be that a bank conducting such business activities that are based on Syariah principles or conventionally where their services includes not to provide in payment traffic. In comparison of the commercial banks the business activities of the rural banks are comparatively have less business activities that collecting only the funds from the public on time deposits savings, credit, and place funds in the form of Bank Indonesia Certificates while the rural banks are not accepting funds in the shape of demands deposits and participate in payment traffic, making business activities in foreign currency, conduct equity participation and business insurance [3].

The Indonesian banking statistics shows that the rural banks are more in number than that of commercial banks in the country, the reason is that the commercial banks have more activities while the rural have less, and the profit of the rural banks is less than that of the commercial banks. The rural banks have less profit than that of commercial banks and still to improve their products and activities [4]. It should be noted that the products of both the banks have not a such big difference and there may be a competition of costumers to get.

Due to such possible competition of rural and commercial banks in Indonesia one can used their dynamical behavior through

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a competition model of Lotka Volterra type and was introduced in 1920 by Alfred J. Lotka and Vito Volterra. This model composed of two equations that describes to compete for food items. This model generalize the logistics model and can be used to modeled such competition between two species in order to predict their outcome efficiently [5]. This Lotka Volterra type model have been utilized recently by many researchers [6–11]. For example the Korean mobile company data has been studied in [6] through competition model. A competition model is used as a technological substitution model in [7]. Similarly, the Korean stock market, dynamics of the markets, modeling and policy implications and banking system have been studied respectively in [8–11]. Recently, a competition model for the bank data has been studied in [12].

The above mathematical models that describes various phenomenon of real life situation has been studied in integer order case except [12]. Sometimes it is not be the case that the integer order model can best describes the dynamics of a particle phenomenon then the people rely on fractional calculus to better understand their behavior. Fractional calculus got a lot of attention from the scientists and researchers when the researchers developed some new fractional operators and thus different mathematical models in science and engineering have been proposed to analyzed the suitability of the fractional operators. The fractional operators that are using the researchers nowadays are the Caputo, Caputo-Fabrizio and the Atangana–Baleanu. Atangana–Baleanu overcome the limitations of Caputo and Caputo–Fabrizio derivative where the kernel was singular and may not have been properly addressed the real world problems. It should be noted that all these operators and many other are used by the researchers to formulate and compared their model with real data to best describe the model and to obtain reasonable parameter estimation [13–21]. All these results describes in the references were effectively used to study different type of models. Some recent fractional mathematical models and their application to different area of science and engineering have been proposed [22–26]. For instant, the authors in [22] considered a mathematical model with experimental data and obtained the model results in the case of arbitrary derivative. A new approach in fractional calculus as a fractal derivative applied to some chaotic models is proposed in [23]. A real data of dengue epidemic with fractional approach is considered in [24], where the authors used various fractional approaches to determine the suitability of fractional derivative to the field data. A new advancement in fractional calculus as a fractal-fractional and their application to a dynamical system is studied in [25]. A mathematical model for ground water problem is considered in fractional calculus is proposed in [26]. More recently, a fractional model for the bank data in the framework of Atangana–Baleanu and Caputo derivative is presented in [12]. This paper generalized the work in [12] by using another CF operators and then used the model parameters estimation by using the least square curve fitting. Also, in this updated paper we not only used the Caputo, CF and AB derivative but also used their fractional parameter model in the sense of Caputo, CF and AB and compare the results with real data. Most recently, some authors used the real data of different nature and obtained the results by using the fractional calculus [25,27–30]. For example, a comparative study with real data in the framework of fractional calculus is explored in [27]. The authors in [28] describes the chickenpox disease with real data application in fractional calculus. The dengue outbreak in the fractional modeling have been proposed in [25]. An fractional epidemic model with two strain is studied in [29] with real data. A real application of fractional derivatives to the blood ethanol concentration is studied in [30].

The above mentioned articles that were proposed by the authors shows the significance of the fractional calculus and their suitability to problems of science and engineering. The results of these papers inspired us to formulated and analyze a real data study of bank data in various fractional approaches for the year 2004–2014. We presented details literature related to the proposed study and also a presented a waste literature on fractional calculus and their application. The rest of the work to the proposed study is partitioned is as follows: Some related fractional calculus related to our study is given in section 2. Model description and their detail analysis is shown in section 3. The application of various fractional approaches to the model is considered in Section 4. A comparison of the fractional operators for different value of α is shown in section 5. Section 6 summarize the fractional approaches used for the real data comparison of bank data.

2. Basics of fractional calculus

The present section recall the fractional derivatives and definitions and related results that will be used later in our fractional bank data studies. We basically, provide here three useful and widely used fractional operators that is Here, we recall Caputo, Caputo–Fabrizio, and the Atangana–Baleanu derivative [14,20,31,32].

Definition 1. Let a function $w:\mathbb{R}^+\to\mathbb{R}$ with fractional order $\alpha>0$, then one can define the fractional integral of order $\alpha>0$ is as follows:

$$I_t^\alpha(w(t)) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\psi)^{\alpha-1}w(\psi)d\psi,$$

here Γ describes the Gamma function and α shows the fractional order parameter.

Definition 2. The Caputo derivative for the given function $w\in C^n$ with order α is described is as follows:

$${}^CD_t^\alpha(w(t)) = I^{n-\alpha}D^n w(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{w^n(\psi)}{(t-\psi)^{\alpha-n+1}}d\psi,$$

and $n-1<\alpha<n\in N$. Clearly, ${}^CD_t^\alpha(w(t))$ tends to $w'(t)$ as $\alpha\to1$.

Definition 3. Suppose $w\in H^1(p,q)$, with $q>p$, and $0\le\alpha\le1$, then the definition of Caputo–Fabrizio derivative is follows as:

$$D_t^\alpha(w(t)) = \frac{\mathcal{P}(\alpha)}{1-\alpha}\int_a^t w'(\psi)\exp\left[-\alpha\frac{t-\psi}{1-\alpha}\right]d\psi, \tag{1}$$

where $\mathcal{P}(\alpha)$ denote the normalized function and holds $\mathcal{P}(0)=\mathcal{P}(1)=1$. If $w\notin H^1(p,q)$ then, the following is suggested:

$$D_t^\alpha(w(t)) = \frac{\alpha\ \mathcal{P}(\alpha)}{1-\alpha}\int_a^t (w(t)-w(\psi))\exp\left[-\alpha\frac{t-\psi}{1-\alpha}\right]d\psi. \tag{2}$$

Let $\nu=\frac{1-\alpha}{\alpha}\in[0,\infty)$, $\alpha=\frac{1}{1+\nu}\in[0,1]$, then equation given by (2) can be expressed is as follows,

$$D_t^\nu(w(t)) = \frac{\mathcal{P}(\nu)}{\nu}\int_a^t w'(\psi)\exp\left[-\frac{t-\psi}{\nu}\right]d\psi,$$

$$\mathcal{P}(0)=\mathcal{P}(\infty)=1. \tag{3}$$

Further,

$$\lim_{\nu\longrightarrow0}\frac{1}{\nu}\exp\left[-\frac{t-\psi}{\nu}\right]=\varphi(x\psi-t). \tag{4}$$

Definition 4. Consider $\alpha\in(0,1)$, for a function $w(\psi)$ then we can write the integral of fractional order α is as follows,

$$I_t^\alpha(w(t)) = \frac{2(1-\alpha)}{(2-\alpha)\mathcal{P}(\alpha)}g(t) + \frac{2\alpha}{(2-\alpha)\mathcal{P}(\alpha)}\int_0^t w(\psi)d\psi,$$
$$t\ge0. \tag{5}$$

Remark 1. In Eq. (4), the remainder of the Caputo type non-integer order integral of the function with order $\alpha\in(0,1)$ is a mean into w with integral of order 1. Thus, it requires,

$$\frac{2}{2\mathcal{P}(\alpha)-\alpha\mathcal{P}(\alpha)}=1, \tag{6}$$

implies that $\mathcal{P}(\alpha) = \frac{2}{2-\alpha}$, $\alpha \in (0, 1)$. Based on Eq. (6), a new Caputo derivative is suggested with $\alpha \in (0, 1)$ and is given by

$$D_t^{\alpha}(w(t)) = \frac{1}{1-\alpha}\int_0^t w'(x)\exp\left[-\alpha\frac{t-\psi}{1-\alpha}\right]d\psi. \tag{7}$$

Definition 5. Consider $w \in H^1(p, q)$, where q greater than p, and $0 \le \alpha \le 1$, then we define the Atangana–Baleanu derivative in the following:

$$ {}^{ABC}_{a}D_t^{\alpha} w(t) = \frac{P(\alpha)}{1-\alpha}\int_a^t w'(\psi)E_{\alpha}\left[-\alpha\frac{(t-\psi)^{\alpha}}{1-\alpha}\right]d\psi. \tag{8}$$

Definition 6. The fractional integral for the Atangana–Baleanu derivative is expressed as follows:

$$ {}^{ABC}_{a}I_t^{\alpha} w(t) = \frac{1-\alpha}{\mathcal{P}(\alpha)}w(t) + \frac{\alpha}{\mathcal{P}(\alpha)\Gamma(\alpha)}\int_a^t f(\psi)(t-\psi)^{\alpha-1}d\psi. \tag{9}$$

One can restore the original function for the case when $\alpha = 0$.

Some results regarding the Atangana–Baleanu derivative is presented in the following:

Theorem 1. [20]. *The following is hold for a function $f \in C[a, b]$:*

$$\|{}^{ABC}_{a}D_t^{\alpha}(w(t))\| < \frac{\mathcal{P}(\alpha)}{1-\alpha}\|w(t)\|,$$
$$\text{where } \|w(t)\| = max_{p \le t \le q}|w(t)|. \tag{10}$$

The Lipschitz condition is satisfied by the Atangana–Beleanu derivative,

$$\|{}^{ABC}_{a}D_t^{\alpha}w_1(t) - {}^{ABC}_{a}D_t^{\alpha}w_2(t)\| < \varpi_1\|w_1(t) - w_2(t)\|. \tag{11}$$

Theorem 2. *A fractional differential is given by the following equation,*

$$ {}^{ABC}_{a}D_t^{\alpha}w(t) = W(t), \tag{12}$$

possess a unique solution given by

$$w(t) = \frac{1-\alpha}{\mathcal{P}(\alpha)}W(t) + \frac{\alpha}{\mathcal{P}(\alpha)\Gamma(\alpha)}\int_a^t F(\psi)(t-\psi)^{\alpha-1}d\psi. \tag{13}$$

3. Model framework

The present section investigates the comparison of commercial and rural banks in Indonesia through Lotka Volterra system. We use the least square curve fitting for each data set to the Lotka Volterra model for best profit data for the period 2004–2014. The Lotka Volterra model consists two classes namely, the profit gained by the commercial bank (y_1) and the profit gained by the rural bank given by (y_2). Some assumptions are under taken for model formulations are given as follows:

- The growth of the population of banks assumed to be logistically,
- Both the banks that is the commercial and the rural have limited annual profit.

The above imposed assumptions leads to the following system of ordinary differential equations that describes the dynamics among commercial and rural banks in Indonesia:

$$\begin{aligned}\frac{dy_1}{dt} &= r_1y_1\left(1-\frac{y_1}{P_1}\right) - \beta_1y_1y_2,\\ \frac{dy_2}{dt} &= r_2y_2\left(1-\frac{y_2}{P_2}\right) - \beta_2y_1y_2,\end{aligned} \tag{14}$$

where r_1 and r_2 respectively denote the growth rate of the profit for commercial and rural bank. The maximum profit for both the banks that is the commercial and rural bank is respectively shown by P_1 and P_2. The parameters β_1 and β_2 are respectively show the coefficients of commercial and rural bank. The unknown parameters in the given model (14) are r_i, P_i and β_i for $i = 1, 2$ that to be estimated through least square curve fitting and then by the fractional operators such as the Caputo, Caputo–Fabrizio (CF) and the Atangana–Baleanu (AB), we will provide some interesting illustrations for data fitting with these operators and will show you which will given the best fitting for the given model (14).

3.1. Parameter estimation

In this subsection we determine the unknown parameters values involved in the model (14) by using the method applied by the authors in [16,33]. We use the real data of banking statistics of Indonesia for the year 2004–2014 to estimate the parameters [4]. The data is derived from the Indonesian banks of rural and commercials where we use the data is in annual profit to best determine the comparison of these two banks. Using the least square curve fitting the parameters fitting to the commercial and rural banks and obtained the parameters values given in Table 1. We

Table 1
Parameters estimation for the rural and commercial banks in Indonesia for the year 2004–2014. (14).

Unknown	Details	Value for commercial bank setting
r_1	Growth rate of the profit of commercial bank	0.5 $year^{-1}$
P_1	Maximum profit of commercial bank	40000 $year^{-1}$
β_1	Coefficient of commercial bank	0.0080 $year^{-1}$
r_2	Growth rate of the profit of rural bank	0.200 $year^{-1}$
P_2	Maximum profit of rural bank	7400 $year^{-1}$
β_2	Coefficient of rural bank	$6.0000e-06$ $year^{-1}$
Unknown	Details	value for rural bank
r_1	Growth rate of the profit of commercial bank	0.2317 $year^{-1}$
P_1	Maximum profit of commercial bank	$3.3237e+05$ $year^{-1}$
β_1	Coefficient of commercial bank	0.8407 $year^{-1}$
r_2	Growth rate of the profit of rural bank	0.7208 $year^{-1}$
P_2	Maximum profit of rural bank	$7.7873e+03$ $year^{-1}$
β_2	Coefficient of rural bank	0.0372 $year^{-1}$
Unknown	Details	value for both the banks
r_1	Growth rate of the profit of commercial bank	0.13243 $year^{-1}$
P_1	Maximum profit of commercial bank	447318.198 $year^{-1}$
β_1	Coefficient of commercial bank	5.6190×10^{-8} $year^{-1}$
r_2	Growth rate of the profit of rural bank	0.19901$year^{-1}$
P_2	Maximum profit of rural bank	7540.6219 $year^{-1}$
β_2	Coefficient of rural bank	2804×10^{-8} $year^{-1}$

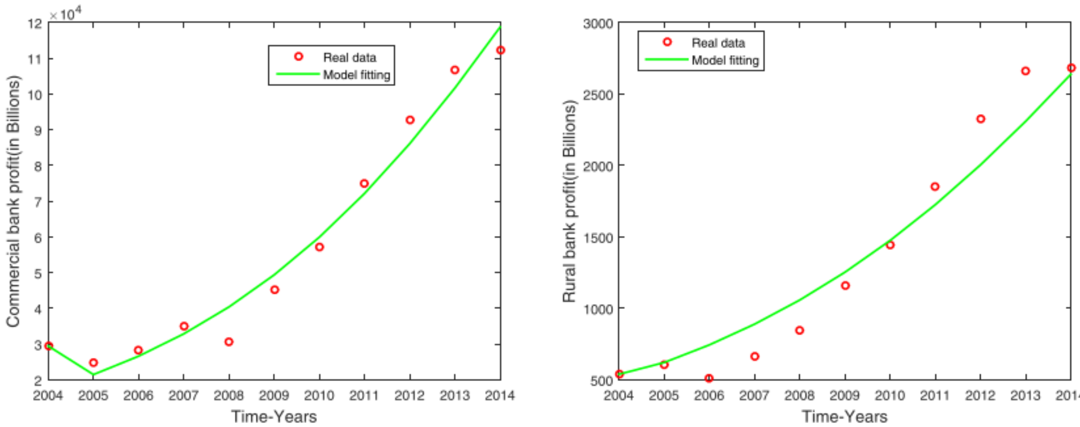


Fig. 1. Model fitting versus rural and commercial bank data for the years 2004–2014, (a) commercial bank data, (b) rural bank data.

first of values given in Table 1 are used for comparison of commercial bank and the fitting of the model versus real data is shown in Fig. 1(a) where the second set of data obtained through least square curve fitting to the rural bank versus model is shown in Fig. 1(b). The third set of data is used for the rest of the simulation results.

First, we write the given model in each operator in details in the following section.

4. Bank model in different fractional operators

Here in this section, we investigate the model (14) in different fractional operators. Initially, we generalize the model (14) in Caputo operator and is given in the following subsection.

4.1. Caputo fractional model

The Caputo fractional model can be obtained by apply the definition of Caputo derivative and the model (14) leads to the following system:

$$\begin{aligned} {}_0^C D_t^\alpha y_1 &= r_1 y_1\left(1 - \frac{y_1}{P_1}\right) - \beta_1 y_1 y_2, \\ {}_0^C D_t^\alpha y_2 &= r_2 y_2\left(1 - \frac{y_2}{P_2}\right) - \beta_2 y_1 y_2, \end{aligned} \tag{15}$$

and the model with fractional Caputo parameters is proposed as

$$\begin{aligned} {}_0^C D_t^\alpha y_1 &= r_1^\alpha y_1\left(1 - \frac{y_1}{P_1^\alpha}\right) - \beta_1 y_1 y_2, \\ {}_0^C D_t^\alpha y_2 &= r_2^\alpha y_2\left(1 - \frac{y_2}{P_2^\alpha}\right) - \beta_2 y_1 y_2. \end{aligned} \tag{16}$$

In the following subsection, we provide a numerical solution to the model (15).

4.2. Numerical solution with Caputo derivative

The present section investigates the numerical solution of the model (15) and using the parameters values described in Table 1. We use the FDE12 method for the solution of the Caputo model described the system (15) and obtain the graphical results shown in Figs. 2 and 3 and with their subgraphs. Fig. (2) shows different approaches used for the numerical solution of the fractional model versus real data fitting. In Fig. 2(a) we compare the real data versus Caputo model for $\alpha = 1$ and then choosing $\alpha = 1, 0.98, 0.96, 0.9$ and shown the results. Further in 2(c) there were observed some deviations of the fractional order $\alpha = 0.98$ versus real data. Also, we give a comparison of the fractional Caputo parameter model and obtained the result in Fig. 2(d) and the future prediction in Fig. 2 (e). In Fig. 3 we used the long time level and presented the dynamics of the Caputo model by using the value of the fractional order parameter $\alpha = 1, 0.9, 0.7, 0.5, 0.3, 0.1$. Thus the results achieved through Caputo model is also valid and can be used for data fitting.

4.3. Caputo–Fabrizio fractional model

The Caputo–Fabrizio fractional model can be obtained by apply the definition of Caputo–Fabrizio derivative and the model (14) leads to the following system:

$$\begin{aligned} {}_0^{CF} D_t^\alpha y_1 &= r_1 y_1\left(1 - \frac{y_1}{P_1}\right) - \beta_1 y_1 y_2, \\ {}_0^{CF} D_t^\alpha y_2 &= r_2 y_2\left(1 - \frac{y_2}{P_2}\right) - \beta_2 y_1 y_2, \end{aligned} \tag{17}$$

and the Caputo–Fabrizio model with fractional parameter is given by

$$\begin{aligned} {}_0^{CF} D_t^\alpha y_1 &= r_1^\alpha y_1\left(1 - \frac{y_1}{P_1^\alpha}\right) - \beta_1 y_1 y_2, \\ {}_0^{CF} D_t^\alpha y_2 &= r_2^\alpha y_2\left(1 - \frac{y_2}{P_2^\alpha}\right) - \beta_2 y_1 y_2. \end{aligned} \tag{18}$$

Next, we investigate the numerical solution of the Caputo–Fabrizio model 17–(18).

4.4. Numerical solution with Caputo–Fabrizio derivative

The present section investigates the numerical solution of the model (17) and using the parameters values described in Table 1. The fraction model of bank data described in system of Caputo–Fabrizio form (17) is used to present a novel numerical approach that is described in [34]. After the implementation of this scheme, we then use it for the numerical solution of the model (17) with many graphical illustrations. Initially, for numerical approach implementation of the fractional model (17), we write it in the form of fractional Volterra type and then applying the fundamental theorem of integration. So, we start the numerical scheme for the first equation of (17) with the application of the fundamental theorem

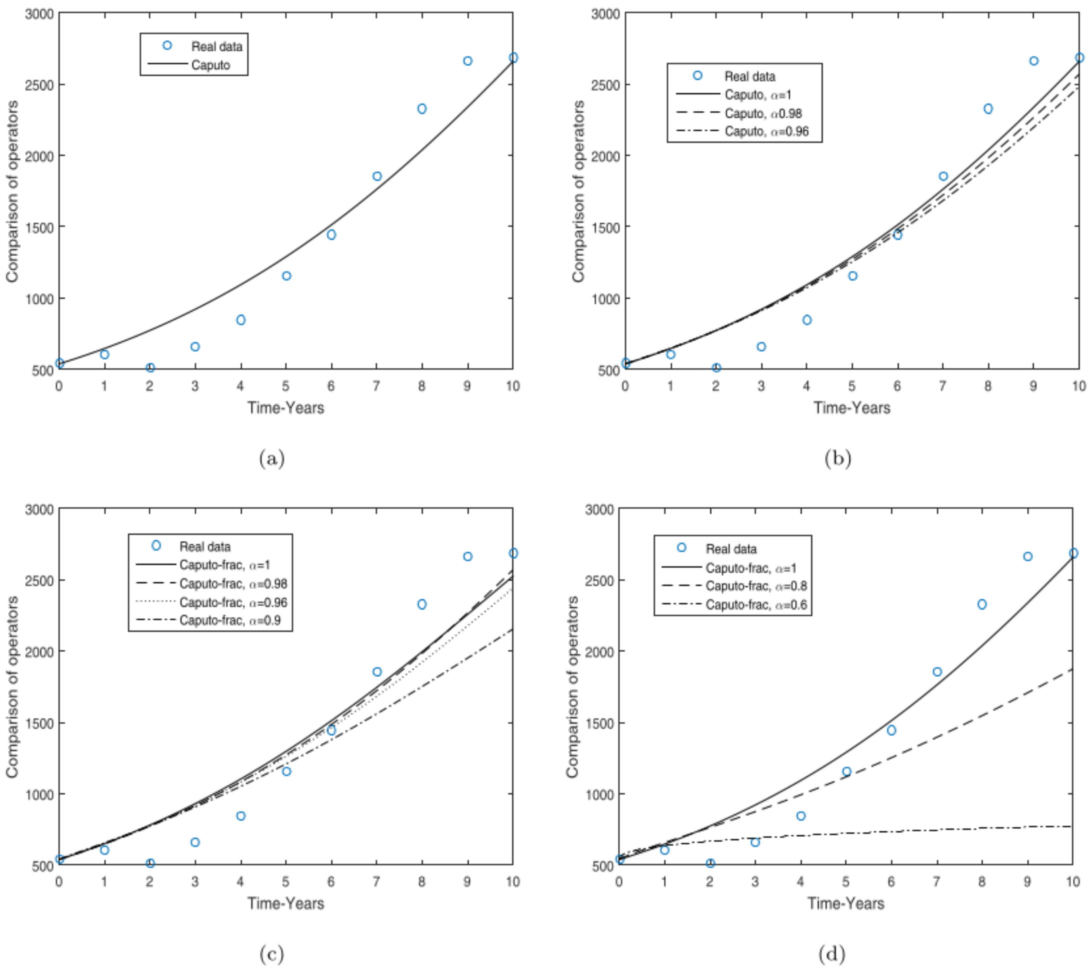


Fig. 2. Model fitting versus rural bank data for the years 2004–2014. (a) real data versus Caputo derivative, (b) real data versus Caputo derivative, $\alpha = 1, 0.98, 0.96$, (c) real data versus Caputo fractional parameter model, $\alpha = 1, 0.98, 0.96, 0.9$, (d) Comparison of Caputo fractional versus data for $\alpha = 1, 0.8, 0.6$.

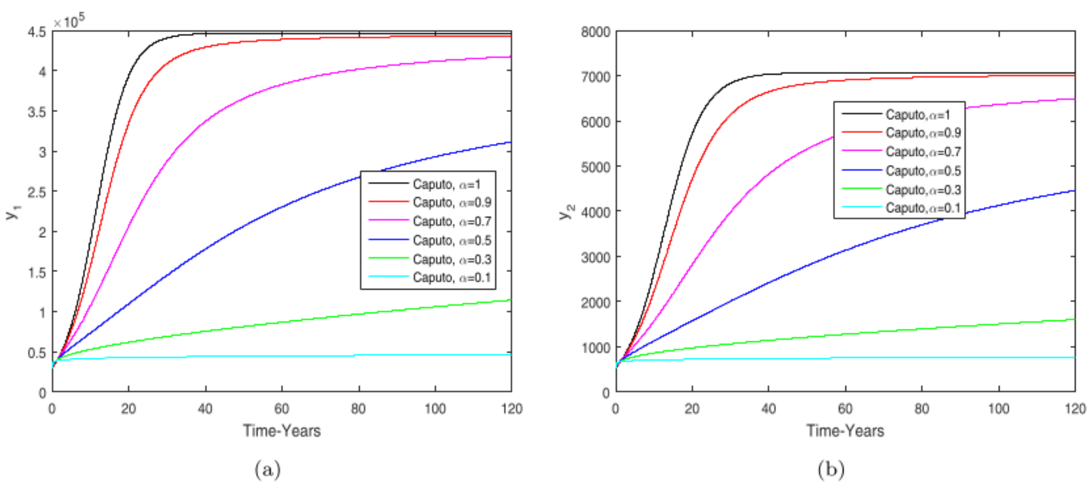


Fig. 3. Simulation of the Caputo model for $\alpha = 1, 0.9, 0.7, 0.5, 0.3, 0.1$.

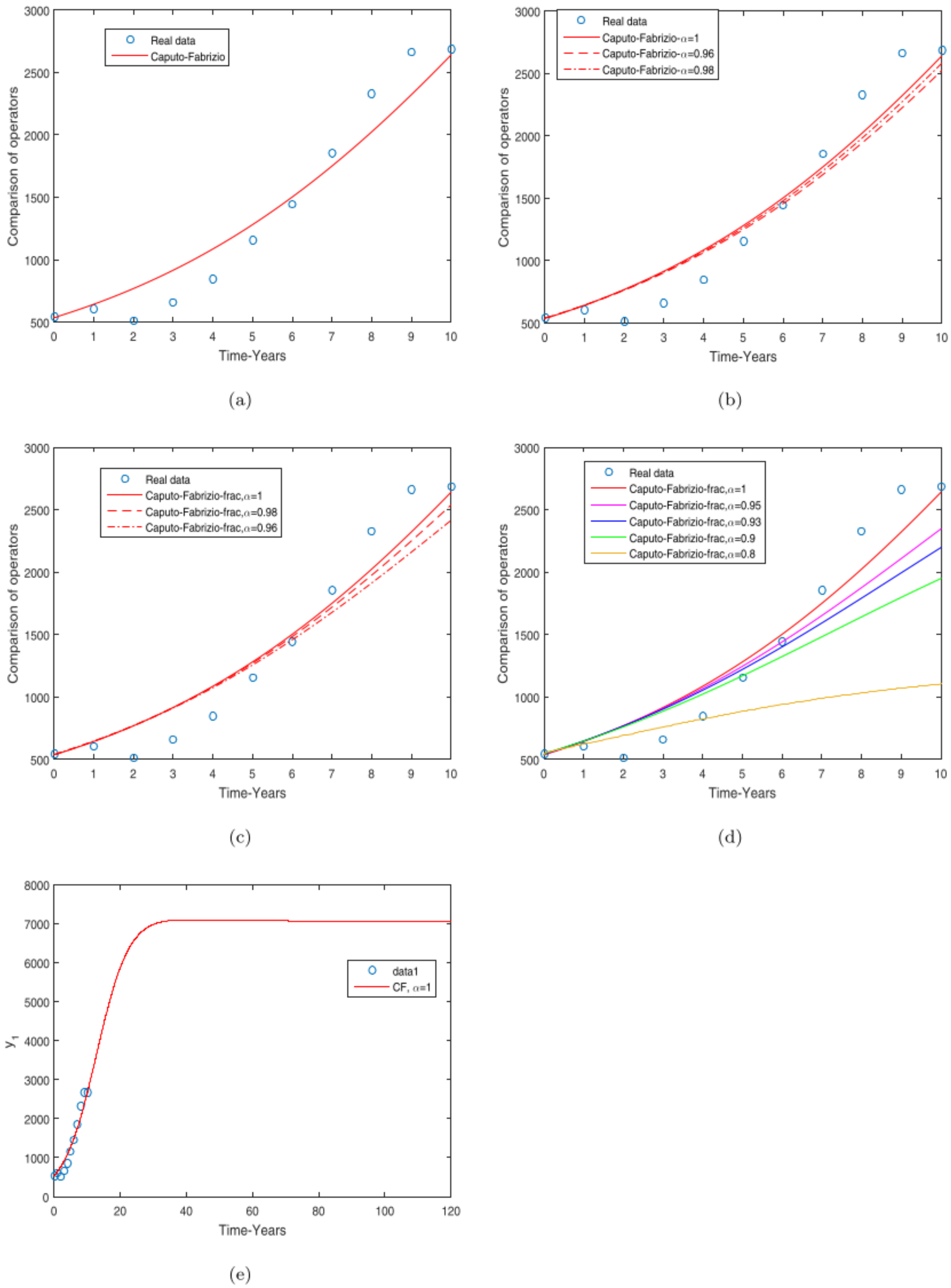


Fig. 4. Model fitting versus rural bank data for the years 2004–2014. (a) real data versus Caputo–Fabrizio derivative, (b) real data versus Caputo–Fabrizio derivative, $\alpha = 1, 0.98, 0.96$, (c) real data versus Caputo–Fabrizio fractional parameter model, $\alpha = 1, 0.98, 0.96$, (d) Comparison of Caputo–Fabrizio fractional versus data for $\alpha = 1, 0.95, 0.93, 0.9, 0.8$, (e) Future prediction of data versus model.

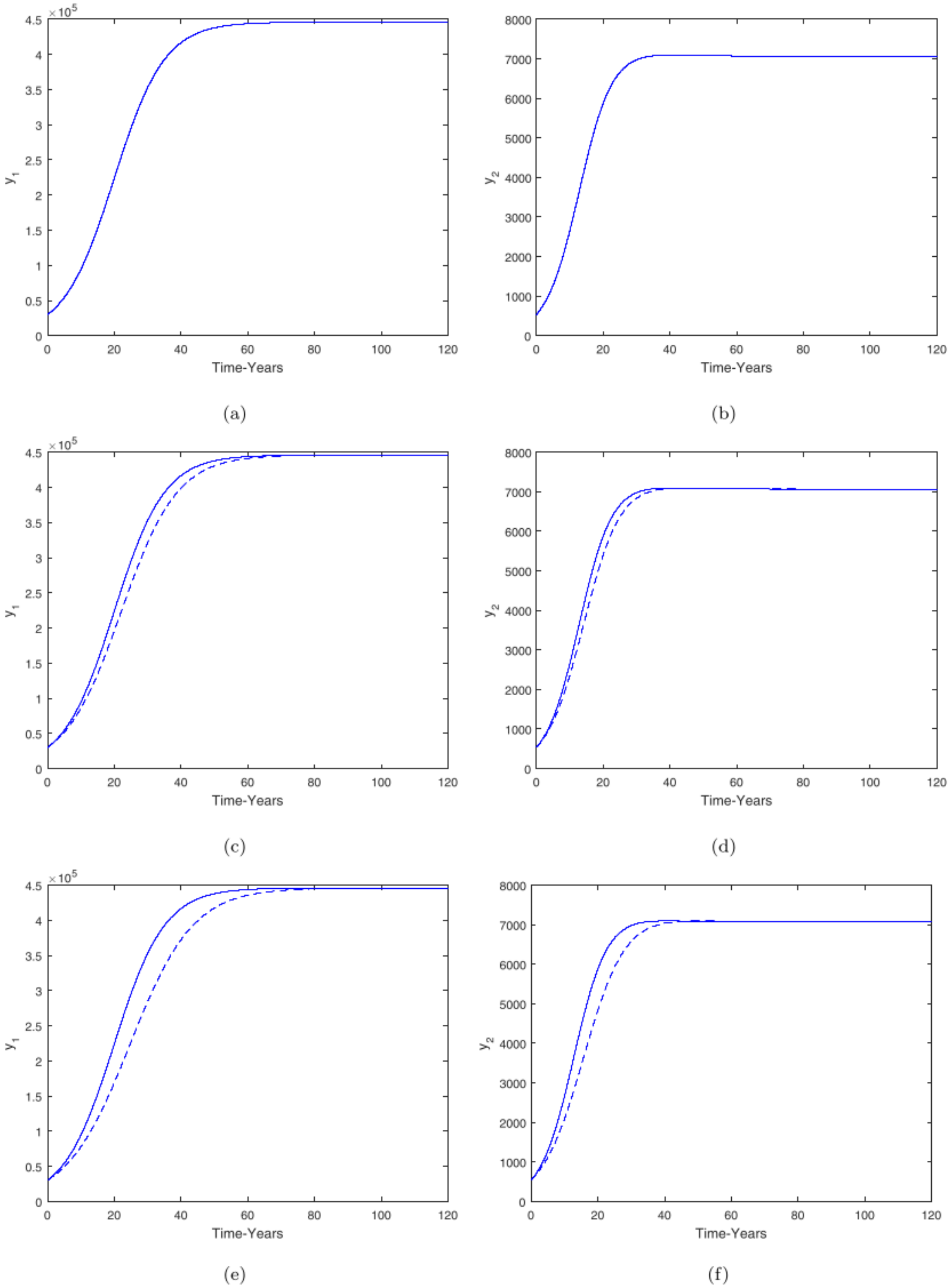


Fig. 5. Simulation of CF model for $\alpha = 1, 0.9, 0.7$, subfigure (a) and (b), $\alpha = 1$, subfigure (c) and (d), $\alpha = 0.9$, and subfigure (e) and (f), $\alpha = 0.7$.

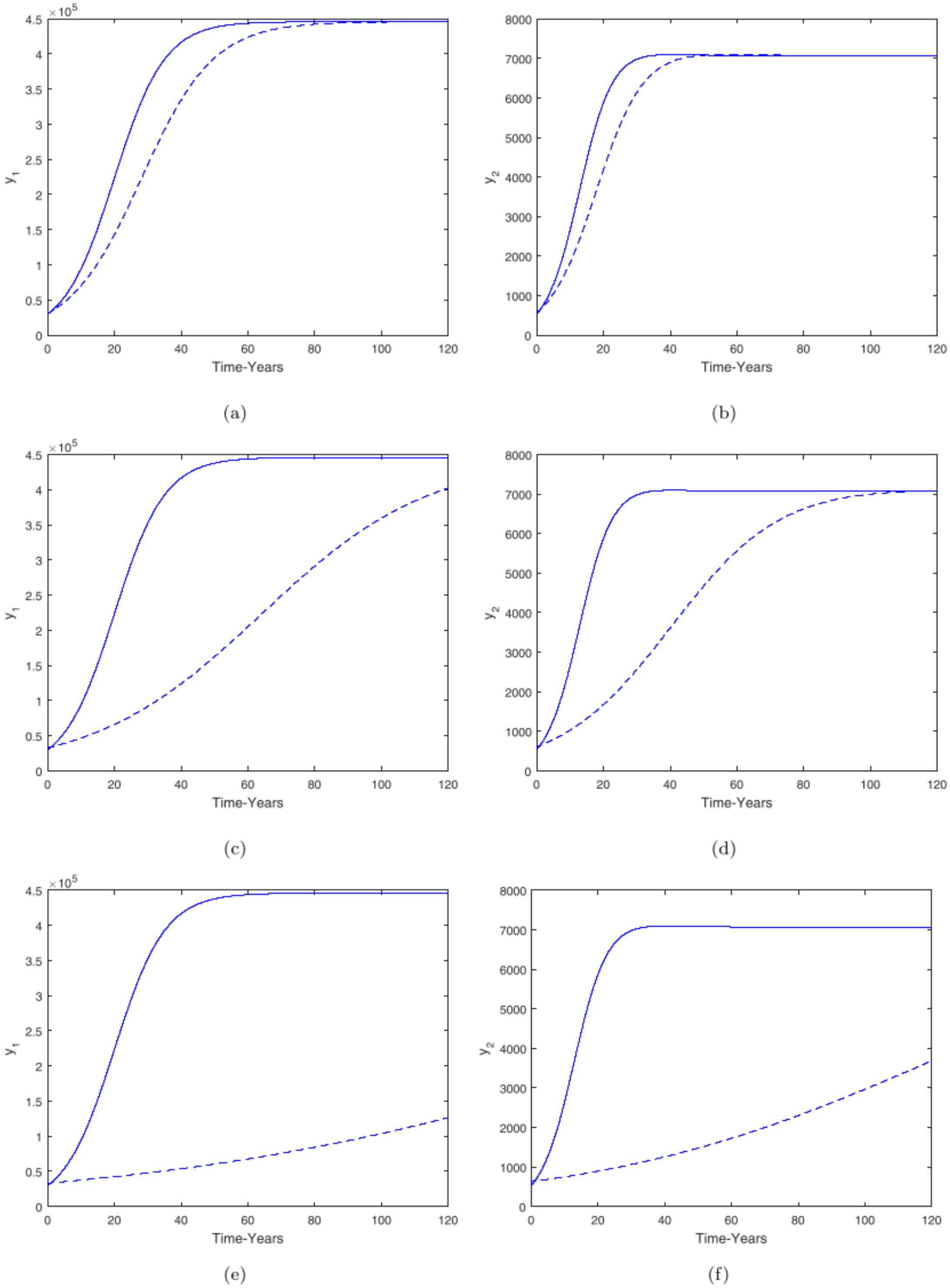


Fig. 6. Simulation of CF model for $\alpha = 0.5, 0.3, 0.1$, subfigure (a) and (b), $\alpha = 0.5$, subfigure (c) and (d), $\alpha = 0.3$, and subfigure (e) and (f), $\alpha = 0.1$.

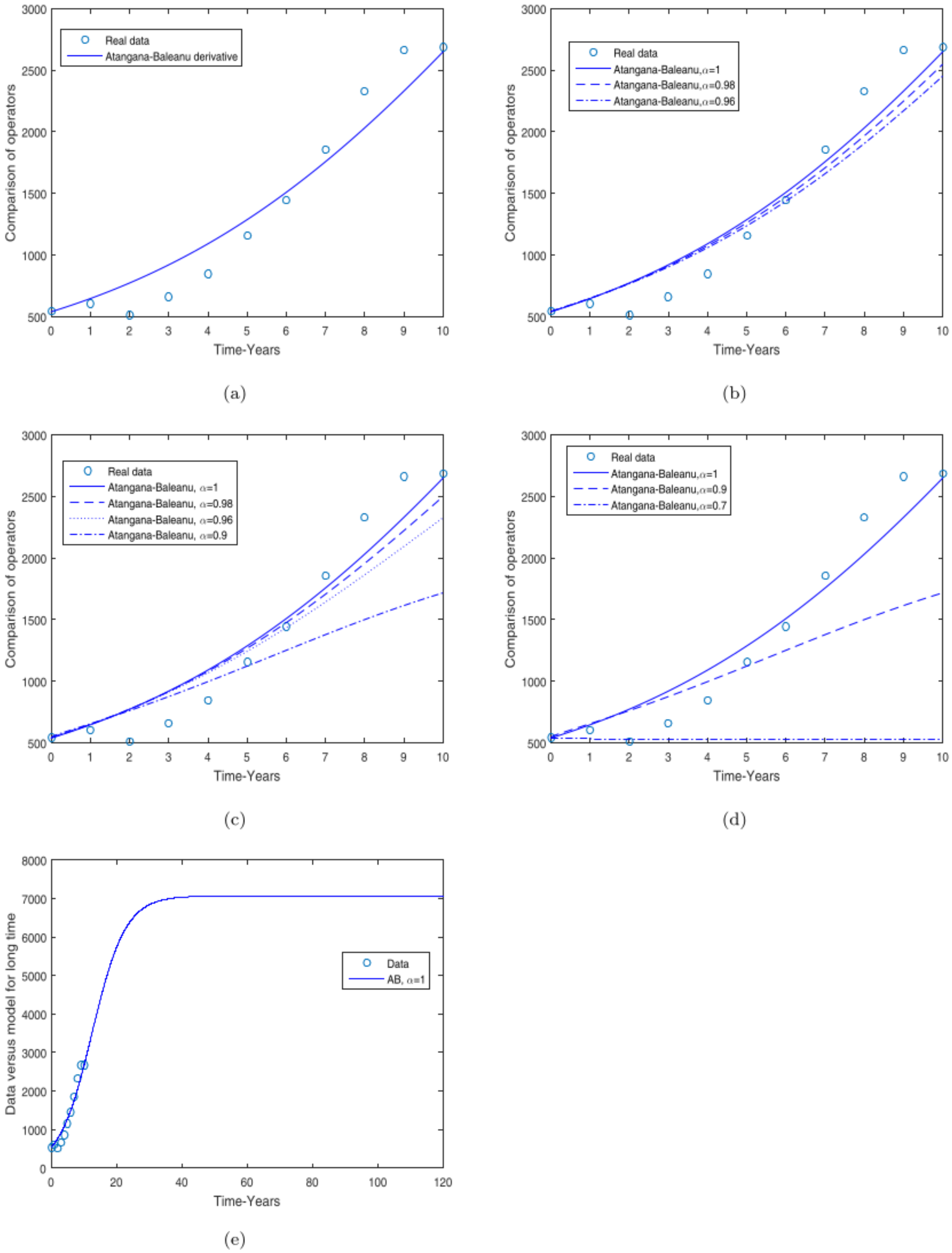


Fig. 7. Model fitting versus rural bank data for the years 2004–2014. (a) real data versus Atangana–Baleanu derivative, (b) real data versus Atangana–Baleanu derivative, $\alpha = 1, 0.98, 0.96$, (c) real data versus Atangana–Baleanu fractional parameter model, $\alpha = 1, 0.98, 0.96, 0.9$, (d) Comparison of Atangana–Baleanu fractional versus data for $\alpha = 1, 0.9, 0.97$.

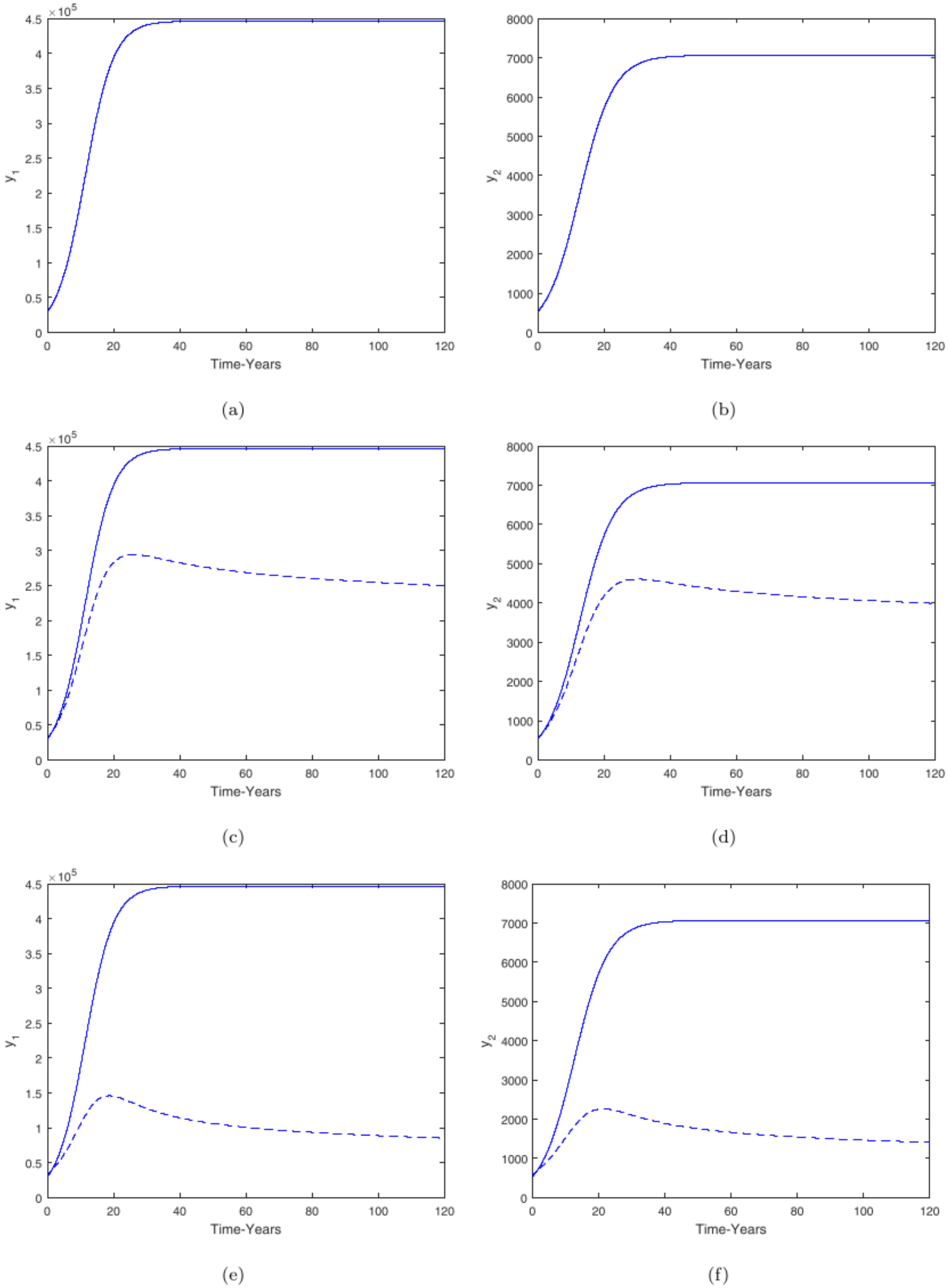


Fig. 8. Simulation of AB model for $\alpha = 1, 0.9, 0.7$, subfigure (a) and (b), $\alpha = 1$, subfigure (c) and (d), $\alpha = 0.9$, and subfigure (e) and (f), $\alpha = 0.7$.

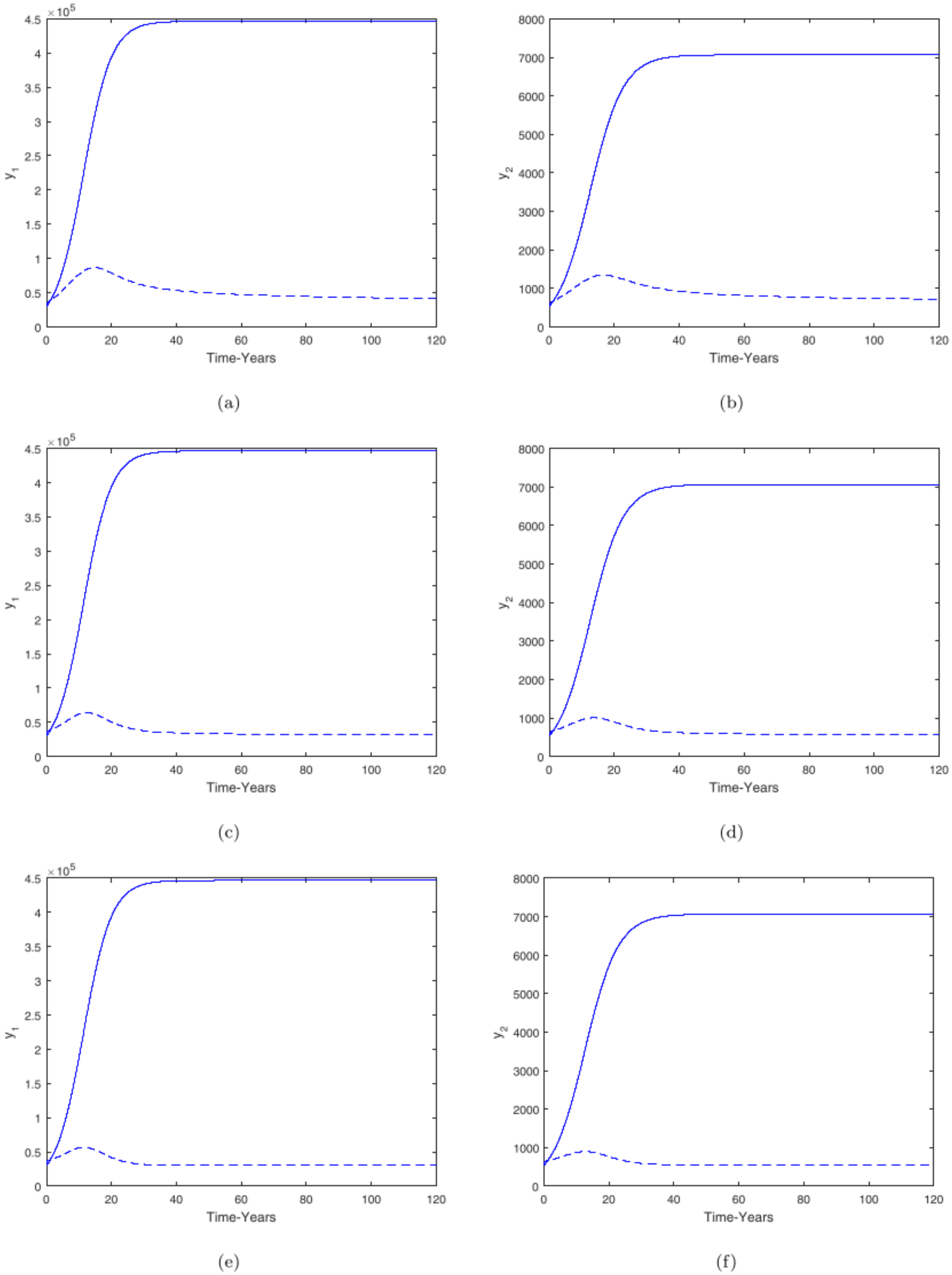
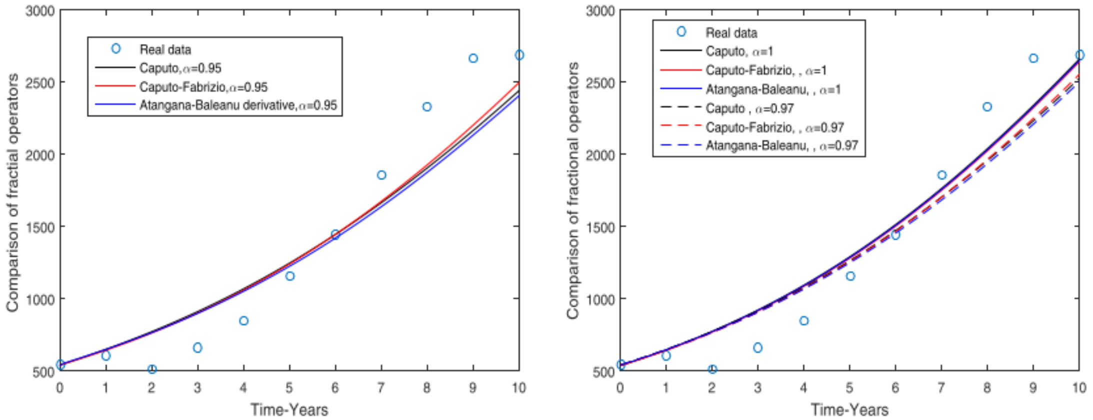


Fig. 9. Simulation of AB model for $\alpha = 0.5, 0.3, 0.1$, subfigure (a) and (b), $\alpha = 0.5$, subfigure (c) and (d), $\alpha = 0.3$, and subfigure (e) and (f), $\alpha = 0.1$.



(a) (b)

Fig. 10. Comparison of Caputo, Caputo–Fabrizio and Atangana–Baleanu operators versus real data, for (a) $\alpha = 1$ and 0.95, whereas for (b) $\alpha = 1$ and 0.97.

of integration and have the following:

$$y_1(t) - y_1(0) = \frac{1-\alpha}{\mathcal{P}(\alpha)}\mathcal{F}_1(t, y_1) + \frac{p}{\mathcal{P}(\alpha)}\int_0^t \mathcal{F}_1(\psi, y_1)d\psi. \quad (19)$$

For $t = t_{n+1}$, where $n = 0, 1, 2, \ldots,$ we obtain

$$y_1(t_{n+1}) - y_{10} = \frac{1-\alpha}{\mathcal{P}(\alpha)}\mathcal{F}_1(t_n, y_{1n}) + \frac{\alpha}{\mathcal{P}(\alpha)}\int_0^{t_{n+1}} \mathcal{F}_1(t, y_1)dt. \quad (20)$$

Further, we obtain the following equation,

$$y_{1n+1} - y_{1n} = \frac{1-\alpha}{\mathcal{P}(\alpha)}\{\mathcal{F}_1(t_n, y_{1n}) - \mathcal{F}_1(t_{n-1}, y_{1n-1})\} + \frac{\alpha}{\mathcal{P}(\alpha)}\int_{t_n}^{t_{n+1}} \mathcal{F}_1(t, y_1)dt. \quad (21)$$

We approximate the function $\mathcal{F}_1(t, y_1)$ by the interpolation polynomial in $[t_k, t_{(k+1)}]$, and have

$$P_k(t) \cong \frac{\mathcal{F}(t_k, y_k)}{h}(t - t_{k-1}) - \frac{\mathcal{F}(t_{k-1}, y_{k-1})}{h}(t - t_k), \quad (22)$$

where $h = t_n - t_{n-1}$. After simplification the equation (21), we have the following,

$$\begin{aligned}\int_{t_n}^{t_{n+1}} \mathcal{F}_1(t, y_1)dt &= \int_{t_n}^{t_{n+1}} \frac{\mathcal{F}_1(t_n, y_{1n})}{h}(t - t_{n-1}) - \frac{\mathcal{F}_1(t_{n-1}, y_{1n-1})}{h}(t - t_n)dt \\ &= \frac{3h}{2}\mathcal{F}_1(t_n, y_{1n}) - \frac{h}{2}\mathcal{F}_1(t_{n-1}, y_{1n-1}). \quad (23)\end{aligned}$$

Using the Eq. (23) in (21), and after some computation leads to the following form,

$$y_{1n+1} = y_{1n} + \left(\frac{1-\alpha}{\mathcal{P}(\alpha)} + \frac{3h}{2\mathcal{P}(\alpha)}\right)\mathcal{F}_1(t_n, y_{1n}) - \left(\frac{1-\alpha}{\mathcal{P}(\alpha)} + \frac{\alpha h}{2\mathcal{P}(\alpha)}\right)\mathcal{F}_1(t_{n-1}, y_{1n-1}). \quad (24)$$

The procedure described above is used in the same manner for the rest of the equation of the fractional model (17), we have:

$$y_{2n+1} = y_{20} + \left(\frac{1-\alpha}{\mathcal{P}(\alpha)} + \frac{3h}{2\mathcal{P}(\alpha)}\right)\mathcal{F}_2(t_n, y_{2n}) - \left(\frac{1-\alpha}{\mathcal{P}(\alpha)} + \frac{\alpha h}{2\mathcal{P}(\alpha)}\right)\mathcal{F}_2(t_{n-1}, y_{2n-1})). \quad (25)$$

The above scheme presented for the solution of the Caputo–Fabrizio model (17) is used to obtain the numerical solution by considering various fractional order values of the parameter α and obtain Figs. 4–6. In Fig. 4 we present a comparison of the real data versus fractional model for the fractional order $\alpha = 1, 0.98, 0.6,$ and we can see that the fitting with Caputo-Fabrizio operator looks good and can be used it for model fitting versus real data. Further, in 23 (e) the future dynamics of the rural bank data give a reasonable fitting to the real data for long time level. The behavior of the system for long time level is presented in 5 and 6 with their subgraphs for the $\alpha = 1, 0.9, 0.7, 0.5, 0.3, 0.1$. For every value of the fractional order α, the CF model (17) provides graphical illustration which is reasonable.

4.5. Atangana–Baleanu fractional model

The Atangana–Baleanu fractional model can be obtained by apply the definition of Atangana–Baleanu derivative and the model (14) leads to the following system:

$$\begin{aligned}{}^{AB}_{0}D^{\alpha}_{t} y_1 &= r_1 y_1\left(1 - \frac{y_1}{P_1}\right) - \beta_1 y_1 y_2, \\ {}^{AB}_{0}D^{\alpha}_{t} y_2 &= r_2 y_2\left(1 - \frac{y_2}{P_2}\right) - \beta_2 y_1 y_2, \quad (26)\end{aligned}$$

and the Atangana–Baleanu model in fractional parameter can be written as

$$\begin{aligned}{}^{AB}_{0}D^{\alpha}_{t} y_1 &= r_1^{\alpha} y_1\left(1 - \frac{y_1}{P_1^{\alpha}}\right) - \beta_1 y_1 y_2, \\ {}^{AB}_{0}D^{\alpha}_{t} y_2 &= r_2^{\alpha} y_2\left(1 - \frac{y_2}{P_2^{\alpha}}\right) - \beta_2 y_1 y_2. \quad (27)\end{aligned}$$

We give in the following subsection the numerical solution of the two systems (26) and (27) wit a novel numerical approach and their comparison with the real data.

4.6. Numerical solution with Atangana-Baleanu derivative

The present section investigates the numerical solution of the model (26) and using the parameters values described in Table 1. Initially, we give a brief details of the numerical procedure for the solution of fractional model given by (26) by adopting the method described in [21]. The selection of this novel approach for the numerical solution of the proposed model (26) is based on the use

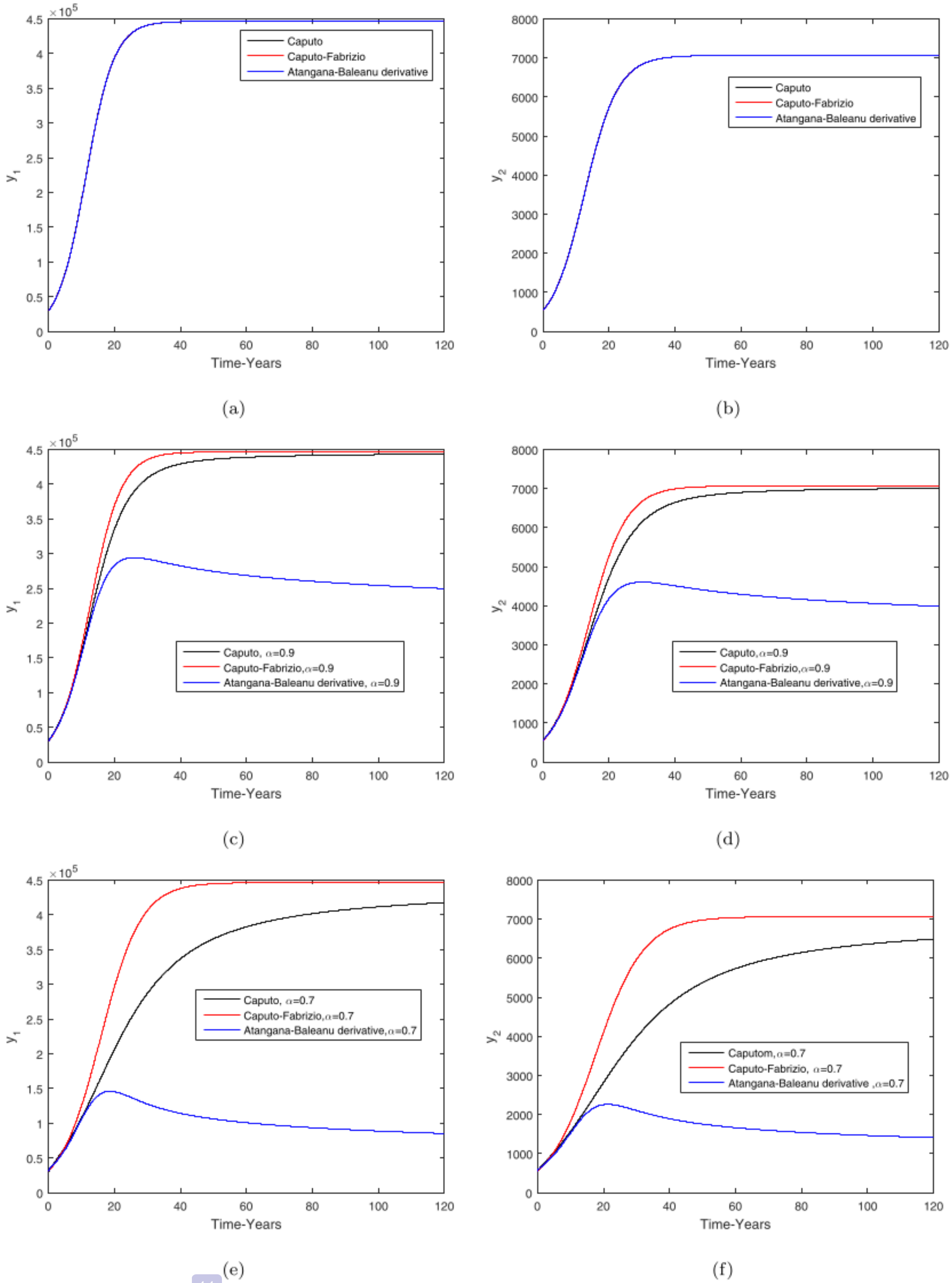


Fig. 11. Comparison of Caputo, Caputo–Fabrizio and Atangana–Baleanu operators for the fractional bank model, when $\alpha = 1, 0.9, 0.7$.

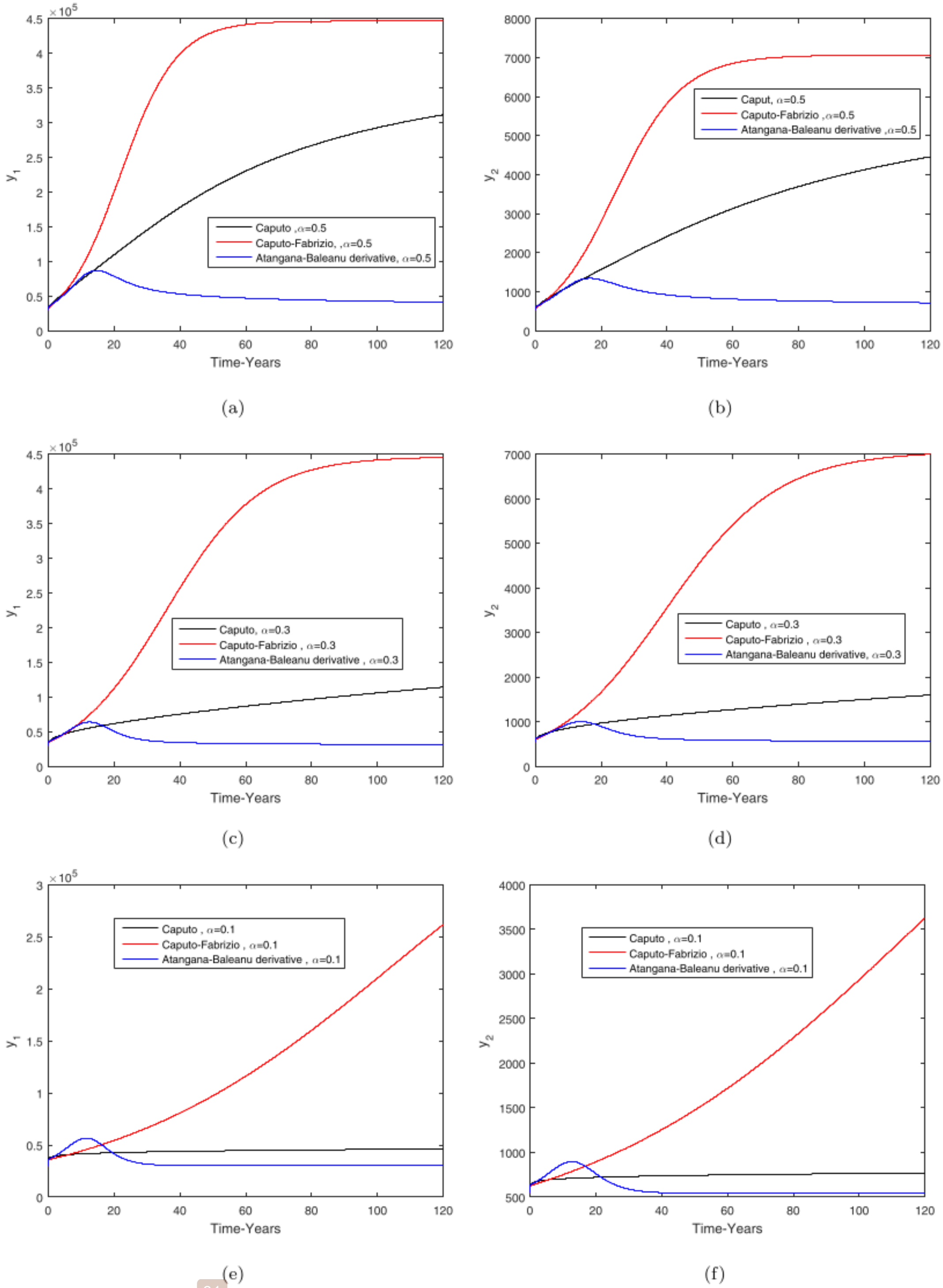


Fig. 12. Comparison of Caputo, Caputo–Fabrizio and Atangana–Baleanu operators for the fractional bank model, when $\alpha = 0.5, 0.3, 0.1$.

of the researchers for many mathematical models of real life phenomenon which can be seen in [35–39] also the reference therein. Therefore, the following procedure is adopted to obtain the numerical solution of the model (26)

Initially, we convert the system given by (26) into the following form by utilizing the fundamental theorem of fractional calculus:

$$f(t)-f(0)=\frac{(1-\alpha)}{ABC(\alpha)}\mathcal{P}(t,f(t))+\frac{\alpha}{ABC(\alpha)\times\Gamma(\alpha)}\int_0^t \mathcal{P}(\psi,y(\psi))(t-\psi)^{\alpha-1}d\psi. \quad (28)$$

For $t=t_{n+1}$, $n=0,1,2,\ldots,$ we have the following,

$$\begin{aligned} f(t_{n+1})-f(0) &= \frac{1-\alpha}{ABC(\alpha)}\mathcal{P}(t_n,f(t_n)) \\ &\quad \frac{\alpha}{ABC(\alpha)\times\Gamma(\alpha)}\int_0^{t_{n+1}}\mathcal{P}(\psi,f(\psi))(t_{n+1}-\psi)^{\alpha-1}d\psi, \\ &= \frac{1-\alpha}{ABC(\alpha)}\mathcal{P}(t_n,f(t_n)) \\ &\quad +\frac{\alpha}{ABC(\alpha)\times\Gamma(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\mathcal{P}(\psi,f(\psi))(t_{n+1}-\psi)^{\alpha-1}d\psi. \quad (29)\end{aligned}$$

The function $\mathcal{P}(\psi,f(\psi))$ is then approximated over the interval $[t_j,t_{j+1}]$, with the interpolation polynomial, and we get the following,

$$\mathcal{P}(\psi,f(\psi))\cong\frac{\mathcal{P}(t_j,f(t_j))}{h}(t-t_{j-1})-\frac{\mathcal{P}(t_{j-1},f(t_{j-1}))}{h}(t-t_j). \quad (30)$$

We put the above into Eq. (29), and then get the following,

$$\begin{aligned} f(t_{n+1}) &= f(0)+\frac{1-\alpha}{ABC(\alpha)}\mathcal{P}(t_n,f(t_n)) \\ &\quad +\frac{\alpha}{ABC(\alpha)\times\Gamma(\alpha)}\sum_{j=0}^{n}\left(\frac{\mathcal{P}(t_j,f(t_j))}{h}\int_{t_j}^{t_{j+1}}(t-t_{j-1})(t_{n+1}-t)^{\alpha-1}dt\right. \\ &\quad \left.-\frac{\mathcal{P}(t_{j-1},f(t_{j-1}))}{h}\int_{t_j}^{t_{j+1}}(t-t_j)(t_{n+1}-t)^{\alpha-1}dt\right). \quad (31)\end{aligned}$$

After the rigorous computation of the Eq. (31), the following approximate solution is obtained:

$$\begin{aligned} f(t_{n+1}) &= f(t_0)+\frac{1-\alpha}{ABC(\alpha)}\mathcal{P}(t_n,f(t_n)) \\ &\quad +\frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\left(\frac{h^\alpha\mathcal{P}(t_j,f(t_j))}{\Gamma(\alpha+2)}((n+1-j)^\alpha(n-j+2+\alpha)\right. \\ &\quad -(n-j)^\alpha(n-j+2+2\alpha))-\frac{h^\alpha\mathcal{P}(t_{j-1},f(t_{j-1}))}{\Gamma(\alpha+2)} \\ &\quad \left.((n+1-j)^{\alpha+1}-(n-j)^\alpha(n-j+1+\alpha))\right). \quad (32)\end{aligned}$$

The above procedure we then applied to our fractional model described in (26), with the following setting,

$$\begin{cases} U_1=\frac{1-\alpha}{ABC(\alpha)}, \\ U_2=((n+1-j)^\alpha(n-j+2+\alpha)-(n-j)^\alpha(n-j+2+2\alpha)), \\ U_3=((n+1-j)^{\alpha+1}-(n-j)^\alpha(n-j+1+\alpha)), \\ U_4=\frac{\alpha}{ABC(\alpha)}, \end{cases} \quad (33)$$

and have,

$$\begin{aligned} y_1(t_{n+1}) &= y_1(t_0)+U_1\mathcal{F}_1(t_n,f(t_n)) \\ &\quad +U_4\sum_{j=0}^{n}\left(\frac{h^\alpha\mathcal{F}_1(t_j,f(t_j))}{\Gamma(\alpha+2)}U_2-\frac{h^\alpha\mathcal{F}_1(t_{j-1},f(t_{j-1}))}{\Gamma(\alpha+2)}U_3\right), \\ y_2(t_{n+1}) &= y_2(t_0)+U_1\mathcal{P}_2(t_n,f(t_n)) \\ &\quad +U_4\sum_{j=0}^{n}\left(\frac{h^\alpha\mathcal{P}_2(t_j,f(t_j))}{\Gamma(\alpha+2)}U_2-\frac{h^\alpha\mathcal{P}_2(t_{j-1},g(t_{j-1}))}{\Gamma(\alpha+2)}U_3\right). \end{aligned} \quad (34)$$

Using the above scheme (34), we obtain the graphical representation shown in Figs. 7–9 for the competition fractional model of bank for the rural and commercial described by the system (26). In Figure (7) and their subgraphs (a-e) we describe briefly the simulation results of the fractional model versus the real data of rural banks by considering different fractional order of α, which indicate that the Atangana–Baleanu fractional operator is used for data setting effectively. In Figs. 8 and 9, and their subgraphs shows the dynamics of the rural and commercial banks described through the system (26) for the values of $\alpha=1, 0.9, 0.7, 0.5, 0.3, 0.1$, which shows that the Atangana–Baleanu derivative gives very flexible and interested graphical results which seems better than that of the Caputo and Caputo–Fabrizio operator. For each Figure in 7–9, we have provided a brief details in figure caption. The description of future prediction of bank data (rural) versus time is plotted in subfigure 7 (e) which shows accurate fitting for long time.

5. Comparison of fractional operators

This section investigates the fractional operators, Caputo, Caputo–Fabrizio and Atangana–Baleanu operators for the numerical solution of the competition model fractional model. We use the parameters given in Table 1 for the numerical results given in Figs. 10 and 12. In Fig. 10 we give a comparison of the real data versus three different fractional operators that is the Caputo, Caputo–Fabrizio and the Atangana–Balenau operator for $\alpha=1, 0.97$ and we can see that their is close similarities between these operators versus real data and it can be seen that these operators are useful to use for the real data modeling. We use the fractional order parameter $\alpha=1, 0.9, 0.7, 0.5, 0.3, 0.1$ and present the graphical results in Figs. 11 and 12. One can see in these graphical results that the results of Atangana–Baleanu fractional operators are so much good when decreasing the fractional order parameter α.

6. Conclusion

We presented the dynamics of the bank data through a competition model with real data. Initially, we formulated the bank model in an integer case and then using the fractional operators such as the Caputo, Caputo–Fabrizio and the Atangana–Baleanu and applied on the integer order system one by one. For each fractional system we give a brief details for the numerical simulation and their data fitting by considering different values of the fractional order parameter. The real data for fractional model versus integer order model are compared and in general all the operators have closed resemblance and any can see that these operators are useful for data fitting of real life situation. But in Caputo case the data fitting for the fractional case some deviation occurred but in general it is not the case. Further, we presented some comparison results of all the three fractional operators versus real data for the case of $\alpha=1, 0.97$ and there we observed some closed resemblance of the operators. Also, for long time behavior all the three fractional operators are compared and some useful results were obtained for various values of the fractional order parameters α. In all these results one can see that the results are in generally of all these operators are good and flexible but the results of Atangana–Baleanu are more and good and provide good graphical behaviors. Thus in all the operators in this study the results of the fractional Atangana–Baleanu operator is excellent. In future this wok can be extended to fractal-fractional derivative with Caputo, fractal-fractional Caputo–Fabrizio and fractal-fractional Atangana–Baleanu derivative, where these new approaches can be tested for the proposed study.

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