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ORIGINAL ARTICLE

A new numerical solution of the competition model among bank data in Caputo-Fabrizio derivative

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Abstract A new numerical scheme for the Caputo-Fabrizio operator is proposed. We initially present a bank model with real data and then present the model in the fractional Caputo-Fabrizio derivative. We estimate and fit the model parameter using the least square curve fitting. The Caputo-Fabrizio model is solved numerically by using three steps Adams-Bashforth method. The proposed scheme is used to obtain graphical results for bank data of rural and commercial. The real data of rural and commercial banks are used to fit with Caputo-Fabrizio model. We show that the Caputo-Fabrizio model show good fitting for the fractional order parameters versus the real data of rural and commercial bank. Further, we show graphical illustration for some values of the fractional order in order to show the effectiveness of the proposed new numerical scheme.

1. Introduction

Mathematical models are considered useful to understand better the dynamics of a real word problem. The mathematical model is not only described the dynamics of the problem but also provide some important future predictions associated. Mathematical models associated to science, social science and engineering attracted many researchers around the globe. The mathematical models associated to social science filed are also getting attentions from researchers. Understanding the dynamics of banking and financial data and its future perspective would be more prominent through a mathematical models. The mathematical models associated to banking finance has vital role to better understand effectively the present and future dynamics. The banking data about the collections and their investment on the citizens are important for their society and economic growth [1]. Rural and commercials banks are the important banking sectors in Indonesia where it is observed that financial affectivities of rural and less than that of commercial activities although with the same products [2–4].

For the economic growth of the country the finance and banking has a vital role. Banking systems with rural and commercial type are the important to do business activities in Indonesia. With the same business products amnion rural and commercial banks in Indonesia, there may possibility of competition among them. The competition among two things, species can be handled effectively by a system of Lotka Volterra system. The importance of Lotka Volterra system and

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its application to practical financial and others related problems are briefly discussed in [5–12].

Now-a-days, fractional orders models getting too much attentions from researchers from different fields of science, social science and engineering. Due to the wide applications of the fractional derivatives, many authors formulated problems in different fields, see for example [14–19]. It documented that the fractional differential equations associated to a practical problem with real data statistics has best parameters and fittings, for example see [20–23]. Regarding some more results about fractional operators and their applications in real world problems are studied in [24–27]. A real statistical data were used to obtain the TB disease analysis through fractional derivative in [24]. The application of the Atangana-Baleanu derivative to Belousov-Zhabotinskii reaction systems is considered in [25]. The dynamics of computer virus in fractional derivative and the fractional calculus with power law is studied by the authors in [26,27]. Some recent and related fractional papers where the authors studied the numerical approximations to the fractional operators, see [28], solution of reaction diffusion equations [29], time fractional diffusion equations [30], applications chaos [31], related to natural phenomenon [32], solution to time MKDV fractional equations [33], fractional Fisher's type equations [34], new derivative with normal distribution kernel [35], application to chemical equations [36], to measles epidemic [37]. In all these papers, fractional operators were chosen differently, such as Caputo, Caputo-Fabrizio and the Atangana-Baleanu. The fractional derivatives have the advantages to capture the dynamics of the problem with different order, but in the case of integer order we cannot see this property. Also, the fractional calculus have been found interesting for data fitting, as we discussed the related references above.

The purpose of this work is to analyze the novel numerical solution for the Caputo-Fabrizio derivative and its application to the real data of banking finance in Indonesia for 2004–2014 [38]. Initially, we taking a competition model and obtained its parameters through estimations techniques of least curve fitting and then formulate the model in fractional derivative of the type of Caputo-Fabrizio. The Caputo-Fabrizio model is then solved numerically by using the three steps Adams-Bashforth rule and provide data fitting results for arbitrary value of fractional order parameter. The section-wise information related to the paper as is follows: The basics related to Caputo-Fabrizio definitions are shown in Section 2. Formulation of competition systems in integer older and fractional order are studied in Section 3. Numerical solution of the fractional model with three steps rule is carried out in Section 4. The results are briefly discussed and summarized respectively in Section 5 and 6.

2. Basics related to Caputo-Fabrizio derivative

We present here the related concept of fractional operator Caputo-Fabrizio in the following.

Definition 1. A $v(t) \in H^1(q_1, q_2)$ and $0 < \tau < 1$. Then, it follows from [39] the definition of Caputo-Fabrizio (CF) operator is,

$$ {}^{CF}D_t^{\tau}(v(t)) = \frac{M(\tau)}{1-\tau} \int_a^t v'(y) \exp\left(\frac{-\tau(t-y)}{1-\tau}\right) dy, \tag{1} $$

where $M(0) = M(1) = 1$ and $M(\tau)$ denote the normalization function. If $v(t)$ does not belong to $H^1(q_1, q_2)$, then another shape of the derivative is,

$$ {}^{CF}D_t^{\tau}(v(t)) = \frac{\tau M(\tau)}{1-\tau} \int_a^t (v(t) - v(y)) \exp\left(\frac{-\tau(t-y)}{1-\tau}\right) dy. \tag{2} $$

Remark 1. Choosing $\beta = \frac{1-\tau}{\tau} \in (0, \infty)$, then $\tau = \frac{1}{1+\beta}$, and so the Eq. (2) becomes,

$$ {}^{CF}D_t^{\tau}(v(t)) = \frac{M(\beta)}{\beta} \int_a^t (v'(y)) \exp\left(\frac{-(t-y)}{\beta}\right) dy, \tag{3} $$

where $M(0) = M(\infty) = 1$,

Remark 2. We present the following property,

$$ \lim_{\beta \to 0} \frac{1}{\beta} \exp\left(\frac{-(t-y)}{\beta}\right) = \mu(y-t), $$

where $\mu(\mathrm{y}-\mathrm{t})$ represent the Dirac delta function. (4)

Definition 2. [40] Later on Losada and Nieto modified this derivative and presented the following,

$$ {}^{CF}D_t^{\tau}(v(t)) = \frac{(2-\tau)M(\tau)}{2(1-\tau)} \int_a^t v'(y) \exp\left(\frac{-\tau(t-y)}{1-\tau}\right) dy, \tag{5} $$

and their integral is given by

$$ {}^{CF}I_t^{\tau}(v(t)) = \frac{2(1-\tau)}{(2-\tau)M(\tau)} v(t) + \frac{2\tau}{(2-\tau)M(\tau)} \int_a^t v(y) dy, \mathrm{t} \geqslant 0. \tag{6} $$

3. Mathematical model

We taking a competition model of Lotka-Volterra system that usually formulated for the competition among two specials for which the compete. Competition among species for food or other financial bodies are important for their servile. Nations around the world can be developed and developing due their competition. For a strong economic growth the banks have important values in their growth. The banks may be rural or commercial or any other type. Therefore, in the present section, we proposed a model of Lotka-Volterra to study the competition among rural and commercial banks in Indonesia by giving the following system of differential equations:

$$ \begin{aligned} \frac{du_1}{dt} &= \theta_1 u_1 \left(1 - \frac{u_1}{K_1}\right) - \psi_1 u_1 u_2, \\ \frac{du_2}{dt} &= \theta_2 u_2 \left(1 - \frac{u_2}{K_2}\right) - \psi_2 u_1 u_2, \end{aligned} \tag{7} $$

with initial conditions $u_1(0) = u_{10}$ and $u_2(0) = u_{20}$. The model (7) described the competition among rural and commercial banks, where the dynamics of the commercial banks at any time of t is given by $u_1(t)$ while the dynamics of rural banks is given by $u_2(t)$. The growth rate of the commercial bank is given by θ_1 while the rural bank growth by θ_2. The parameter K_1 and K_2 respectively show the maximum profit gained by commercial and rural banks. The parameters ψ_1 represent the coefficient for commercial while ψ_2 is for rural bank. It is clear that the parameters is θ_i, K_i and ψ_i for $i = 1, 2$ are positive.

3.1. Stability of the equilibrium points of the model

We obtain the equilibrium points for the model given in (7), by solving the equations below in the absence of the rate of change,

$$\frac{du_1}{dt} = 0, \quad \frac{du_2}{dt} = 0, \tag{8}$$

which leads to the following equations,

$$\begin{aligned} &\theta_1 u_1\left(1 - \tfrac{u_1}{K_1}\right) - \psi_1 u_1 u_2 = 0, \\ &\theta_2 u_2\left(1 - \tfrac{u_2}{K_2}\right) - \psi_2 u_1 u_2 = 0. \end{aligned} \tag{9}$$

The following equilibrium points exists for our model (13) and (7),

$$\begin{aligned} &Z_0 = (0,0), \quad Z_1 = (0, K_2), \quad Z_2 = (K_1, 0), \\ &Z_3 = \left(\tfrac{K_1\theta_2(\psi_1 K_2 - \theta_1)}{\psi_1\psi_2 K_1 K_2 - \theta_1\theta_2}, \tfrac{K_2\theta_1(\psi_2 K_1 - \theta_2)}{\psi_1\psi_2 K_1 K_2 - \theta_1\theta_2}\right), \end{aligned}$$

among these fixed points, we get $\psi_1 K_2 - \theta_1 > 0, \psi_2 K_1 - \theta_2 > 0$ and $\psi_1\psi_2 K_1 K_2 - \theta_1\theta_2 > 0$. If the solutions lines at equilibrium point Z_3 of the model lies in the first quadrant and interest each other, then the model will have positive equilibrium point. We show the stability of the model (7) at these points below: The Jacobian matrix in of the system is given by:

$$J = \begin{pmatrix} \left(1 - \frac{u_1}{K_1}\right)\theta_1 - \frac{u_1\theta_1}{K_1} - u_2\psi_1 & -u_1\psi_1 \\ -u_2\psi_2 & \left(1 - \frac{u_2}{K_2}\right)\theta_2 - \frac{u_2\theta_2}{K_2} - \psi_2 u_1 \end{pmatrix}. \tag{10}$$

The stability at the point $Z_0 = (0,0)$, we obtain two positive eigenvalues from the Jacobian matrix J, that is, θ_1, θ_2, which shows instability at Z_0. At the equilibrium point $Z_1 = (0, K_2)$, we get the eigenvalues $-\theta_2, \theta_1 - K_2\psi_1$. The equilibrium point would be stable at Z_1 if $\theta_1 < K_2\psi_1$. At the equilibrium point Z_2, we get the eigenvalues $-\theta_1 < 0, \theta_2 - K_1\psi_2$. If $\theta_2 < K_1\psi_2$, then the equilibrium point Z_2 is stable. For the equilibrium point Z_3, we get the following characteristics equation:

$$\lambda^2 + a_1\lambda + a_2 = 0, \tag{11}$$

where

$$\begin{aligned} a_1 &= \tfrac{\theta_1\theta_2(\theta_1 + \theta_2 - K_2\psi_1 - K_1\psi_2)}{\theta_1\theta_2 - K_1 K_2\psi_1\psi_2}, \\ a_2 &= \tfrac{\theta_1\theta_2(\theta_1 - K_2\psi_1)(\theta_2 - K_1\psi_2)}{\theta_1\theta_2 - K_1 K_2\psi_1\psi_2}. \end{aligned} \tag{12}$$

The coefficients a_1 and a_2 can be positive if $\theta_2 > K_1\psi_2, \theta_1 > K_2\psi_1$ and $\theta_1\theta_2 > K_1K_2\psi_1\psi_2$, then the equilibrium point Z_3 becomes locally asymptotically stable.

3.2. A Caputo-Fabrizio fractional model

We apply the definition of the Caputo-Fabrizio derivative on the model (7) that leads to the following shape:

$$\begin{aligned} &{}^{CF}D_t^{\tau}(u_1(t)) = \theta_1 u_1\left(1 - \tfrac{u_1}{K_1}\right) - \psi_1 u_1 u_2, \\ &{}^{CF}D_t^{\tau}(u_2(t)) = \theta_2 u_2\left(1 - \tfrac{u_2}{K_2}\right) - \psi_2 u_1 u_2, \end{aligned} \tag{13}$$

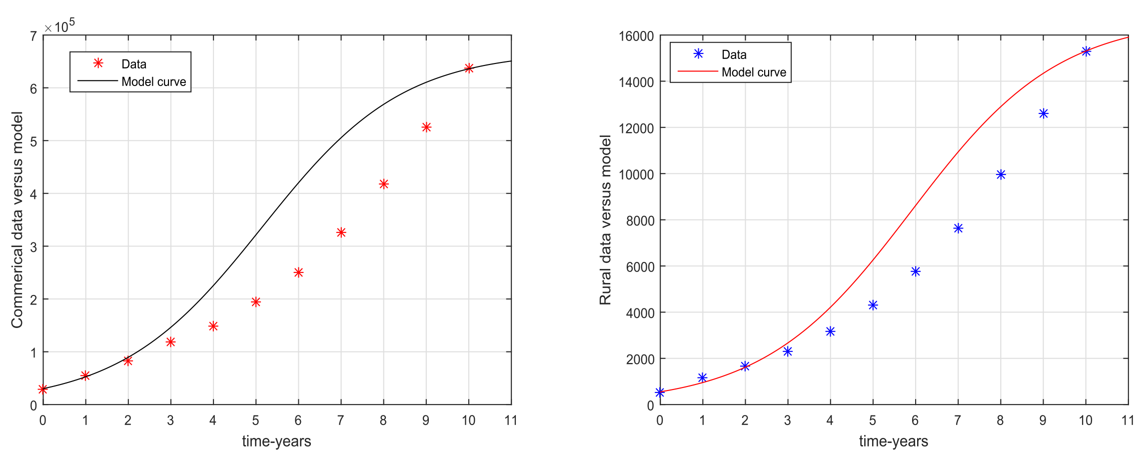
with initial conditions $u_1(0) = u_{10}$ and $u_2(0) = u_{20}$, and τ denotes the fractional order of Caputo-Fabrizio operator.

3.3. Estimations of parameters

The parameters estimated for the banking data of rural and commercial banks since 2004–2014 for the fractional model (13) at $\tau = 1$. We use the least square method to obtain best fitting to our data. For the best parameters values and their fitting, we present the following and objective functions,

$$\oplus = argmin \sum_{j=1}^{n} \left[u_{tj} - \overline{u_{tj}}\right]^2,$$

where $\overline{u_{tj}}$ determines the actual banking data and u_{tj} explains the solution associated to the model at t_j, the actual data points is measured by n. Utilizing this method, we present the parameters estimations for our considered model (13) are $\psi_1 = 3.90 \times 10^{-10}, \psi_2 = 3.9 \times 10^{-8}, K_1 = 669318.198$, $K_2 = 17540.6219, \theta_1 = 0.6$ and $\theta_2 = 0.58$ and depict the fitting results are given in Fig. 1. These estimated parameters will be



(a) (b)

Fig. 1 Model versus data, (a) commercial data, (b) rural data.

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used further to obtain graphical results for the model using the numerical scheme of Adams-Bashforth three steps described below.

4. Numerical solution using Three-step Adams-Bashforth scheme

The numerical solution of a fractional nonlinear differential equations are difficult to solve exactly, therefore, the researcher from time to developed and developing some novel numerical methods for their solution. The numerical methods to handle problems of fractional differential equations have been developed to obtain the approximate solution. Usually these methods are based on the discretization. In these methods, we include here Adams-Bashforth-Moulton type predictor-corrector methods, finite difference methods, and finite element methods [41–44]. In the present work, we utilize and follow the method explained in [45] for our proposed problem (13). This methods is based on the rule of Adams–Bashforth with three steps.

Consider a general fractional differential equations in Caputo-Fabrizio derivative,

$$ {}^{CF}D_t^{\tau}(v(t)) = g(t, v(t)), \quad 0 < \tau < 1, \tag{14} $$

where ${}^{CF}D_t^{\tau}(.)$, represents the CF derivative defined in Eq. (1). We apply the fractional integral shown below:

$$ {}^{CF}I_t^{\tau}(g(t)) = \frac{1-\tau}{M(\tau)}g(t) + \frac{\tau}{M(\tau)}\int_0^t g(x)dx, \tag{15} $$

to both sides of the Eq. (14), we get

$$ \begin{aligned} &{}^{CF}I_t^{\tau\,CF}D_t^{\tau}(v(t)) = {}^{CF}I_t^{\tau}(g(t, v(t))), \\ &v(t) - v(0) = {}^{CF}I_t^{\tau}(g(t, v(t))), \\ &\quad = \tfrac{1-\tau}{M(\tau)}g(t, v(t)) + \tfrac{\tau}{M(\tau)}\int_0^t g(s, v(s))ds. \end{aligned} \tag{16} $$

Now discretizing the interval $[0, t]$ of time with h steps and have the sequence $t_0 = 0, t_{k+1} = t_k + h, k = 0, 1, 2, ..., n-1, t_n = t$. It follows from Eq. (16), we get the recursive formula in the following:

$$ v(t_{k+1}) - v(0) = \frac{1-\tau}{M(\tau)}g(t_k, v(t_k)) + \frac{\tau}{M(\tau)}\int_0^{t_{k+1}} g(t, v(t))dt \tag{17} $$

and

$$ v(t_k) - v(0) = \frac{1-\tau}{M(\tau)}g(t_{k-1}, v(t_{k-1})) + \frac{\tau}{M(\tau)} \times \int_0^{t_k} g(t, v(t))dt. \tag{18} $$

It follows from (17) and (18), we get

$$ v(t_{k+1}) - v(t_k) = \frac{1-\tau}{M(\tau)}[g(t_k, v_k) - g(t_{k-1}, v_{k-1})] + \frac{\tau}{M(\tau)} \times \int_{t_k}^{t_{k+1}} g(t, v(t))dt. \tag{19} $$

Now, we explain and deriving in details the procedure of three-step Adams–Bashforth rule. To do this, we approximate the integral in Eq. (19) given by

$$ \int_{t_k}^{t_{k+1}} g(t, v(t))dt, $$

by the approximation $\int_{t_k}^{t_{k+1}} P_2(t)dt$, where $P_2(t)$ denote the interpolation polynomial associated to Lagrange with degree that passes through the given three points $(t_{k-2}, g(t_{k-2}, v(t_{k-2}))), (t_{k-1}, g(t_{k-1}, v(t_{k-1})))$, and $(t_k, g(t_k, v(t_k)))$. That is,

$$ P_2(t) = \sum_{i=0}^{2} g(t_{k-i}, v_{k-i})L_i(t), \tag{20} $$

where $L_i(t)$ for the three points (t_{k-2}, t_{k-1}, t_k) are the Lagrange basis polynomials. Using the change of variable $s = \frac{t_{k+1}-t}{h}$, and using the Lagrange basis polynomials and their integrating, we have

$$ \begin{aligned} \int_{t_k}^{t_{k+1}} g(t, v(t))ds &= h\int_0^1 \Big[\tfrac{(s-2)(s-3)}{(1-2)(1-3)}g(t_k, v_k) + \tfrac{(s-1)(s-3)}{(2-1)(2-3)}g(t_{k-1}, v_{k-1}) \\ &\quad + \tfrac{(s-2)(s-1)}{(3-2)(3-1)}g(t_{k-2}, v_{k-2})\Big]ds, \\ &= h\big[\tfrac{23}{12}g(t_k, v_k) - \tfrac{4}{3}g(t_{k-1}, v_{k-1}) + \tfrac{5}{12}g(t_{k-2}, v_{k-2})\big], \end{aligned} \tag{21} $$

where $v_{k-2} = v(t_{k-2}), v_{k-1} = v(t_{k-1})$, and $v_k = v(t_k)$. Then, using (21) into (19), we present the following iterative formula:

$$ \begin{aligned} v_{k+1} &= v_k + \tfrac{1}{M(\tau)}\big[(1-\tau) + \tfrac{23}{12}h\tau\big]g(t_k, v_k) \\ &\quad - \tfrac{1}{M(\tau)}\big[(1-\tau) + \tfrac{4}{3}h\tau\big]g(t_{k-1}, v_{k-1}) \\ &\quad + \tfrac{5h\tau}{12M(\tau)}g(t_{k-2}, v_{k-2}). \end{aligned} \tag{22} $$

If we consider $\tau = 1$ in the above expression (22), then, we get a classical Adams–Bashforth three-steps rule. One can obtain the truncation error for the given three steps rule with the Lagrange interpolating polynomial by to estimate the error, say

$$ g(t, v(t))dt = P_2(t) + E_2(t), \tag{23} $$

where

$$ E_2(t) = \frac{g^3(\xi_k, v(\xi_k))}{3!}(t - t_k)(t - t_{k-1})(t - t_{k-2}), \xi_k \in (t_{k-2}, t_k). \tag{24} $$

Then we have

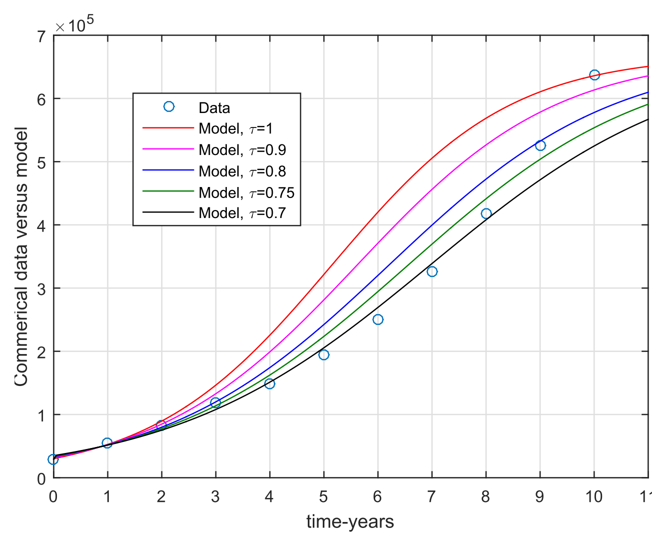


Fig. 2 Bank data of commercial banks versus model at $\tau = 1, 0.9, 0.8, 0.75, 0.7$.

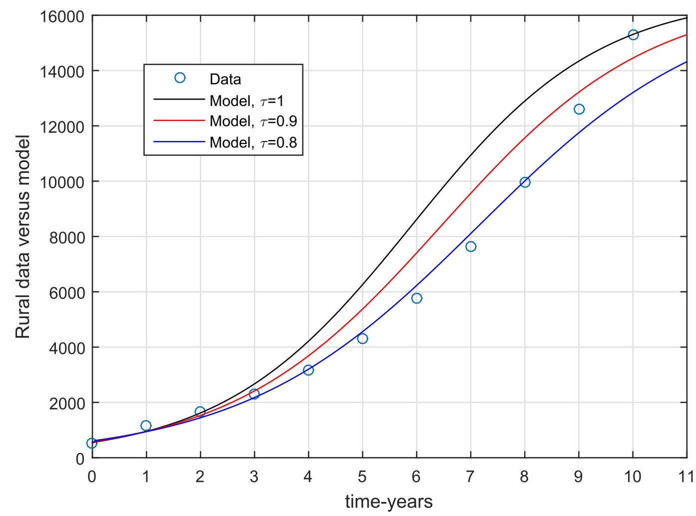
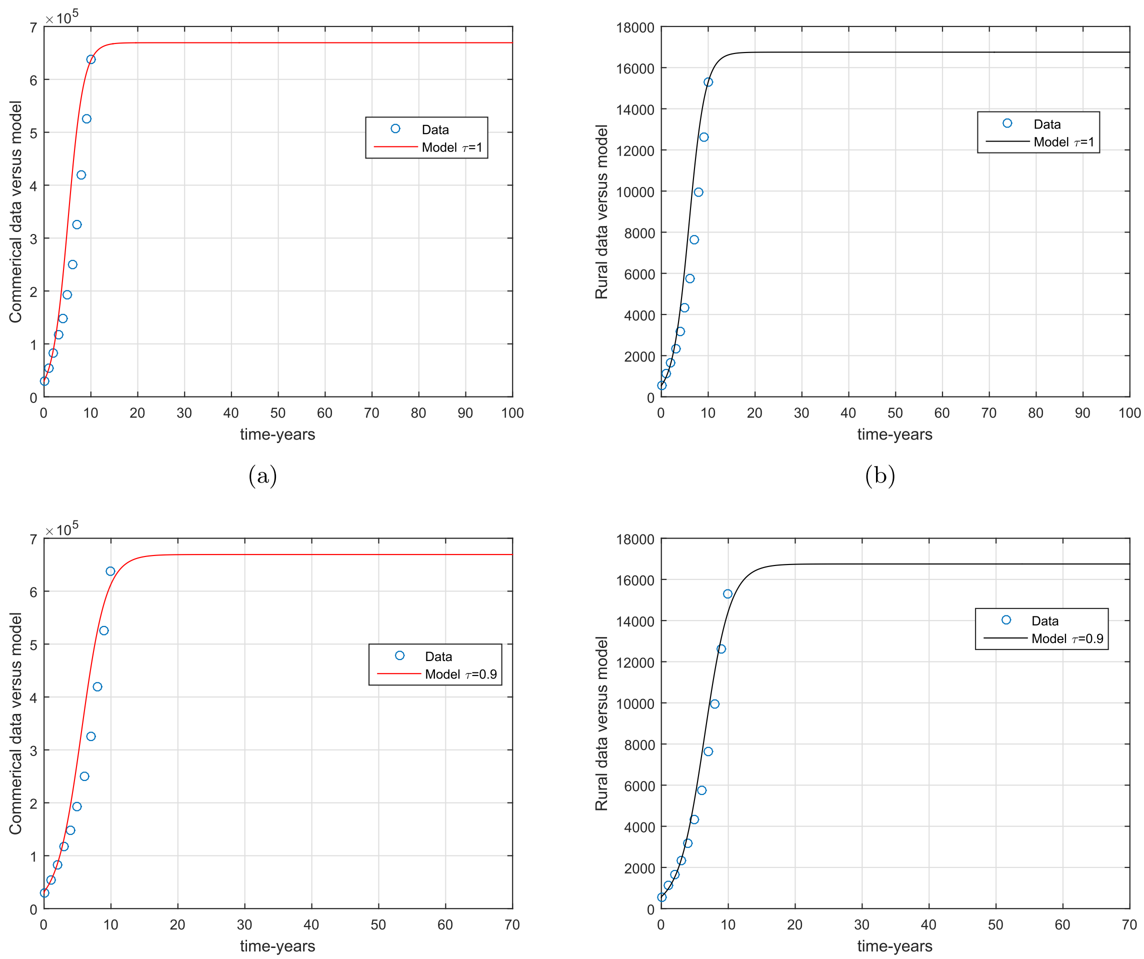


Fig. 3 Bank data of rural banks versus model at $\tau = 1, 0.9, 0.8$.



(a) (b)

(c) (d)

Fig. 4 Model predictions of with data for long time behavior when $\tau = 1, 0.9$.

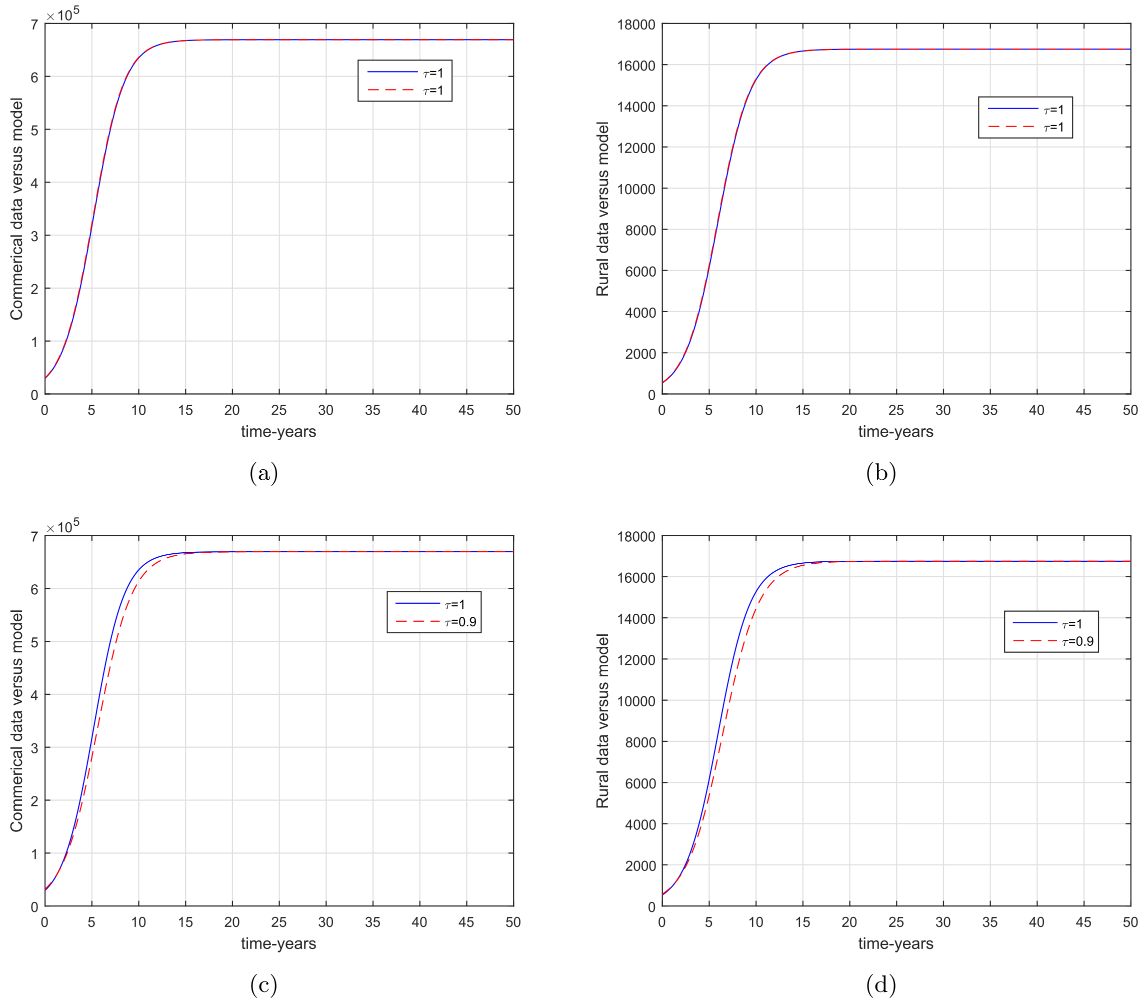


Fig. 5 Numerical solution of the model for $\tau = 1, 0.9$.

$$\begin{aligned}\int_{t_k}^{t_{k+1}} E_2(t)dt &= \int_{t_k}^{t_{k+1}} \tfrac{g^3(\xi_k, v(\xi_k))}{3!}(t-t_k)(t-t_{k-1})(t-t_{k-2})dt,\\ &\approx -\tfrac{h^4 g^3(\mu_k, v(\mu_k))}{6.3!}\int_0^1 (s-1)(s-2)(s-3)ds,\\ &= \tfrac{3}{8}h^4 g^3(\mu_k, v(\mu_k)),\end{aligned} \tag{25}$$

where $\mu_k \in (t_{k-2}, t_{k+1})$, the mean value theorem is used for the approximation of the integral.

If we denote expression on the right side of Eq. (22) by $\hat{v}_k$, then, we have

$$v_{k+1} = \hat{v}_k + \frac{3}{8}h^4 g^{(3)}(\mu_k, v(\mu_k)).$$

The following is the formula to determine the truncation error associated to Eq. (22), given by

$$\begin{aligned}\tfrac{v_{k+1}-\hat{v}_k}{h} &= \tfrac{\tau}{M(\tau)}\cdot\tfrac{\frac{3}{8}h^4 g^{(3)}(\mu_k, v(\mu_k))}{h},\\ &= \tfrac{3}{8M(\tau)}\tau h^3 g^3(\mu_k, v(\mu_k)).\end{aligned} \tag{26}$$

Next, we utilize the fractional three-steps rule of Adams–Bashforth scheme given by (22) to obtain the solution of model (13) numerically. In vector form the system can be written as:

$${}^{CF}D_t^{\tau}(\mathbf{v}(t)) = \mathbf{g}(t, \mathbf{v}(t)), 0 < \tau < 1, \tag{27}$$

where

$$\mathbf{v}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \mathbf{g}(t, \mathbf{v}(t)) = \begin{bmatrix} g_1(t, \mathbf{v}(t)) \\ g_2(t, \mathbf{v}(t)) \end{bmatrix}. \tag{28}$$

The scalar functions g_1 and g_2 shows the right side of the model Eqs. (13) given by

$$\begin{aligned} g_1(t, \mathbf{v}(\mathbf{t})) &= \theta_1 u_1\left(1 - \tfrac{u_1}{K_1}\right) - \psi_1 u_1 u_2,\\ g_2(t, \mathbf{v}(\mathbf{t})) &= \theta_2 u_2\left(1 - \tfrac{u_2}{K_2}\right) - \psi_2 u_1 u_2.\end{aligned}$$

The fractional integral shown in Eq. (15) is applied on both sides of the Eq. (27), we get

$$\begin{aligned}\mathbf{v}(t) - \mathbf{v}(0) &= {}^{CF}I_t^{\tau}(\mathbf{g}(t, \mathbf{v}(t))),\\ &= \tfrac{1-\tau}{M(\tau)}\mathbf{g}(t, v(t)) + \tfrac{\tau}{M(\tau)}\int_0^t \mathbf{g}(s, v(s))ds.\end{aligned} \tag{29}$$

We have the iterative formula by applying the scheme presented in Eq. (23)–(30):

$$\begin{aligned}\mathbf{v}_{k+1} = \mathbf{v}_k &+ \frac{1}{M(\tau)}\left[(1-\tau) + \frac{23}{12}h\tau\right]\mathbf{g}(t_k, \mathbf{v}_k)\\ &- \frac{1}{M(\tau)}\left[(1-\tau) + \frac{4}{3}h\tau\right]\mathbf{g}(t_{k-1}, \mathbf{v}_{k-1})\\ &+ \frac{5h\tau}{12M(\tau)}\mathbf{g}(t_{k-2}, \mathbf{v}_{k-2}),\end{aligned} \tag{30}$$

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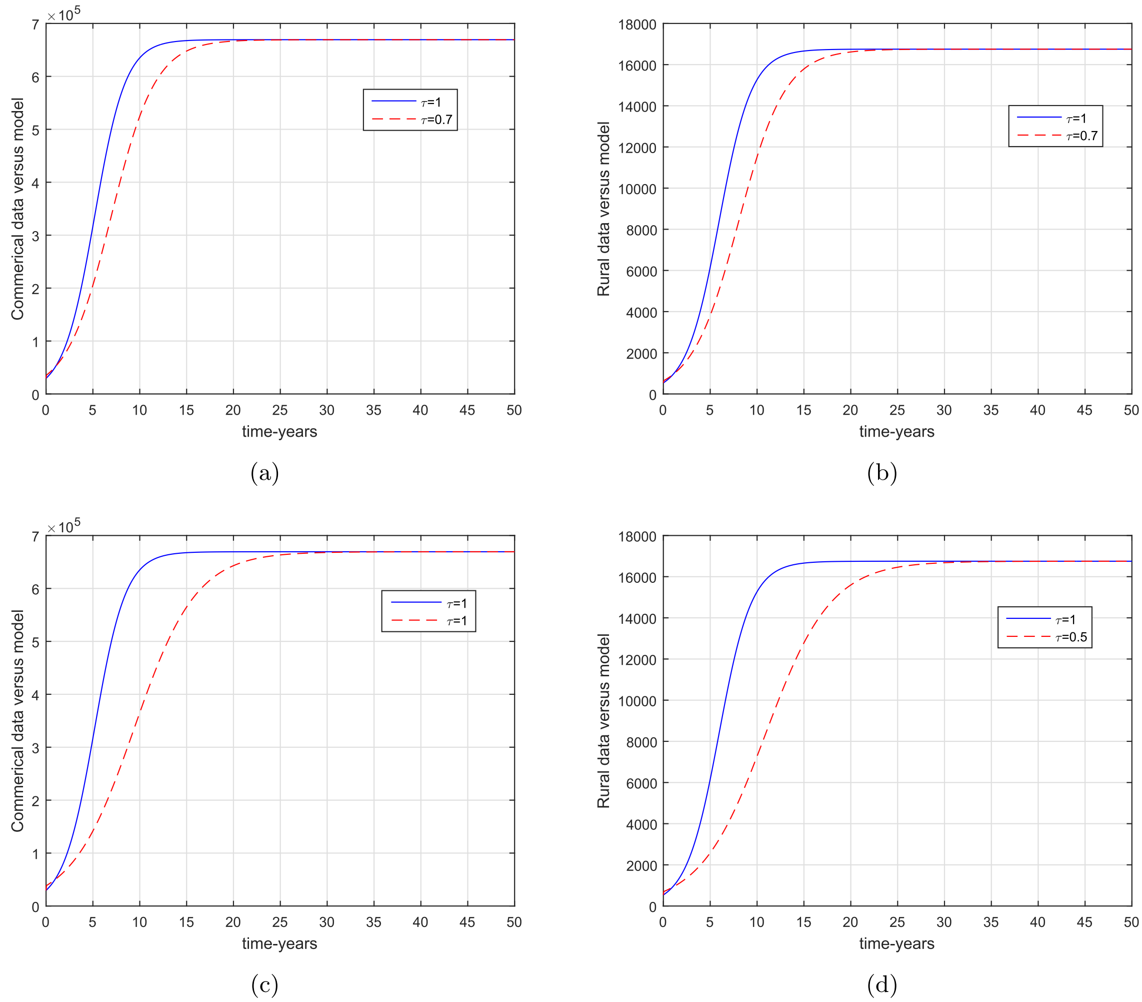

Fig. 6 Numerical solution of the model for $\tau = 0.7, 0.5$.

where $\mathbf{v}_{k-2} = \mathbf{v}(t_{k-2})$, $\mathbf{v}_{k-1} = \mathbf{v}(t_{k-1})$, and $\mathbf{v}_k = \mathbf{v}(t_k)$ and $v_0 = v(t_0) = [u_1(t_0), u_2(t_0)]^T$.

5. Numerical results

We used the numerical scheme for the solution of fractional differential equation model of competition among rural and commercial banks. The estimated set of parameter values for the bank data of Indonesia for the given years 2004–2014 are: $\psi_1 = 3.90 \times 10^{-10}$, $\psi_2 = 3.9 \times 10^{-8}$, $K_1 = 669318.198$, $K_2 = 17540.6219$, $\theta_1 = 0.6$ and $\theta_2 = 0.58$. The time unit of these parameters are per year. Using these parameters we utilized the three steps Adams-Bashforth method above and obtained the graphical results shown in Figs. 2–6. In Fig. 2, we compared the real data of commercial banks with model and obtained good fitting for $\tau = 0.7$. Fig. 3 is obtained for the rural bank when comparing the real data versus model, and showed that the model provide reasonable fitting when $\tau = 0.8$. The long term dynamics of the rural and commercial banks are depicted in Fig. 4, for $\tau = 1, 0.9$. In Fig. 4(a) and (b), we consider the time-level 100 years and the data is fitted accurately but for Fig. 4(c) and (d), the data is accurately fitted for 70 years when taking $\tau = 0.7$, which shows the importance of the fractional derivatives. Further, to show the novelty and importance of this new numerical solution, we depicted graphical results for the given model considering many values of the fractional order τ, see Figs. 5 and 6.

6. Conclusion

The present work explored the numerical solution of fractional differential equation model formulated in Caputo-Fabrizio derivative. Initially, we consider a competition model of bank data in Indonesia of real data since 2004–2014 and obtained its parameters estimations. The estimated parameters are then used to obtain the numerical solution of the model using a novel numerical technique of Adams-Bashforth with three steps. The novel numerical procedure is tested for the bank model with real data for different values of the fractional order parameters τ. We depicted the real data of rural and commercial bank versus model for suggested value of τ. We proved for $\tau = 0.7$, the commercial bank showed good fitting for $\tau = 0.7$ and for the rural bank its provide good fitting when $\tau = 0.8$. Further, we showed more graphical results for the illustration of this novel numerical techniques considering some arbitrary values of τ.

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Declaration of Competing Interest

The authors declare that no conflict of interest exists regarding the publication of this work.

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