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ORIGINAL ARTICLE

Modeling and analysis of competition model of bank data with fractal-fractional Caputo-Fabrizio operator

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KEYWORDS

Fractal-fractional Caputo-Fabrizio model;
Banking data model;
Actual data;
Simulation results

Abstract The present paper consider a newly introduced operator known as fractal-fractional where the fractional operator considered is Caputo-Fabrizio. We consider a competition system and propose the field data of banks for 2004–2014 of Indonesia banks of the type rural and commercial. The model with fractal-fractional operator known as fractal-fractional Caputo-Fabrizio derivative is formulated and show their analysis. We give a novel method to solve the model numerically and present the graphical results. We consider different values of the fractal and fractional order parameters and compare the results with integer order fitting for real data. We show keeping fractal order fix and varying fractional order, keeping fractional order fix and varying fractal, varying both fractional and fractal order for fixed values, varying values arbitrarily, and for long term, we achieve better results for fitting than that of integer order for commercial and rural data. This new definition of fractal-fractional in the form of Caputo-Fabrizio derivative provide better results than that of the ordinary integer order.

1. Introduction

Mathematical modeling considered effective tool to understand the dynamics of real-life problems in science and engineering, see for example [1–3]. It is obvious that mathematical models characterize in the best way the real-world problem [4,5]. Mathematical models are not only used for the science and engineering problems but also gaining much attention in banking and finance. Banking-finance has a vital role in the development of country economy. Banks are defined to be the place for a particular region to collect the amount from their individuals and then spent bank on their people for their betterment that lead to better society [6]. The amount collected from the individuals in a particular region stored in banks is used further to facilitate their peoples in the shape of lending etc. Rural and commercial banks doing such activities of amount collecting and spending on people for betterment of the individuals of a particular area, such banks

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may be Islamic or convectional or both of these [6]. The banks that their activities are on the basis of Syariah law or conventionally are known as the rural banks according to the Act No. 10 of 1998. In Indonesia, it is observed that rural banks have comparatively fewer activities of business than the commercial banks. Commercial banks collecting funds from the people for some fixed time and then credit but the rural banks do no do this [7,8]. The reports of the Indonesian banks statistics show that commercial banks are high in number compared to the rural one and the reason is that the rural have fewer activities than that of the commercial ones. Due to the high number of commercial banks, but their products are the same and still improving their business activities [9]. With the same business activities and products, it is observed that there is no huge difference among these banks products, so in the future, there may be possibilities to have more costumers and possibilities of competition among these.

The above discussion about the rural and commercial banks in Indonesia suggests that among these two banks have no such reasonable difference between there business products, so there may have competition between these two. The competition among banks can be formulated through differential equations system known as Lotka Volterra system. This Lotka Volterra system is the generalization of the logistic equation which consisting of two equations and is useful for competitions between two species [10]. Many researchers around the world used Lotka Volterra system for different practical problems, such as [11–18] and the references therein. For instant, for mobile compony data [11], technological substitution [12], stock market of Korean, modeling and policy implications, dynamics of the markets, and banking data applications [13–16]. A very recent application to bank data in Indonesia is discussed in [17–20].

The above-described works are the application of Lotka Volterra system to different practical problems with integer-order except [17–20]. The fractional-order system is considered more superior than the integer-order, because of their heredity and memory properties, and much more. For integer order system we have just integer order with limited studies while we have a variety of choices for the order of the fractional system. The fractional system may be obtained with different operators, Caputo, Caputo-Fabrizio (CF) and Atangana-Baleanu operators (AB), etc. These operators are proposed by the researchers for variety of problems [21–27]. Formulation of

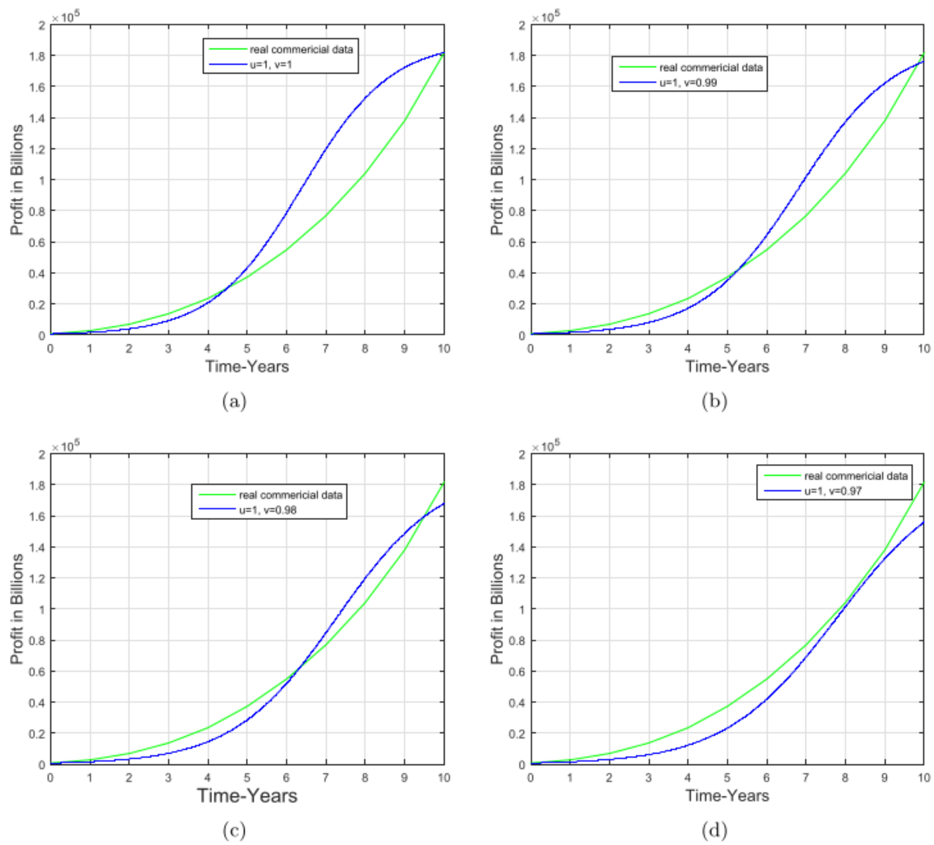

Fig. 1 Model versus commercial data fitting when $u = 1$ fixed: (a) $v = 1$, (b) $v = 0.99$, (c) $v = 0.98$, (d) $v = 0.97$.

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real-life problems and to identify the accurate parameters are useful for future predictions. It is proven by researchers that the fractional-order system gives the best setting for parameter estimations, and one has reliable information for their proposed problems [28–31]. In the past, some researchers have shown the importance of data fitting using the fractional system, for example, dengue data [28], groundwater flow [29], parameters estimation for dengue data [30]. Chickenpox model with real data [31], banking data models [17,18]. In all these works the fractional models are considered by applying one operator, or more. Some more results about the fractional derivatives and their applications to real-world problems can be seen in [32–38] and the work cited inside it. In these works, the authors presented the engineering and physical problems with fractional derivatives and provide effective results. Recently, fractal-fractional model and their application to banking and finance has been studied in [19,20]. The fractal-fractional operators are considered recently in [39] and their applications to chaotic different problems. The fractal-fractional is shown by the authors for Caputo, Caputo-Fabrizio and the Atangana-Baleanu. The advantage of this new operator to the real data problem is discussed briefly in [19,20].

The main target of the work is to apply the fractional operator is a combination with the fractal operator. We use the new definition of fractal-fractional where the fractional operator is considered Caputo-Fabrizio. The fractal-fractional Caputo-Fabrizio has fractal and fractional orders and we show that its results are reliable for data fitting. We present some important application of this newly proposed operator to the banks data and show that this new idea gives the best fitting to real data than the integer one. For the proposed study, we consider Indonesian banking data for the period of 2004–2014 considered in [17,18], and provide many illustrations in the form of simulations to show that this new idea is powerful than the integer-order. In order to do this, we divide the work section-wise and is follows: Basic results associated with fractal-fractional operators as well as model descriptions are given in Section 2 and 3 respectively. A numerical approximation for the fractal-fractional Caputo-Fabrizio is shown in Section 4. Numerical results with brief discussion is given in Section 5 while the achievement is shown in Section 6.

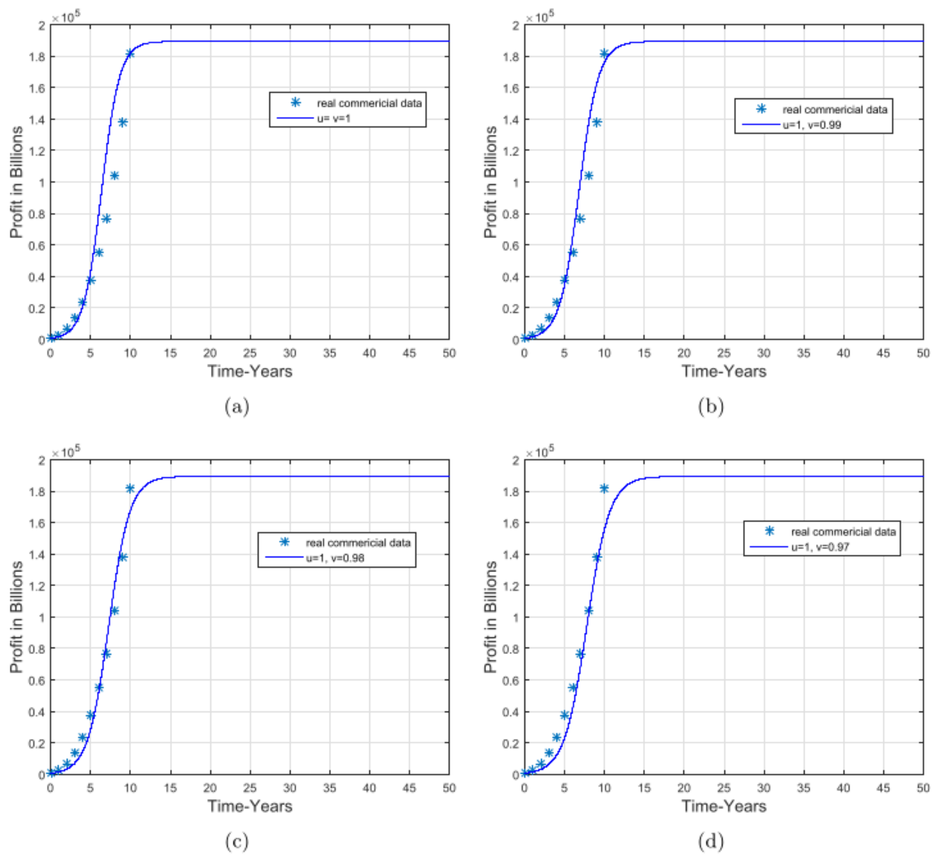

Fig. 2 Model versus commercial data fitting for long time when $u = 1$ fixed: (a) $v = 1$, (b) $v = 0.99$, (c) $v = 0.98$, (d) $v = 0.97$.

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2. Preliminaries

This section contains the important new definitions of fractal-fractional(FF) in the sense of different fractional operators described very recently in [39,40].

Definition 1. A continuous and differentiable function, say $z(t)$ in the interval (p_1, p_2) with fractal order v, then the definition of FF derivative of $z(t)$ with fractional order u in Riemann-Liouville sense with power law kernel is presented by:

$$^{FFP}D^{u,v}_{0,t}(z(t)) = \frac{1}{\Gamma(m-u)}\frac{d}{dt^v}\int_0^t (t-s)^{m-u-1}z(s)ds, \quad (1)$$

with $m-1 < u, v \leqslant m \in \mathbb{N}$ and $\frac{dz(s)}{ds^v} = \lim_{t\to s}\frac{z(t)-z(s)}{t^v-s^v}$.

Definition 2. A continuous and differentiable function, say $z(t)$ in the interval (p_1, p_2) with fractal order v, then the definition of FF derivative of $z(t)$ with fractional order u in Riemann-Liouville sense with exponentially decaying kernel is presented by:

$$^{FFE}D^{u,v}_{0,t}(z(t)) = \frac{M(u)}{1-u}\frac{d}{dt^v}\int_0^t exp\left(-\frac{u}{1-u}(t-s)\right)z(s)ds, \quad (2)$$

with $u > 0, v \leqslant m \in \mathbb{N}$ and $M(0 = M(1) = 1$.

Definition 3. A continuous and differentiable function, say $z(t)$ in the interval (p_1, p_2) with fractal order v, then the definition of FF derivative of $z(t)$ with fractional order u in Riemann-Liouville sense with generalized Mittag-Leffler kernel is presented by:

$$^{FFM}D^{u,v}_{0,t}(z(t)) = \frac{AB(u)}{1-u}\frac{d}{dt^v}\int_0^t E_u\left(-\frac{u}{1-u}(t-s)^u\right)z(s)ds, \quad (3)$$

with $u > 0, v \leqslant 1 \in \mathbb{N}$ and $AB(u) = 1 - u + \frac{u}{\Gamma(u)}$.

Definition 4. A continuous function, say $z(t)$ in the interval (p_1, p_2), then the definition of FF integral of $z(t)$ with fractional order u and with power law kernel is presented by:

$$^{FFP}J^{u,v}_{0,t}(z(t)) = \frac{v}{\Gamma(u)}\int_0^t (t-s)^{u-1}s^{v-1}z(s)ds. \quad (4)$$

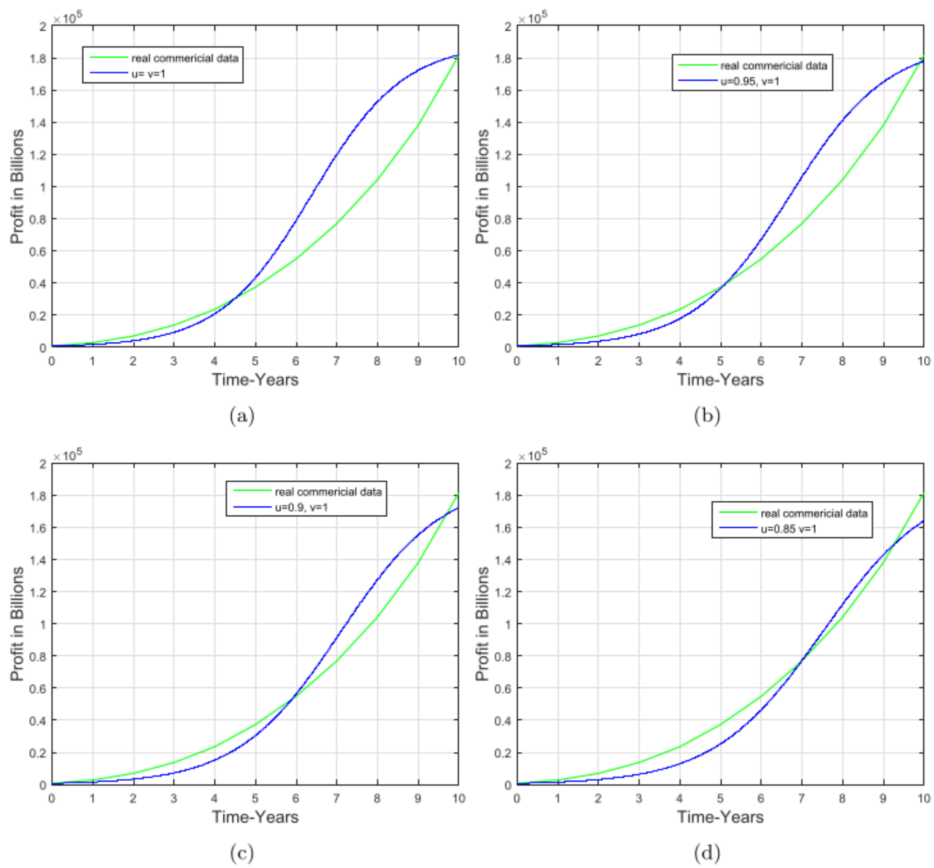

Fig. 3 Commercial data versus model, $v = 1$: (a) $u = 1$, (b) $u = 0.95$, (c) $u = 0.9$, (d) $u = 0.85$.

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Definition 5. A continuous function, say $z(t)$ in the interval (p_1, p_2), then the FF integral of $z(t)$ with order u has the exponentially decaying kernel is presented by:

$$ {}^{FFE}J^{u}_{0,t}(z(t)) = \frac{uv}{M(u)} \int_0^t s^{u-1} z(s) ds + \frac{v(1-u)t^{v-1} z(t)}{M(u)}. \tag{5}$$

Definition 6. A continuous function, say $z(t)$ in the interval (p_1, p_2), then the FF integral of $z(t)$ with order u has the generalized Mittag-Leffler kernel is presented by:

$$ {}^{FFM}J^{u,v}_{0,t}(z(t)) = \frac{uv}{AB(u)} \int_0^t s^{v-1} (t-s)^{u-1} z(s) ds + \frac{v(1-u)t^{v-1} z(t)}{AB(u)}. \tag{6}$$

3. Mathematical model

We present the model construction by taking the Lotka–Volterra model that consists of two equations to present the model descriptions. We consider here the competition system that based on two equations which are considered to be more useful for the dynamics of competition among two species, commodities etc over time. The species or commodities over time are considered in this work are the banks with kind rural and commercial. In Indonesia the study in [17,18] demonstrates the existing of not hugely differences in the products among these banks types, so the possibilities among these banks exist for the competition. We consider the maximum profit that have by the commercial banks at any time t is given by $X_1(t)$ while for the rural banks is $X_2(t)$. We assumed that the banks have limited funds and grows logistically both the banks. With the suggested assumptions the dynamics among rural and commercial banks described in the following though the system of nonlinear differential equations:

$$\begin{aligned} \frac{dX_1}{dt} &= r_1 X_1 \left(1 - \frac{X_1}{C_1}\right) - \phi_1 X_1 X_2, \\ \frac{dX_2}{dt} &= r_2 X_2 \left(1 - \frac{X_2}{C_2}\right) - \phi_2 X_1 X_2, \end{aligned} \tag{7}$$

where model (7) represents the dynamics of a positive quantity, so the initial conditions associated are

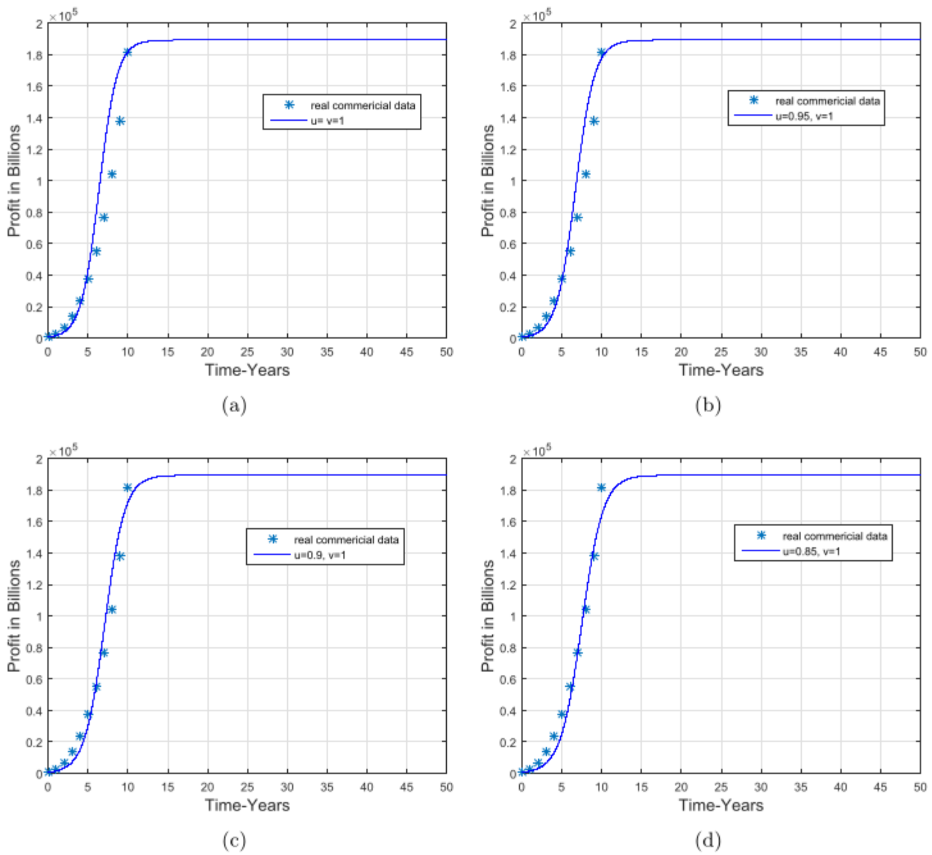

Fig. 4 Model versus commercial data fitting for long time, $v = 1$ fixed: (a) $u = 1$, (b) $u = 0.95$, (c) $u = 0.9$, (d) $u = 0.85$.

$X_1(0) = X_{10} \geqslant 0, X_2(0) = X_{20} \geqslant 0$. Further, r_i for $i = 1, 2$ represent banks growth for commercial and rural respectively, the maximum profit gained by the commercial and rural is shown respectively by C_1 and C_2, where the coefficients ϕ_i for $i = 1, 2$ is used for respectively commercial and rural banks. It should be noted that the parameters given (7) are related banks which are clearly positive, that is $r_1, r_2, C_1, C_2, \phi_1, \phi_2 > 0$. In the following subsection, we apply the new idea of fractal-fractional to the above described model (7), where the fractional operator is chosen to be Caputo-Fabrizio.

3.1. Fractal-fractional Caputo-Fabrizio model

In order to have a model formulation in the sense of a new definition known as fractal-fractional in the sense of Caputo-Fabrizio for the dynamics of competition among rural versus commercials banks are described by the non-linear fractal-fractional differential equations:

$$\begin{aligned} {}^{CF}D^{u,v}_{0,t}(X_1(t)) &= r_1 X_1\left(1 - \frac{X_1}{C_1}\right) - \phi_1 X_1 X_2, \\ {}^{CF}D^{u,v}_{0,t}(X_2(t)) &= r_2 X_2\left(1 - \frac{X_2}{C_2}\right) - \phi_2 X_1 X_2, \end{aligned} \tag{8}$$

where u is the fractional-order while v is used for fractal order and the rest of the parameters are defined already above. To have some properties of the model (8) such as their fixed points, we present it in detail in the following subsections.

3.2. Equilibria

The equilibria of the FF model (8) can obtained by setting,

$${}^{FF}D^{u,v}_{0,t}(X_1(t)) = 0, \quad {}^{FF}D^{u,v}_{0,t}(X_2(t)) = 0. \tag{9}$$

The imposed conditions on Eqs. (9), leads to the following:

$$\begin{aligned} r_1 X_1\left(1 - \frac{X_1}{C_1}\right) - \phi_1 X_1 X_2 = 0, \\ r_2 X_2\left(1 - \frac{X_2}{C_2}\right) - \phi_2 X_1 X_2 = 0. \end{aligned} \tag{10}$$

Solution of (10) leads to the equilibrium points below:

$$\begin{aligned} Q_0 &= (0,0), \; Q_1 = (0, C_2), \; Q_2 = (C_1, 0), \\ Q_3 &= \left(\frac{C_1 r_2(\phi_1 C_2 - r_1)}{\phi_1\phi_2 C_1 C_2 - r_1 r_2}, \frac{C_2 r_1(\phi_2 C_1 - r_2)}{\phi_1\phi_2 C_1 C_2 - r_1 r_2}\right), \end{aligned}$$

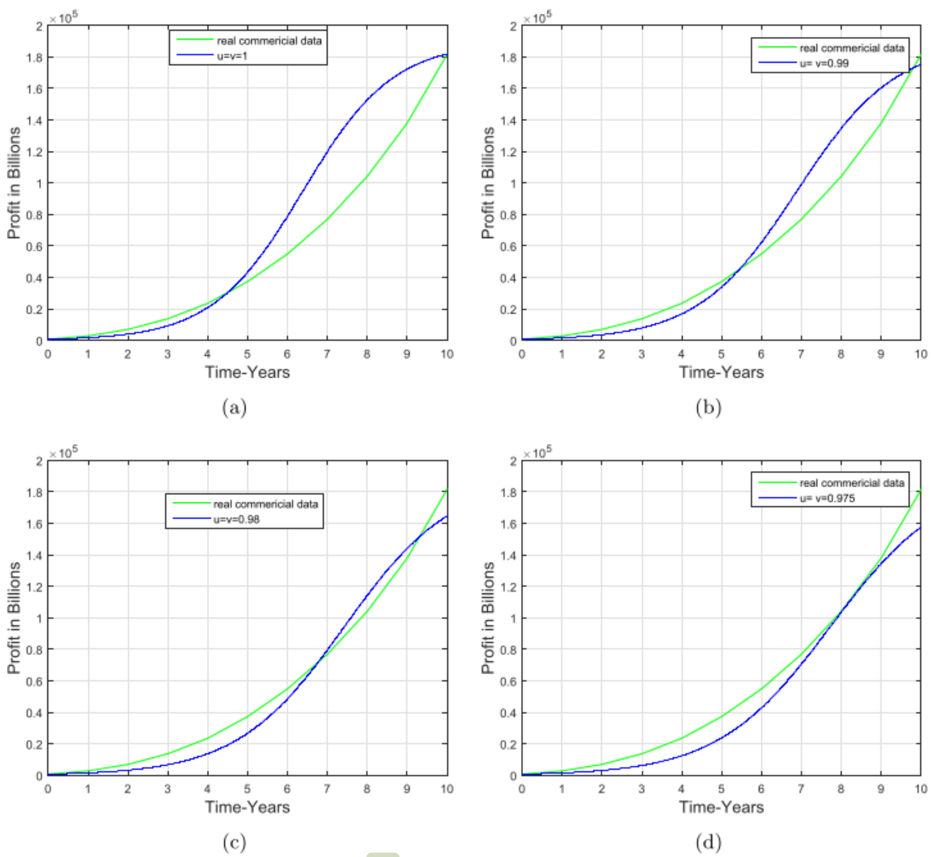

Fig. 5 Commercial data versus model: (a) $u = v = 1$, (b) $u = v = 0.99$, (c) $u = v = 0.98$, (d) $u = v = 0.975$.

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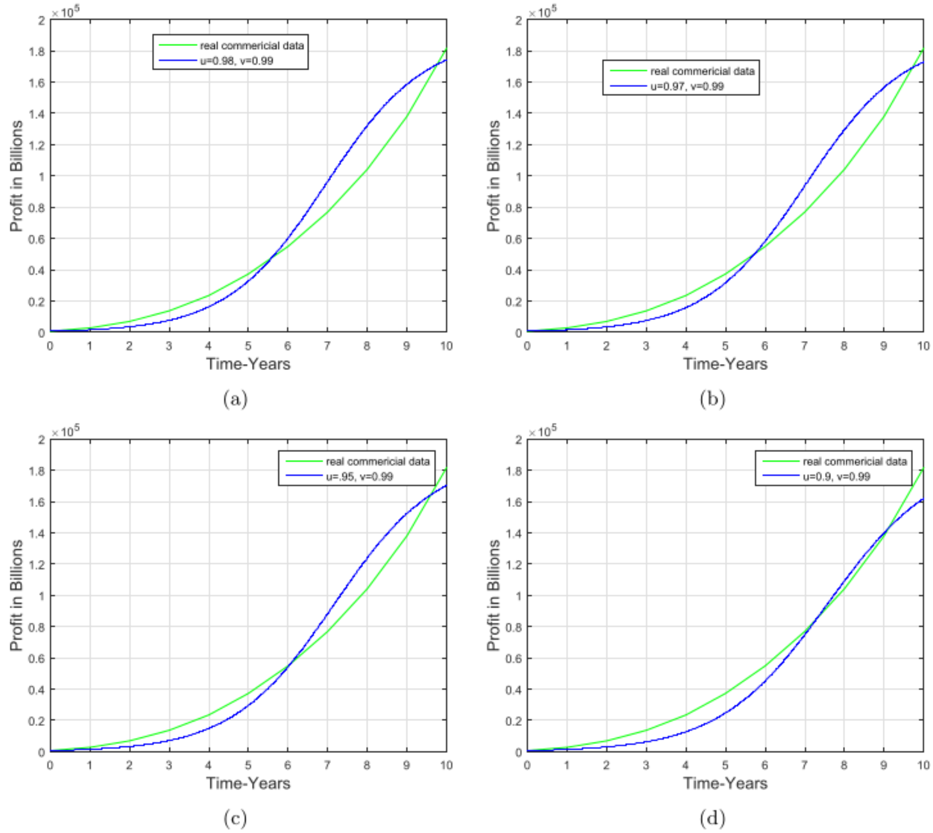

Fig. 6 Model versus commercial data fitting: (a) $u = 0.98, v = 0.99$, (b) $u = 0.97, v = 0.98$, (c) $u = 0.95, v = 0.99$, (d) $u = 0.9, v = 0.99$.

with these equilibrium points, we have $\phi_1 C_2 - r_1 > 0, \phi_2 C_1 - r_2 > 0$ and $\phi_1\phi_2 C_1 C_2 - r_1 r_2 > 0$, then ${}^{FF}D^{u,v}_{0,t}(X_1(t)) > 0$ and ${}^{FF}D^{u,v}_{0,t}(X_2(t)) > 0$ otherwise not. The model will have an equilibrium if its solution line in given in Q_3 cut each other in quadrant 1. It should be noted that with some reasonable set of values of the parameters the stability of the model will be determined.

4. Solution procedure for FF model

The numerical approach to solve the given model (8), we here provide a novel approach which is based on the Adams-Bashforth method. To have a scheme for the fractal-fractional model (8), we convert the model (8) to the following form:

$$\begin{aligned} {}^{CF}D^{u}_{0,t}(X_1(t)) &= vt^{v-1} f_1(X_1, X_2, t), \\ {}^{CF}D^{u}_{0,t}(X_2(t)) &= vt^{v-1} f_2(X_1, X_2, t). \end{aligned} \tag{11}$$

We apply the CF integral on Eq. (11) which leads to the following:

$$\begin{aligned} X_1(t) &= X_1^0 + \tfrac{vt^{v-1}(1-u)}{M(u)} f_1(X_1, X_2, t) + \tfrac{uv}{M(u)} \int_0^t \lambda^{v-1} f_1(X_1, X_2, \lambda) d\lambda, \\ X_2(t) &= X_2^0 + \tfrac{vt^{v-1}(1-u)}{M(u)} f_2(X_1, X_2, t) + \tfrac{uv}{M(u)} \int_0^t \lambda^{v-1} f_2(X_1, X_2, \lambda) d\lambda. \end{aligned} \tag{12}$$

where

$$\begin{aligned} f_1(X_1, X_2, \lambda) &= r_1 X_1 \left(1 - \tfrac{X_1}{C_1}\right) - \phi_1 X_1 X_2, \\ f_2(X_1, X_2, \lambda) &= r_2 X_2 \left(1 - \tfrac{X_2}{C_2}\right) - \phi_2 X_1 X_2. \end{aligned}$$

Using $t = t_{n+1}$, the following is established,

$$\begin{aligned} X_1^{n+1}(t) &= X_1^n + \tfrac{vt^{v-1}(1-u)}{M(u)} f_1(X_1^n, X_2^n, t_n) + \tfrac{uv}{M(u)} \int_0^{t_{n+1}} \lambda^{v-1} f_1(X_1, X_2, \lambda) d\lambda, \\ X_2^{n+1}(t) &= X_2^n + \tfrac{vt^{v-1}(1-u)}{M(u)} f_2(X_1^n, X_2^n, t) + \tfrac{uv}{M(u)} \int_0^{t_{n+1}} \lambda^{v-1} f_2(X_1, X_2, \lambda) d\lambda. \end{aligned} \tag{13}$$

Further, we have the following:

$$\begin{aligned} X_1^{n+1}(t) &= X_1^n + \tfrac{vt^{v-1}(1-u)}{M(u)} f_1(X_1^n, X_2^n, t_n) - \tfrac{vt_{n-1}^{v-1}(1-u)}{M(u)} f_1(X_1^{n-1}, X_2^{n-1}, t_{n-1}) \\ &\quad + \tfrac{uv}{M(u)} \int_{t_n}^{t_{n+1}} \lambda^{v-1} f_1(X_1, X_2, \lambda) d\lambda, \\ X_2^{n+1}(t) &= X_2^n + \tfrac{vt^{v-1}(1-u)}{M(u)} f_2(X_1^n, X_2^n, t_n) - \tfrac{vt_{n-1}^{v-1}(1-u)}{M(u)} f_2(X_1^{n-1}, X_2^{n-1}, t_{n-1}) \\ &\quad + \tfrac{uv}{M(u)} \int_{t_n}^{t_{n+1}} \lambda^{v-1} f_2(X_1, X_2, \lambda) d\lambda. \end{aligned} \tag{14}$$

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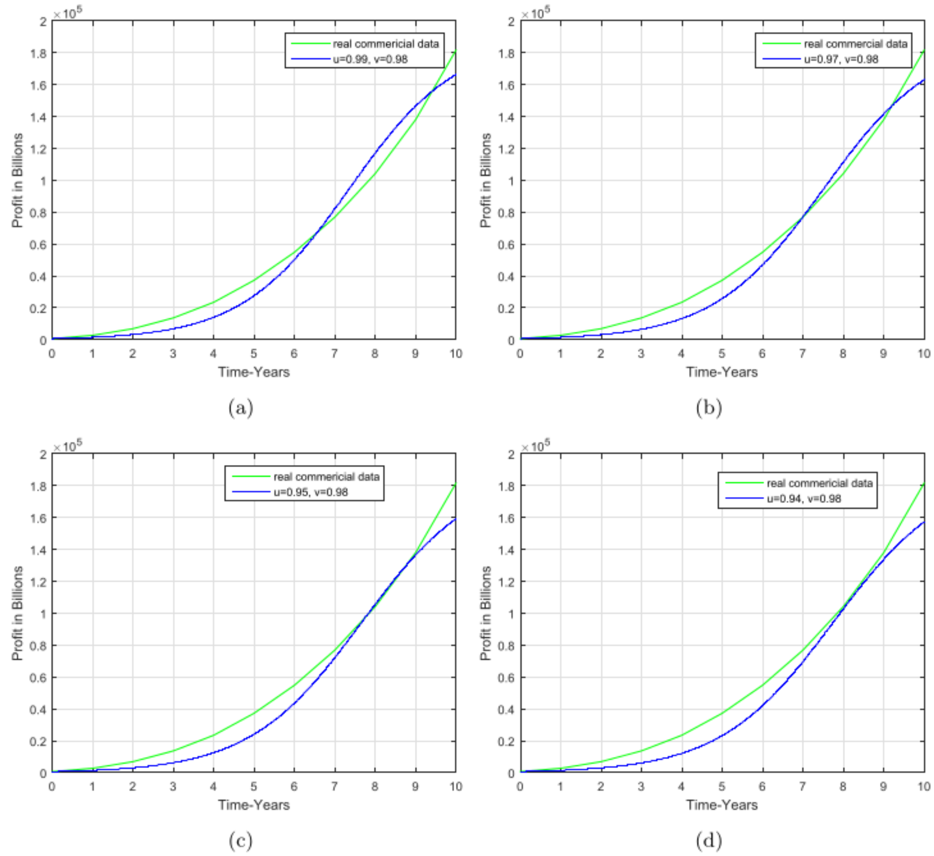

Fig. 7 Commercial data versus model: (a) $u = 0.99, v = 0.98$, (b) $u = 0.97, v = 0.98$, (c) $u = 0.95, v = 0.98$, (d) $u = 0.94, v = 0.98$.

It follows from the Lagrange polynomial interpolation and integrating the following expressions:

$$\begin{aligned} X_1^{n+1}(t) = & X_1^n + \frac{vt^{v-1}(1-u)}{M(u)} f_1(X_1^n, X_2^n, t_n) - \frac{vt_{n-1}^{v-1}(1-u)}{M(u)} f_1(X_1^{n-1}, X_2^{n-1}, t_{n-1}) \\ & + \frac{uv}{M(u)}\left[\frac{3h}{2} t_n^{v-1} f_1(X_1^n, X_2^n, t_n) - \frac{h}{2} t_{n-1}^{v-1} f_1(X_1^{n-1}, X_2^{n-1}, t_{n-1})\right], \\ X_2^{n+1}(t) = & X_2^n + \frac{vt^{v-1}(1-u)}{M(u)} f_2(X_1^n, X_2^n, t_n) - \frac{vt_{n-1}^{v-1}(1-u)}{M(u)} f_2(X_1^{n-1}, X_2^{n-1}, t_{n-1}) \\ & + \frac{uv}{M(u)}\left[\frac{3h}{2} t_n^{v-1} f_2(X_1^n, X_2^n, t_n) - \frac{h}{2} t_{n-1}^{v-1} f_2(X_1^{n-1}, X_2^{n-1}, t_{n-1})\right]. \end{aligned} \tag{15}$$

Further simplifications of (15) leads to the following:

$$\begin{aligned} X_1^{n+1}(t) = & X_1^n + vt_n^{v-1}\left(\frac{1-u}{M(u)} + \frac{3uh}{2M(u)}\right) f_1(X_1^n, X_2^n, t_n) \\ & - vt_{n-1}^{v-1}\left(\frac{1-u}{M(u)} + \frac{uh}{2M(u)}\right) f_1(X_1^{n-1}, X_2^{n-1}, t_{n-1}), \\ X_2^{n+1}(t) = & X_2^n + vt_n^{v-1}\left(\frac{1-u}{M(u)} + \frac{3uh}{2M(u)}\right) f_2(X_1^n, X_2^n, t_n) \\ & - vt_{n-1}^{v-1}\left(\frac{1-u}{M(u)} + \frac{uh}{2M(u)}\right) f_2(X_1^{n-1}, X_2^{n-1}, t_{n-1}). \end{aligned} \tag{16}$$

5. Numerical results

We consider the efficient numerical scheme described above for the numerical solution of the bank model in fractal-fractional operators in the sense of Caputo-Fabrizio given in (8). We obtain the simulation results by considering the novel approach presented above using these parameters values: $\phi_1 = 2.90 \times 10^{-10}$, $\phi_2 = 3.9 \times 10^{-7}$, $C_1 = 189318.198$, $C_2 = 17540.6219$, $\gamma_1 = 0.88$, and $\gamma_2 = 0.58$. These values are the best suitable values which describe effectively fitting to the data of commercial and rural banks. We analyze here the dynamics of two different data of commercial and rural banks in Indonesia through a competition model in this simulation. First, we will give brief details of the commercial real data versus a model with various values of fractal-fractional order u and v, then we will analyze the model fitting to the real data of rural banks with fractal-fractional order parameters u and

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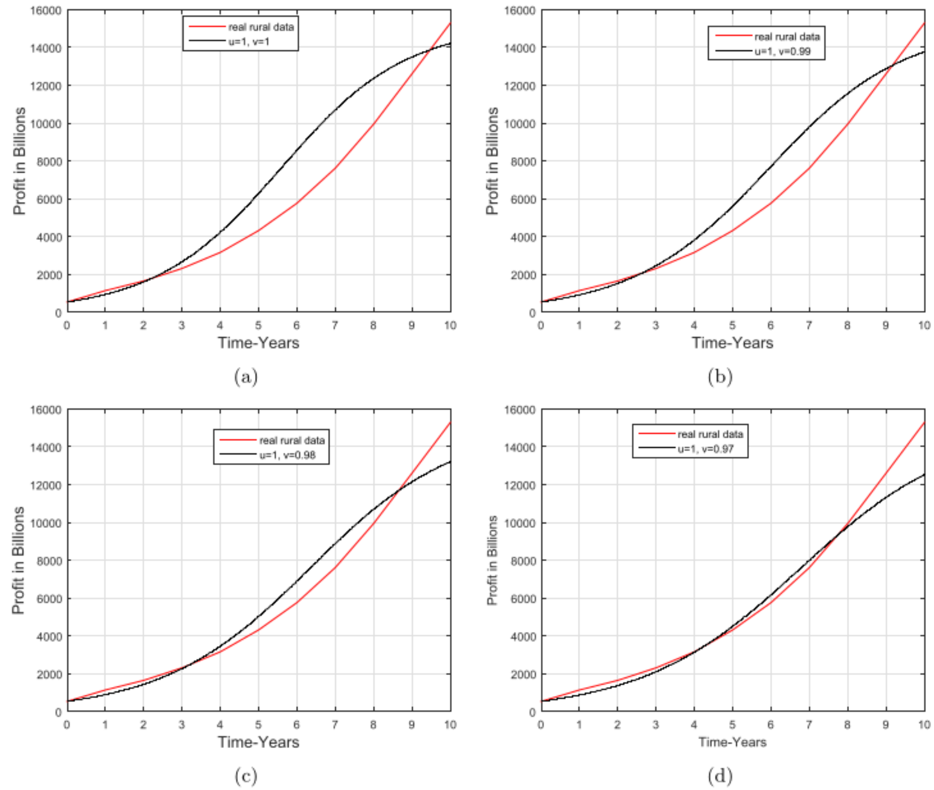

Fig. 8 Rural data versus model fitting, $u = 1$ fixed: (a) $v = 1$, (b) $u = 0.99$, (c) $u = 0.98$, (d) $u = 0.97$.

v. The fractional operator considered here is the Caputo-Fabrizio and shown in the graphics by u while the fractal derivative is shown by v. The useful scheme presented above is used to obtain the graphical results for different values of fractal and fractional are shown in Figs. 1–14.

The graphical results in Figs. 1–7 describe the dynamics of commercial data versus model fitting while the graphical results in Figs. 8–14 show rural data versus model fitting. We simulate first the data of commercial banks and fix the fractional-order parameter $u = 1$ and vary the fractal order parameter v ($v = 1, 0.99, 0.98, 0.97$) see Fig. 1. Here, one can see that decreasing the fractal order parameter v the fitting of the model gets close and close to real data, and for $v = 0.97$, we have some good fitting for the commercial data. The future prediction of the commercial bank data versus model is simulated in Fig. 2 with 50 years. It can be seen very clearly that the decrease in the fractal order v could be the best fitting for the real data. Fig. 3 is obtained by setting $v = 1$ fixed and the fractional-order parameter u is varying. Decreasing the fractional-order parameter u from $1, 0.95, 0.9, 0.85$, the fitting to data refining and we have some reasonable fitting for $u = 0.85$. Fig. 4 is obtained for a long time with the same parameters of u and $v = 1$, where the fractional parameter over a long time shows the best fit compare to the integer case. We have Figs. 5–7, where we limited ourself to give fix values to u and v (see Fig. 5) changing u and v see Figs. 6 and 7. The graphical results when varying both the u and v give more reasonable setting for the data versus model. In Figs. 8–14, we present the rural data versus model by considering various values of u and v. In Fig. 8 we fix the fractional-order parameter $u = 1$ fix and varying the fractal parameter v with $v = 1, 0.99, 0.98, 0.97$. The decrease in v gives good fitting for rural data versus model. A more clear result is shown in Fig. 9 for data comparison over long time. Keeping the value of the fractal order v fixed and varying the fractional-order u, we have some reasonable better result for non-integer order than that of integer order, see Fig. 10. The variation in u and keeping v fix we have Fig. 11 which shows better fitting than that of integer order. Keeping both the parameters fix to some values we get a better fitting, see Fig. 12. Changing the values of the fractional-order u and fractal order v we have Figs. 13 and 14. Fig. 14 explains the comparison of rural data with the model with different values of the fractal and fractional orders. We observe that varying both the fractal

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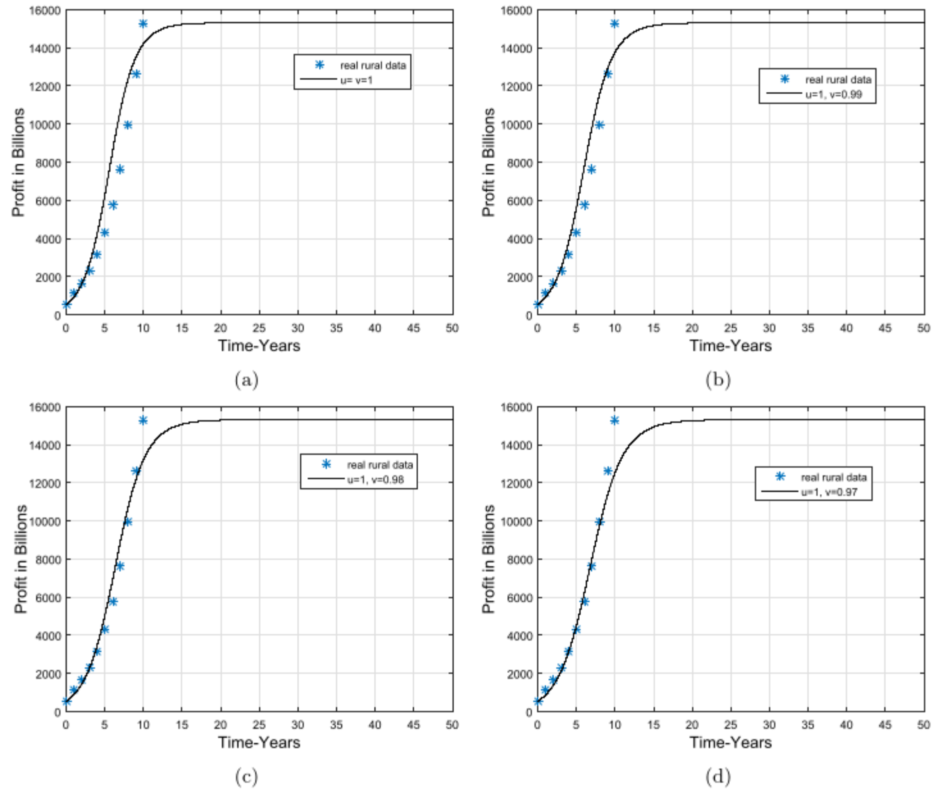

Fig. 9 Future prediction for rural data versus model fitting, $u = 1$ fixed: (a) $v = 1$, (b) $u = 0.99$, (c) $v = 0.98$, (d) $v = 0.97$.

and fractional order parameters values the real data coincides better with the model curve and thus generate good fitting results. From these graphical results, we conclude that this new idea of fractal-fractional to the real world problem give more and more best fitting and will be more useful for the models arising in science and engineering.

6. Conclusion

We presented the dynamics of bank data through differential equations as a competition among banks that is rural and commercial banks for the specific period of years 2004–2014. We considered a competition model first and then applied a very recent introduced fractional operator namely fractional-fractional Caputo-Fabrizio. We applied the fractal-fractional operator and the model was formulated. The necessary results for the model have been obtained. For numerical simulation of the competition system, we constructed a very useful scheme and the results were obtained and discussed in detail. We have taken various values of the fractional and fractal order and presented better results for the data fitting. We observed that the data of commercial and rural banks for the fractal-fractional orders parameters provide interesting fitting than that of integer order. It is worthy to mention that the results of the fractal-fractional operators are more reliable and better than ordinary operators. The applications fractal-fractional

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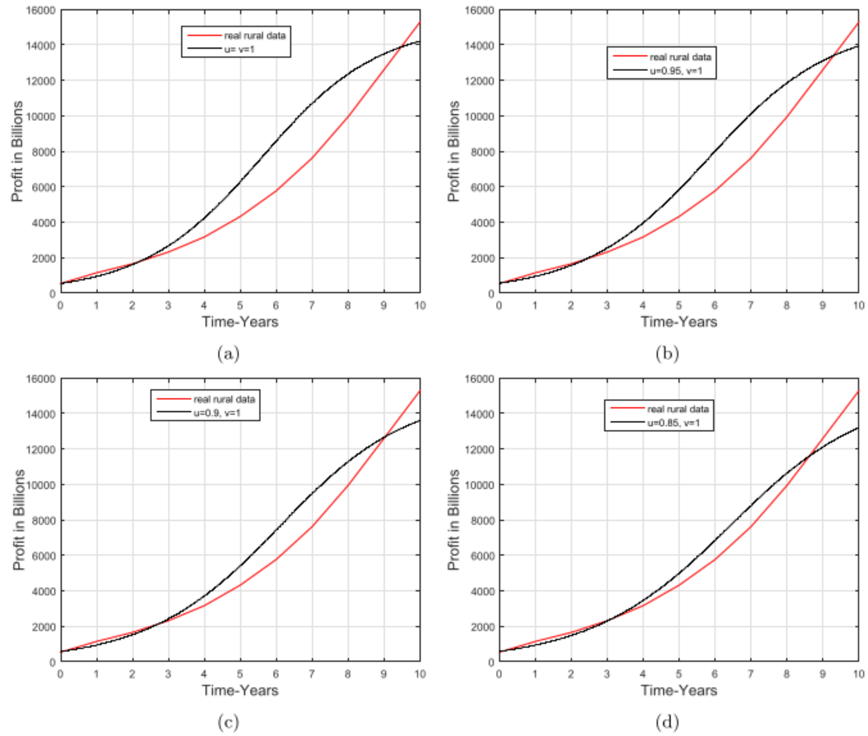

Fig. 10 Rural data versus model fitting, $v = 1$ fixed: (a) $u = 1$, (b) $u = 0.95$, (c) $u = 0.9$, (d) $u = 0.85$.

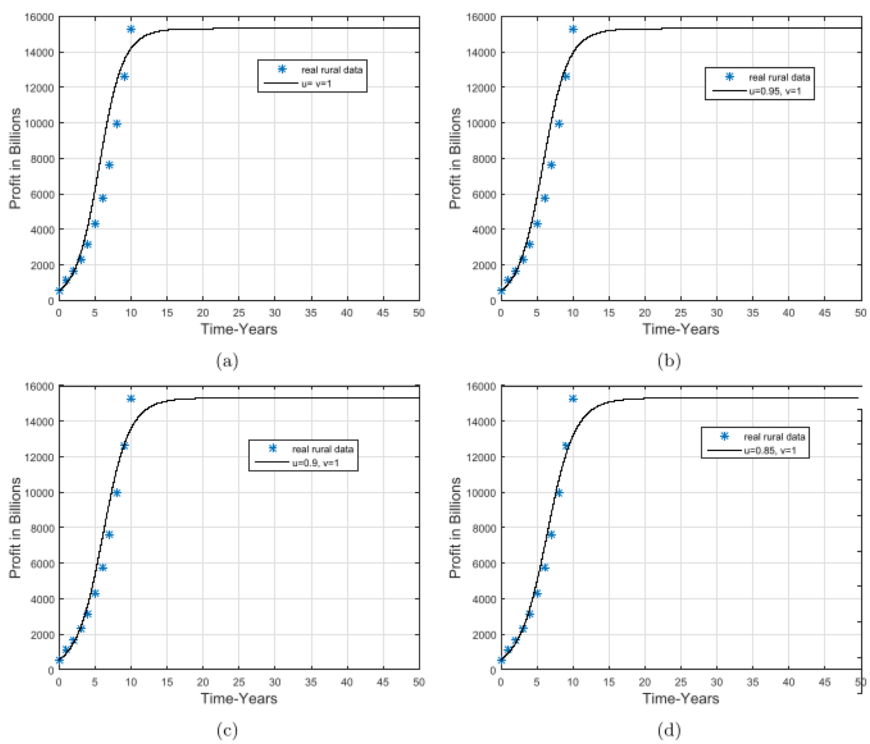

Fig. 11 Future prediction for rural data versus model fitting, $v = 1$ fixed: (a) $u = 1$, (b) $u = 0.95$, (c) $u = 0.9$, (d) $u = 0.85$.

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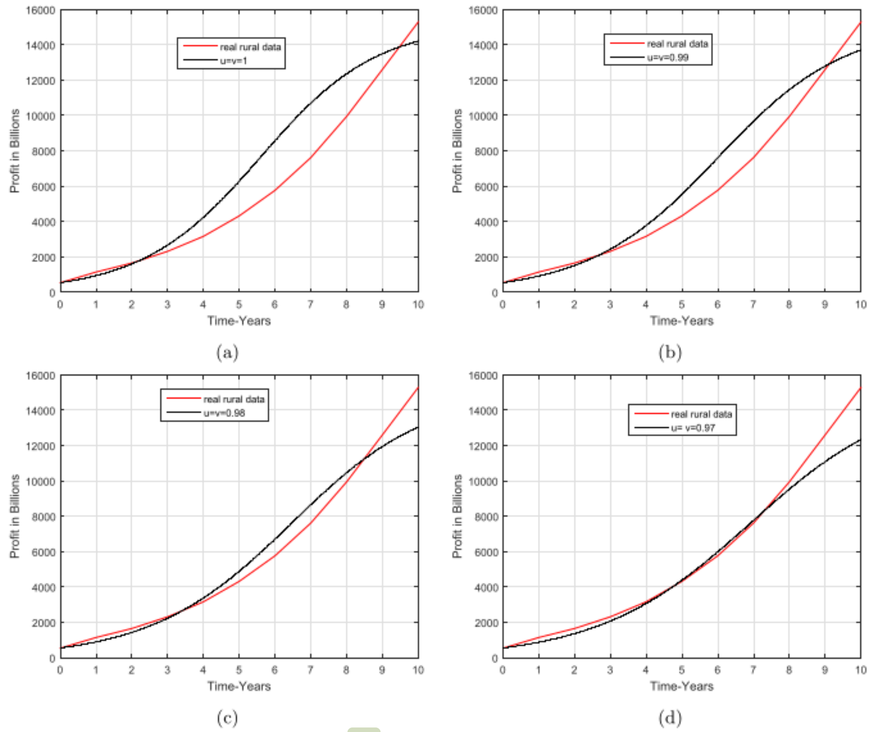

Fig. 12 Rural data versus model fitting: (a) $u = v = 1$, (b) $u = v = 0.99$, (c) $u = v = 0.98$, (d) $u = v = 0.97$.

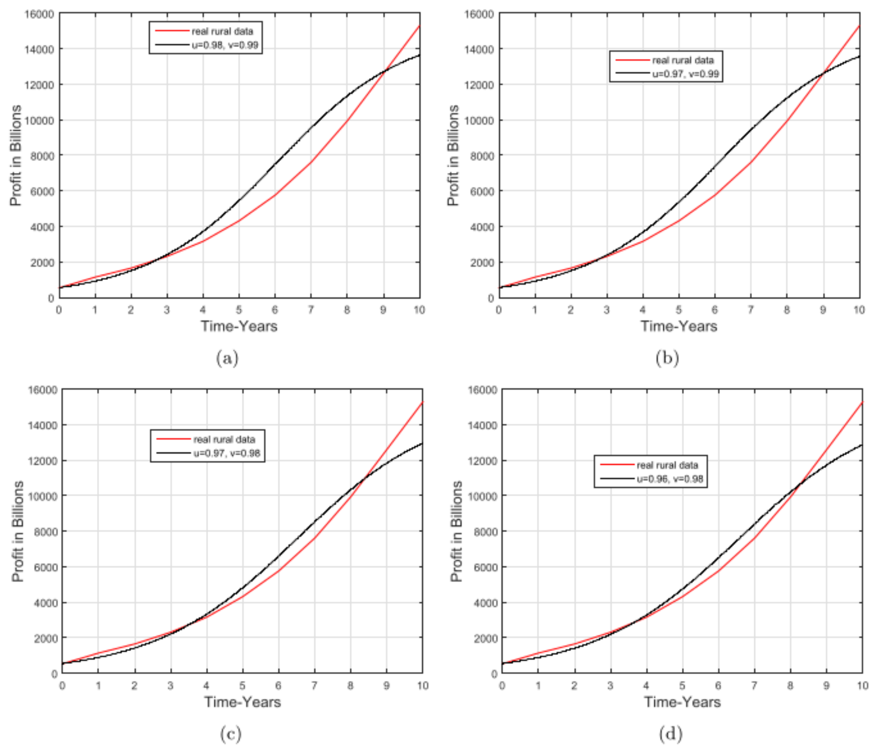

Fig. 13 Rural data versus model fitting: (a) $u = 0.98, v = 0.99$, (b) $u = 0.97, v = 0.99$, (c) $u = 0.97, v = 0.98$, (d) $u = 0.96, v = 0.98$.

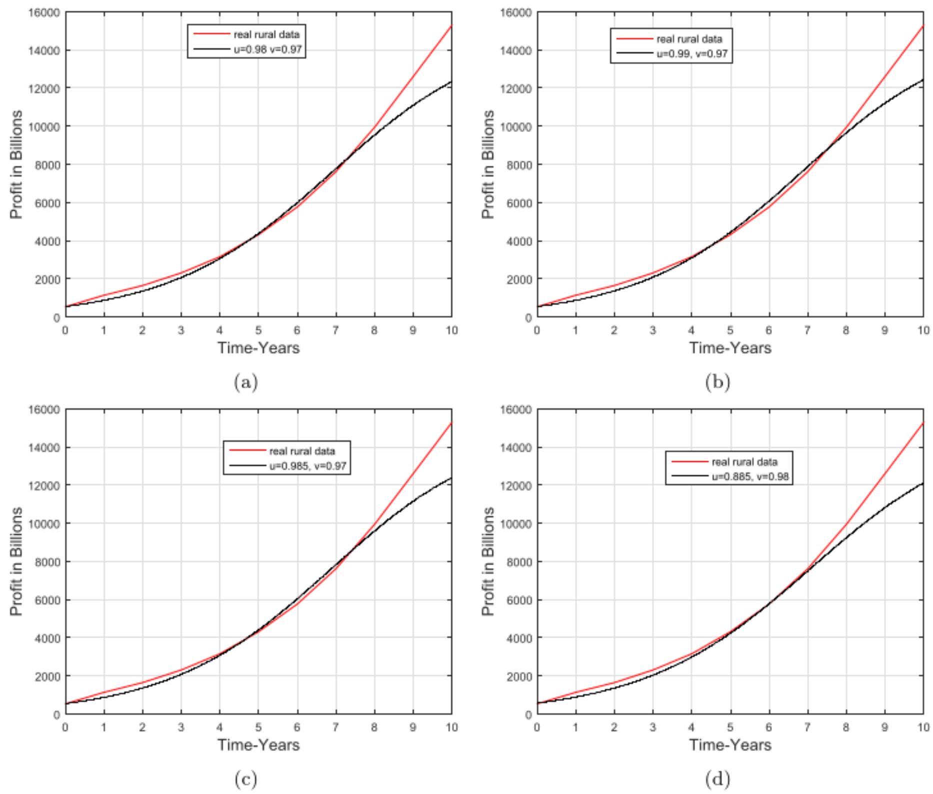

Fig. 14 Rural data versus model fitting: (a) $u = 0.98, v = 0.97$, (b) $u = 0.99, v = 0.97$, (c) $u = 0.985, v = 0.97$, (d) $u = 0.885, v = 0.98$.

operators with the real data in the field of science and engineering and other social sciences sector can get benefit from these. These new data fitting results with the new idea of fractal-fractional Caputo-Fabrizio derivative for the real data model suggested in this work will open new doors to the researchers who work in fractional calculus.

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