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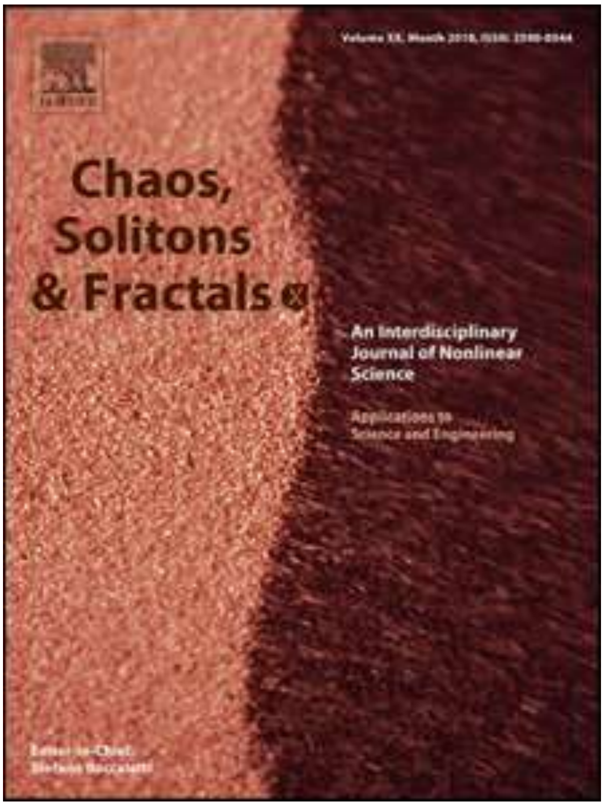

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A fractional model for the dynamics of competition between commercial and rural banks in Indonesia

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ABSTRACT

In the present paper, we propose a mathematical model that describes the dynamics of competition between commercial and rural banks in Indonesia through two different fractional operators Atangana-Baleanu and Caputo. We present a parameter estimation of the Lotka–Volterra competition model by using the genetic algorithm method. Parameter estimation is done based on annual profit data of commercial and rural banks in Indonesia. The estimation results capable to predict the profit of commercial and rural banks every year which is not much different from the real data. Next, the competition model between commercial and rural banks in Indonesia is explored in the fractional sense of Atangana–Baleanu and Caputo derivative. The fractional model is examined through the Atangana–Baleanu and Caputo fractional derivative and present the results. A recent numerical procedure is used to obtain the graphical results using various values of the fractional order parameter for the dynamics of the model. A comparison of both the operators for various values of the fractional order parameters are given. We discussed briefly the results and then summarized briefly in section conclusion.

1. Introduction

Banks are the business entities that collecting funds from the people of a particular region, area or country and save it and then distribute this funds to the public of that particular area etc in the form of credit or in cash for many different aspects to improve the lifestyle of inhabitants [1]. The bank's business is not only as a store of funds and lenders but also a tool for the government to stabilize monetary and encourage the national economic growth rate. According to Act of The Republic of Indonesia Number 10 of 1998 concerning banking, the types of banks consist of commercial bank and rural bank [1].

The commercial bank is the bank that carries out business activities conventionally or based on Syariah Principles which in their activities provide services in payment traffic. In providing banking services, its business activities can be conventional, such as banks that apply in general, but can also be based on Islamic principles based on Islamic religious norms [1]. Commercial banks aim to support the implementation of national development in order to improve equity, economic growth and national stability towards increasing the welfare of the people [2].

Definition of rural bank in accordance with Act No. 10 of 1998 concerning banking is a bank conducting business activities conventionally or based on Syariah principles which in its activities do not provide services in payment traffic. Business activities undertaken by the rural bank are very limited compared to commercial banks, which only includes the collection of funds from the public in the form of time deposits, savings, credit, and place funds in the form of Bank Indonesia Certificates. Rural banks are not allowed to accept deposits in the form of demand deposits, participate in payment traffic, conduct business activities in foreign currencies, conduct equity participation and conduct insurance business [3].

Based on Indonesian banking statistics published by the Financial Services Authority, the number of commercial banks in Indonesia is less than the number of rural banks [4]. This is because commercial bank activities are more than those of rural banks. Even though, the profit of commercial banks is greater than the profit of rural banks. Although rural banks have lower profits compared to commercial banks, rural banks can still survive to develop their products [4]. Products issued by commercial banks and rural banks are not much different, so there will be a competition to get customers.

The competition between commercial banks and rural banks in Indonesia can be analyzes through the Lotka–Volterra competition model. The Lotka–Volterra model was first introduced by

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Alfred J. Lotka and Vito Volterra in 1920. Lotka–Volterra competition equation is a model that describes the competition of two species to obtain food sources. This competition can occur between the same species and different species. Lotka–Volterra competition model is a modification of the logistics model and can be used to predict the outcome of a competition [5]. In recent years, the Lotka–Volterra equation has been used to analyze the competition of market shares such as mobile phone market [6], technological forecasting market [7,8] and also operating systems market [9,10]. The author in [11] has been studied banking system using a three-level Lotka–Volterra model.

A mathematical model cannot be interpreted in real cases if the parameter values of the model is unknown. The first aim of this paper is to estimate the parameter values of the competition model between commercial banks and rural banks in Indonesia. Parameter estimation is based on annual profit data of commercial banks and rural banks in Indonesia from 2004 to 2014. The parameter estimation technique can be done by using the genetic algorithm optimization method. The value of the guessing parameter will be used to solve the numerical solution of the model, then it will be matched against the real data using the objective function of absolute error. We use the genetic algorithm to minimize the error. The genetic algorithm is a branch of artificial intelligence in the form of optimization and search techniques based on the principles of genetics and selection. This algorithm can be used to find solutions to problems with one or many variables, both continuous and discrete problems, and not only provide a single solution [12]. The genetic algorithm method has been previously applied to estimate parameters of the non-linear ordinary differential equations system model [9,13–15].

Fractional order (FO) derivatives and fractional integrals have been an important concept in the study of fractional calculus and have been used as a useful tool in the modeling of a various complex problem which is unable through classical integer derivative. Due to various reasons, the model with FO is more useful than the ordinary order derivative. For example, the FO models have the property of memory and hereditary properties and gives a reasonable fitting to the real incidence cases of diseases and other data of experimental, see the reference [16]. The author's in [16] used a TB model with fractional order and using the real incidence cases and compare the results with fractional order parameter by using various values of the fractional order parameter. Further, FO models are more helpful, gives deeper information to explore the complexity of the system and allow greater degrees of freedom in the model. The idea of classical FO derivatives and related work has been presented in [17] and is applied in various systems with memory which exists in most biological systems including Lotka–Volterra model [18–22]. The classical Caputo FO derivative has a serious disadvantage that their kernel has a singularity and therefore cannot accurately describe the various real-world phenomenon. To avoid this inconvenience, a new FO operator is known as Caputo-Fabrizio (CF) based on generalized exponent law with the non-singular kernel is presented in [23]. In recent years a number of models using CF operator in various fields have been developed which can be found in [24–26]. In the recent era, a new fractional derivative that could address better the local dynamics of the models and the behavior of crossover for many realistic situations a Mittag-Leffler the concept is used in Atangana and Baleanu [27]. In this new invention regarding the fractional operator, that is the Atangana–Baleanu derivative [27] it is shown that it has non-local and non-singular and it is proven in many applications of models that applied this derivative to realistic problems which show the effectiveness of the operator [28,29]. Further application of the Atangana–Baleanu derivative and their properties have been discussed in detail in [30–32]. For example, a new derivative with Mittag Leffler functions has been discussed in detail in [30]. A series of solutions for fractional operators using Mittag-Leffler functions are analyzed briefly in [31]. Further, a vector-borne disease model with the application of the newly Atangana–Baleanu derivative is proposed by the authors in [32] and the numerical results are provided which is show the importance of the new operators.

The authors are inspired by this new operator which is applied to many real-life problems and aims to develop a new fractional Lotka–Volterra model to study the dynamics of competition between commercial and rural banks in Indonesia using real data. The model parameters are fitted from the real data published by the Financial Services Authority Republic of Indonesia from 2004 to 2014 [4], using a genetic algorithm. Further sections are organized as follows: The fundamental concepts of the fractional derivative are given in Section 2. We formulate the model and discuss the basic results associated to it in Section 3. In Section 4, parameter estimation is explored. Iterative solution and numerical simulations are given in Section 5. Finally, the present work is concluded in Section 6.

2. Basic concepts of A–B derivative and Caputo derivative

In this section, we provide a basic concept of Caputo and A–B derivative. Initially, we recall the basic concept of Caputo derivative and the relevant results in the following [17,33,34].

Definition 1. For a function $g: R^{+} \rightarrow R$ one can write the fractional integral of order $\alpha > 0$ as

$$I_t^{\alpha}(h(t)) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\chi)^{\alpha-1}h(\chi)d\chi,$$

where the symbol shows the Γ function.

Definition 2. For a function $h \in C^n$ of order α, the Caputo fractional order derivative can be shown as

$$^{C}D_t^{\alpha}(h(t)) = I^{n-\alpha}D^n h(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{h^n(\chi)}{(t-\chi)^{\alpha+n-1}}d\chi,$$

where $n-1 < \alpha < n \in N$.

Now we present the basic definitions of A-B fractional derivative which will be used later in our study [27].

Definition 3. Suppose $h \in F^1(a, b)$, $b > a$, $\alpha \in [0, 1]$ then in Caputo sense the newly fractional derivative can be written as follows:

$$^{AB}_{a}D_t^{\alpha}(h(t)) = \frac{B(\alpha)}{1-\alpha}\int_a^t h'(\chi)E_{\alpha}\Big[-\alpha\frac{(t-\chi)^{\alpha}}{1-\alpha}\Big]d\chi.$$

Definition 4. Consider $h \in F^1(a, b)$, $b > a$, $\alpha \in [0, 1]$, (not necessary differentiable) then, in Riemann–Liouville (ABR) sense, one can express the newly derivative knows as Atangana–Baleanu fractional derivative as is follows:

$$^{ABR}_{a}D_t^{\alpha}(h(t)) = \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_a^t h(\chi)E_{\alpha}\Big[-\alpha\frac{(t-\chi)^{\alpha}}{1-\alpha}\Big]d\chi.$$

Definition 5. For the Atangana–Baleanu fractional derivative one can express the fractional integral wit non local kernel as follows:

$$^{AB}_{a}I_t^{\alpha}(h(t)) = \frac{1-\alpha}{B(\alpha)}h(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_a^t h(y)(t-y)^{\alpha-1}dy.$$

We obtain the initial function and the classical integral respectively, when the fractional order is zero and 1.

Theorem 1 [27]**.** *On* $[a, b]$, *the following inequality holds for f when the function h is continuous on* $[a, b]$.

$$\|^{ABR}_{a}D_t^{\alpha}(h(t))\| < \frac{B(\alpha)}{1-\alpha}\|h(x)\|, \text{ where } \|h(x)\| = max_{a\leq x\leq b}|h(x)|. \quad (1)$$

Theorem 2 [27]**.** *Atangana–Baleanu and Atanagan–Balwanu-R derivatives both fulfill the condition Lipschitz given in the following:*

$$\|{}_a^{AB}D_t^{\alpha} h_1(t) - {}_a^{ABC}D_t^{\alpha} h_2(t)\| < K\|h_1(t) - h_2(t)\|, \quad (2)$$

also for ABR derivative we have

$$\|{}_a^{ABR}D_t^{\alpha} h_1(t) - {}_a^{ABR}D_t^{\alpha} h_2(t)\| < K\|h_1(t) - h_2(t)\|. \quad (3)$$

Theorem 3 *[27]***.** *For the FDE*

$${}_a^{AB}D_t^{\alpha} h(t) = s(t), \quad (4)$$

one can obtain a unique solution by using the inverse Laplace transform and the convolution result [27]:

$$h(t) = \frac{1-\alpha}{AB(\alpha)} s(t) + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)} \int_a^t s(\xi)(t-\xi)^{\alpha-1} d\xi. \quad (5)$$

3. Methods

In this section, we discuss the Lotka–Volterra model which use to describe the competition between commercial banks and rural banks in Indonesia. We then explain the genetic algorithm method to fit profit data of commercial banks and rural banks in Indonesia. We also give the annual profit data source from 2004 to 2014.

3.1. Lotka–Volterra model

Lotka–Volterra model approach to capturing the competition between the commercial bank and rural bank in Indonesia consists of two components (populations), namely profits of commercial bank (x_1) and profits of rural bank (x_2). The assumptions used to build the model are as follows

1. Population growth is assumed to grow logistically.
2. The annual profit of commercial banks and rural banks is limited.

It follows from the discussions above, the dynamics of competition between commercial banks and rural banks in Indonesia can be expressed in the following differential equations:

$$\begin{aligned} \frac{dx_1}{dt} &= R_1 x_1\left(1 - \frac{x_1}{K_1}\right) - \alpha_1 x_1 x_2, \\ \frac{dx_2}{dt} &= R_2 x_2\left(1 - \frac{x_2}{K_2}\right) - \alpha_2 x_1 x_2, \end{aligned} \quad (6)$$

with R_1 and R_2 are the growth rates of profit for commercial and rural banks respectively, α_1, α_2 are the coefficients of commercial and rural banks respectively, while K_1 and K_2 are the maximum profit of the commercial and rural banks respectively. Based on the model (6), the unknown parameter values are six parameters, namely R_1, K_1, R_2, K_2, α_1 and α_2. These parameters will be estimated in value using genetic algorithms.

3.2. Model with Atangana–Baleanu derivative

Researchers have suggested several mathematical tools called differential operators. Some have been known to depict processes following the power law. However, the one with the power law was not able to capture a dynamical system in different layers, for instance, a problem that follows at the same time power law and exponential decay law. The mathematical operator based on power law does not also capture in particular statistical setting like, for instance, the Gaussian distribution which is observed in many physical problems like the one under investigation. Thus some new differential operators have been suggested like the Caputo–Fabrizio derivative that is able to capture fading memory, and the Atangana–Baleanu fractional derivative that is able to capture a waiting time displaying power law and exponential decay. Additionally, this operator possesses some statistical setting as was revealed that the operator can describe physical problem presenting some Gaussian behavior. More importantly, the Atangana–Baleanu derivative can capture accurately the random walk and also the Brownian motion which is a great achievement in the field of the fractional differential operator. We, therefore, use this operator in this paper to include into the mathematical model the cross-over behavior in waiting time distribution and more precisely we give the model the property of queueing since the generalized Mittag-Leffler possesses it. Another important reason relies on the fact that, there is a beginning and an end to model, however, the power law kernel is equipped with the artificial singularity that renders each model singular even those with no sign of the singularity.

Therefore, based on these discussions, we feel that the newly derivative is appropriate to apply on our proposed problem (6). So, we reformulate the above model (6) by replacing classical integer order derivative by A–B fractional derivative of order α. The resulting Lotka–Volterra fractional model with A–B derivative is as follows:

$$\begin{aligned} {}_0^{AB}D_t^{\alpha} x_1 &= R_1 x_1\left(1 - \frac{x_1}{K_1}\right) - \alpha_1 x_1 x_2, \\ {}_0^{AB}D_t^{\alpha} x_2 &= R_2 x_2\left(1 - \frac{x_2}{K_2}\right) - \alpha_2 x_1 x_2. \end{aligned} \quad (7)$$

3.3. Model with Caputo derivative

We express the model (6) in Caputo derivative as given by,

$$\begin{aligned} {}^{C}D_t^{\alpha} x_1 &= R_1 x_1\left(1 - \frac{x_1}{K_1}\right) - \alpha_1 x_1 x_2, \\ {}^{C}D_t^{\alpha} x_2 &= R_2 x_2\left(1 - \frac{x_2}{K_2}\right) - \alpha_2 x_1 x_2. \end{aligned} \quad (8)$$

3.4. Data of profit commercial and rural bank

The Indonesia Banking Statistic published by the Financial Services Authority Republic of Indonesia is a publication media that provides data of Indonesia Banking [4]. The data used in the Indonesian Banking Statistics is derive from commercial bank monthly reports, and rural bank monthly reports. Based on Financial Services Authority, we use the annual profit data of commercial banks and rural banks in Indonesia from 2004 to 2014 to estimate parameters of the model (6) [4]. The complete data can be seen in Fig. 1.

3.5. Genetic algorithm estimation

Genetic Algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection [12]. Genes are basic instructions for building GA. Genes can represent a problem variable. Solutions are represented by chromosomes (individuals) [35]. The population is a collection of chromosomes. Chromosomes are formed in each generation and then evaluated using several measures of cost function. The cost function of a chromosome in GA is the value of the objective function of the phenotype. Selection is a process that randomly selects chromosomes from the population according to the value of the cost function. The higher cost function will give chromosomes a higher chance of being selected. The selection process selects two mains from the population for crossing. The purpose of selection is to choose good chromosomes in a population that are expected to have offspring who have higher fitness [35]. Crossover is the process of crossing between two parents to get a new chromosome that inherits the properties of the two parents, while mutation is a random process of changing some individual traits that produce new genetic structures [36].

The parameter estimation procedure for the model (6) using GA is as follows

a. Determine the maximum generation number (*ngen*), population size (*npop*), crossover probability (P_c), mutation

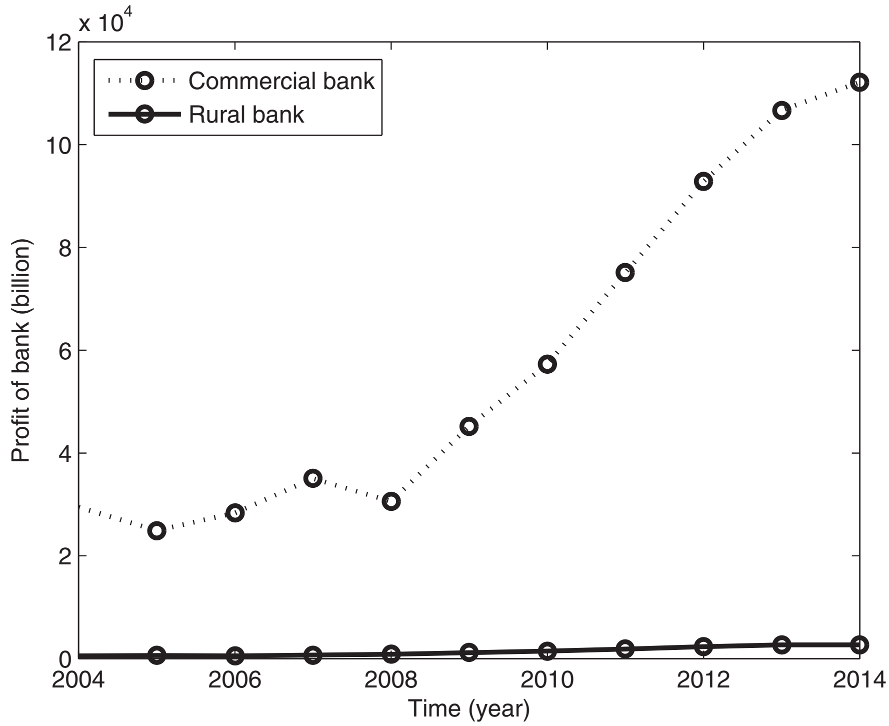

Fig. 1. Profit of commercial and rural bank.

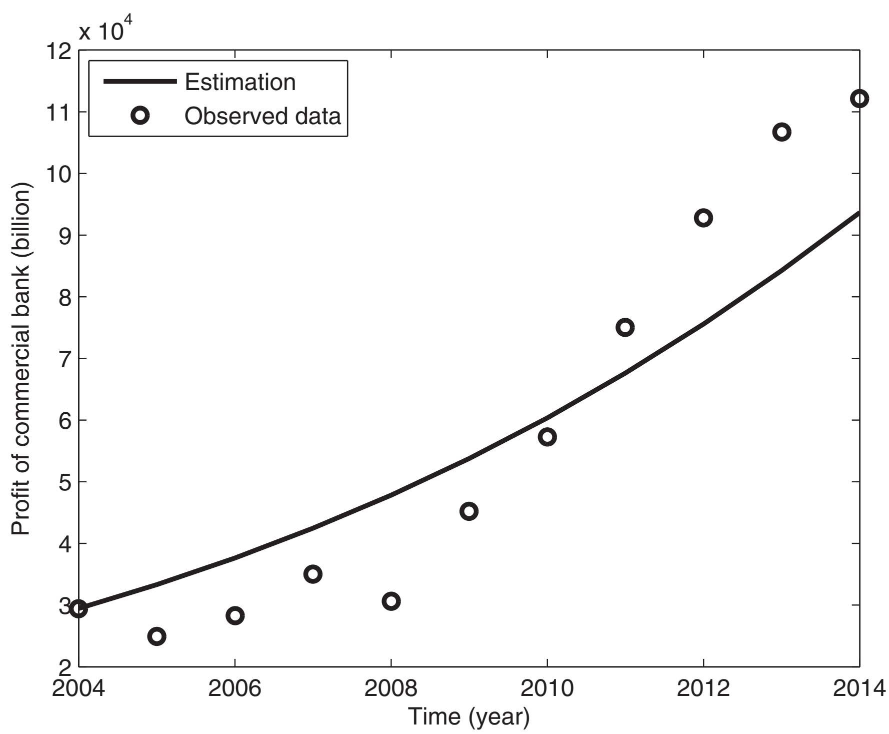

Fig. 2. Estimated profit of commercial bank to the data.

probability (P_m). Crossover probability is the proportion of the population in the current generation that is maintained in the next generation. The probability of crossover is chosen as 0.5.

b. Generate chromosome elements in intervals [0,1] as many parameters as estimated, and continue to be raised as much as *npop*. The chromosomes are formed, made as the initial population.

c. Evaluate the value of the objective function for each chromosome in the population. The value of the objective function used is to minimize

$$e = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\frac{\hat{x}_{1,i} - x_{1,i}}{x_{1,i}}\right| + \left|\frac{\hat{x}_{2,i} - x_{2,i}}{x_{2,i}}\right|\right), \quad (9)$$

with n is the number of data, $x_{1,i}$ and $x_{2,i}$ is the real data profit of commercial and rural bank respectively, while $\hat{x}_{1,i}$ and $\hat{x}_{2,i}$ is

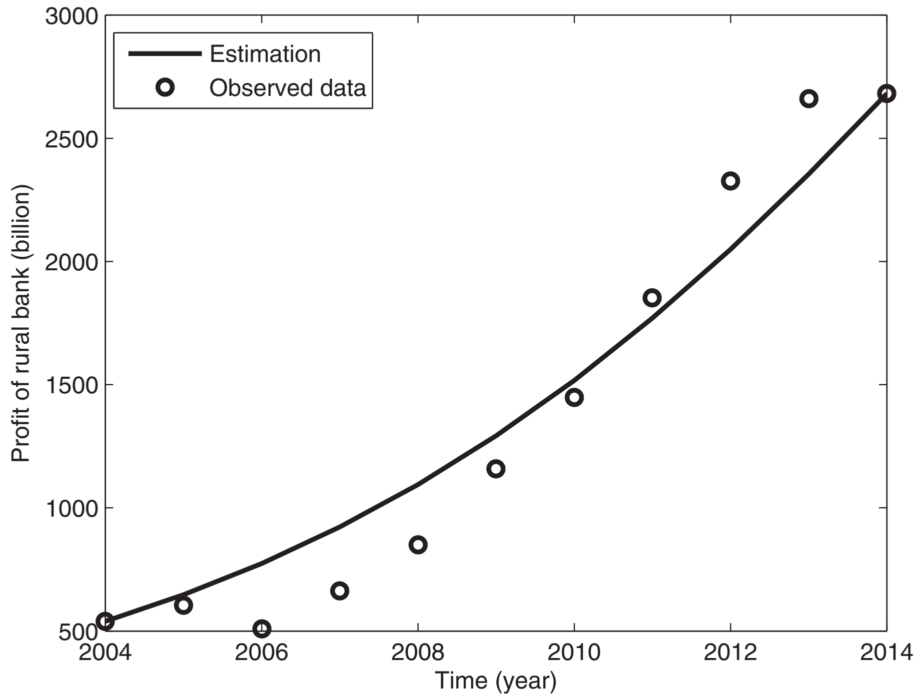

Fig. 3. Estimated profit of rural bank to the data.

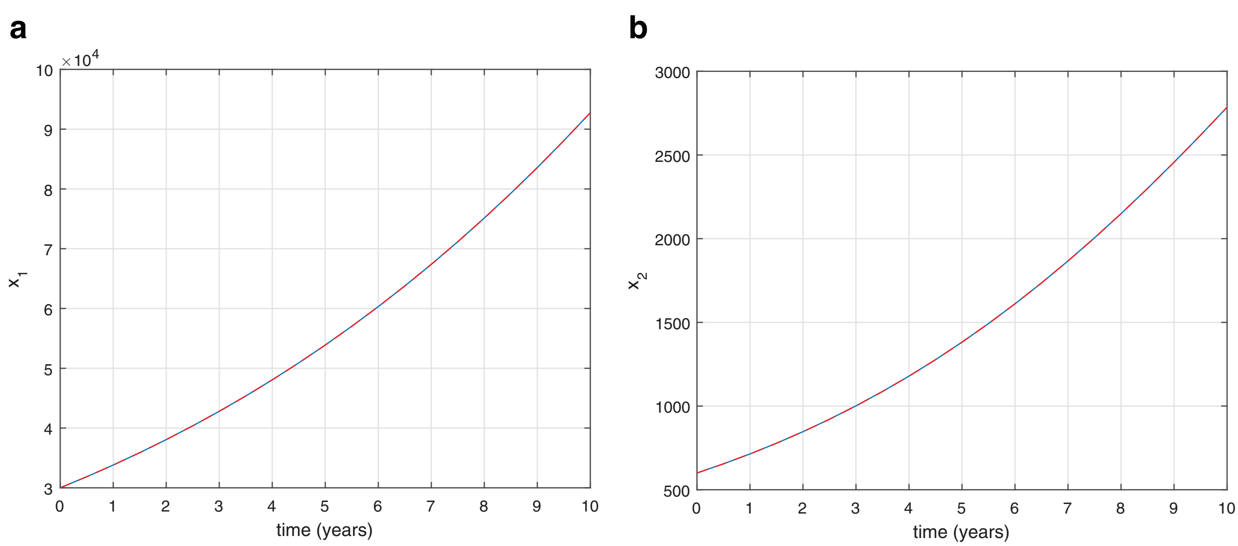

Fig. 4. The behavior of x_1 and x_2 for $\alpha = 1$.

the component of the solution of the differential Eq. (6) using the Runge–Kutta method.

d. Calculates the value of the objective function for each individual based on Eq. (9).
e. Select a number of individuals in the population to become individual parent sub-populations.
f. Choose a number of pairs obtained from individual parent sub-populations. The selection of pairs in this case uses the Roulette-Whell method.
g. Perform a crossover process by looking for a parent chromosome that produces random values between 0 and 1. Offspring can be generated from the parent chromosome.
h. Perform a random mutation process for a number of individuals for variable division (genes) in the population.
i. Stop the algorithm process after the maximum number of generations is reached. If not, go back to step d.

4. Parameter estimation of Lotka–Volterra model

The parameter estimation process in the mathematical model approach to the dynamics of competition between commercial banks and rural banks in Indonesia consists of six genes. The first gene represents the parameter R_1, the second gene represents the parameter α_1, the third gene represents the parameter K_1, the fourth gene represents the parameter R_2, the fifth gene represents the parameter α_2 and the sixth gene represents the parameter K_2. The parameter values is selected if the mean of relative error (9) has minimum value. Here, the mean of relative error in Eq. (9) is chosen as the cost function in the implementation of GA.

In this paper, *npop* is used for 100 populations to get results that are close to the optimum results. Furthermore, for the probability of crossing probability, *xrate*, and the probability of mutation is not specifically determined. For practical purposes, the

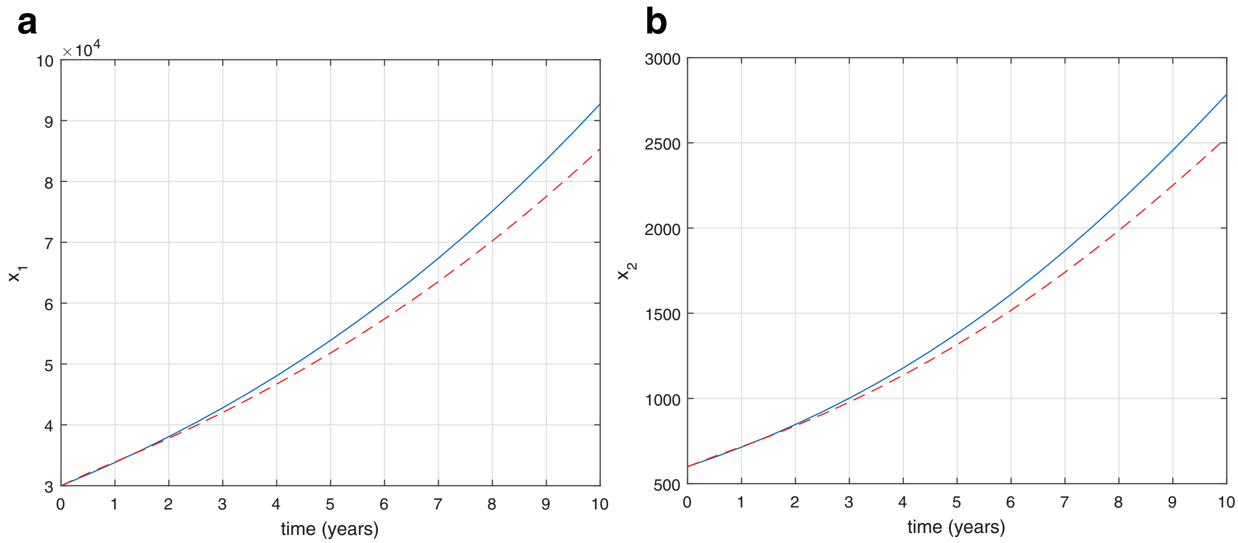

Fig. 5. The behavior of x_1 and x_2 for $\alpha = 0.95$.

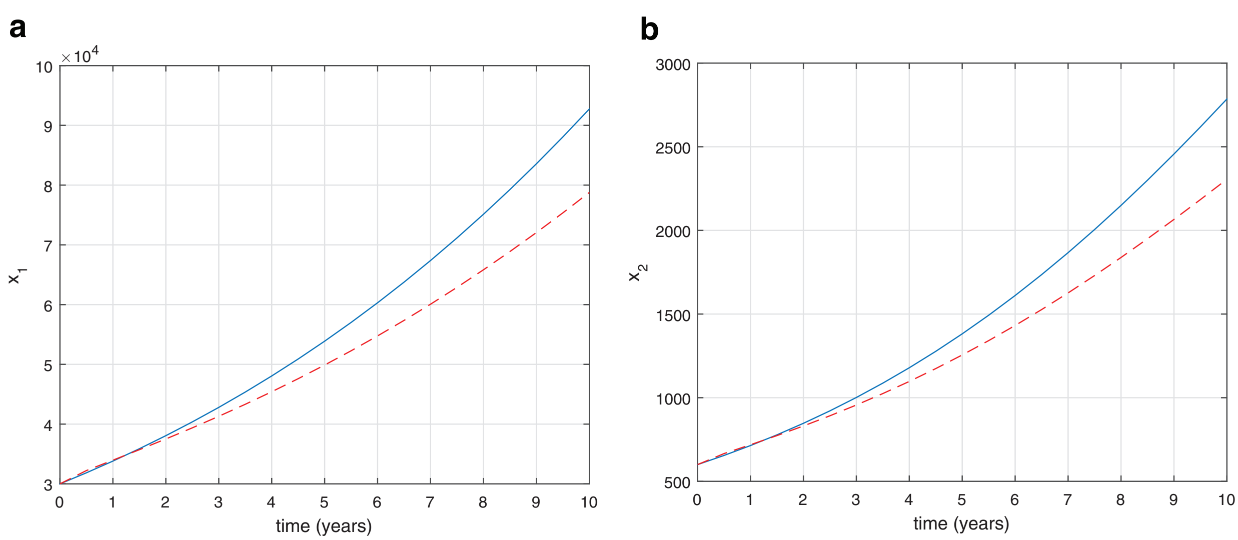

Fig. 6. The behavior of x_1 and x_2 for $\alpha = 0.90$.

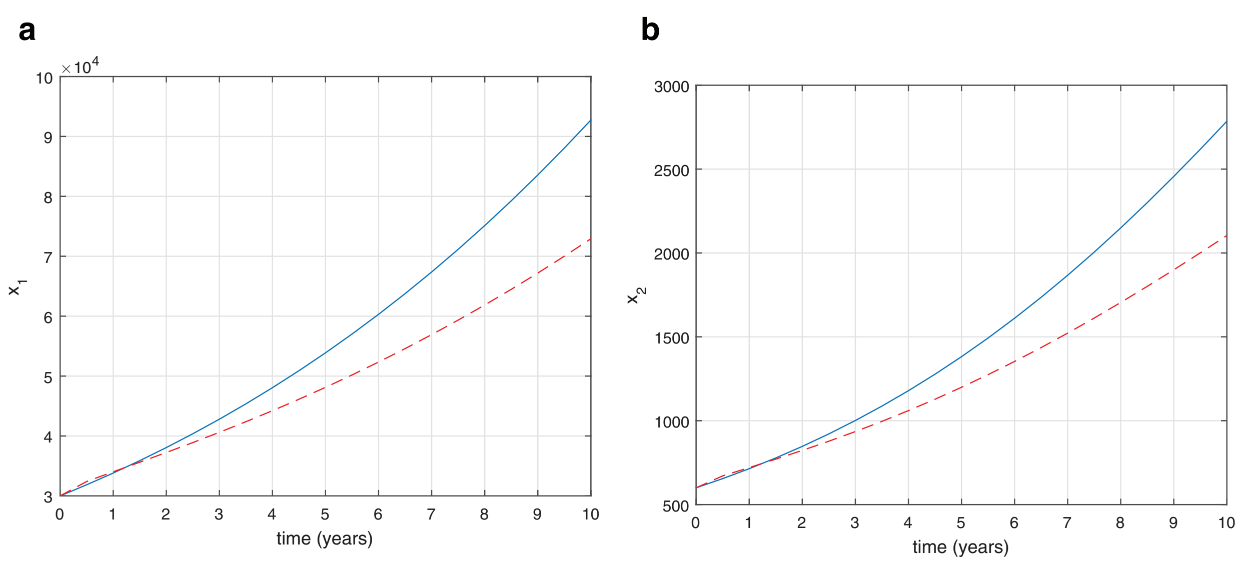

Fig. 7. The behavior of x_1 and x_2 for $\alpha = 0.85$.

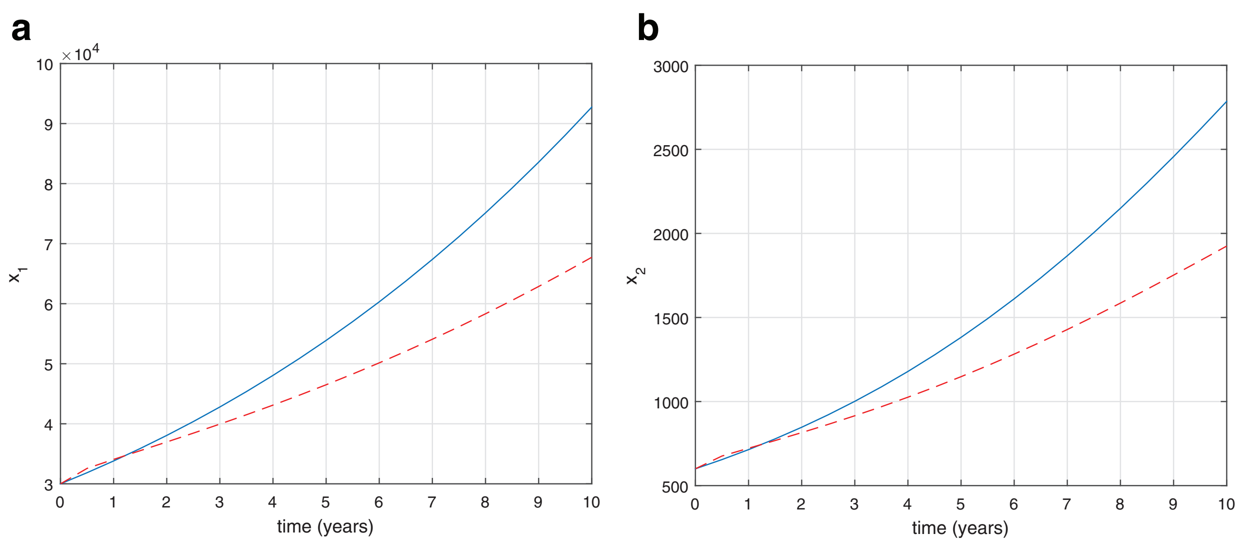

Fig. 8. The behavior of x_1 and x_2 for $\alpha = 0.80$.

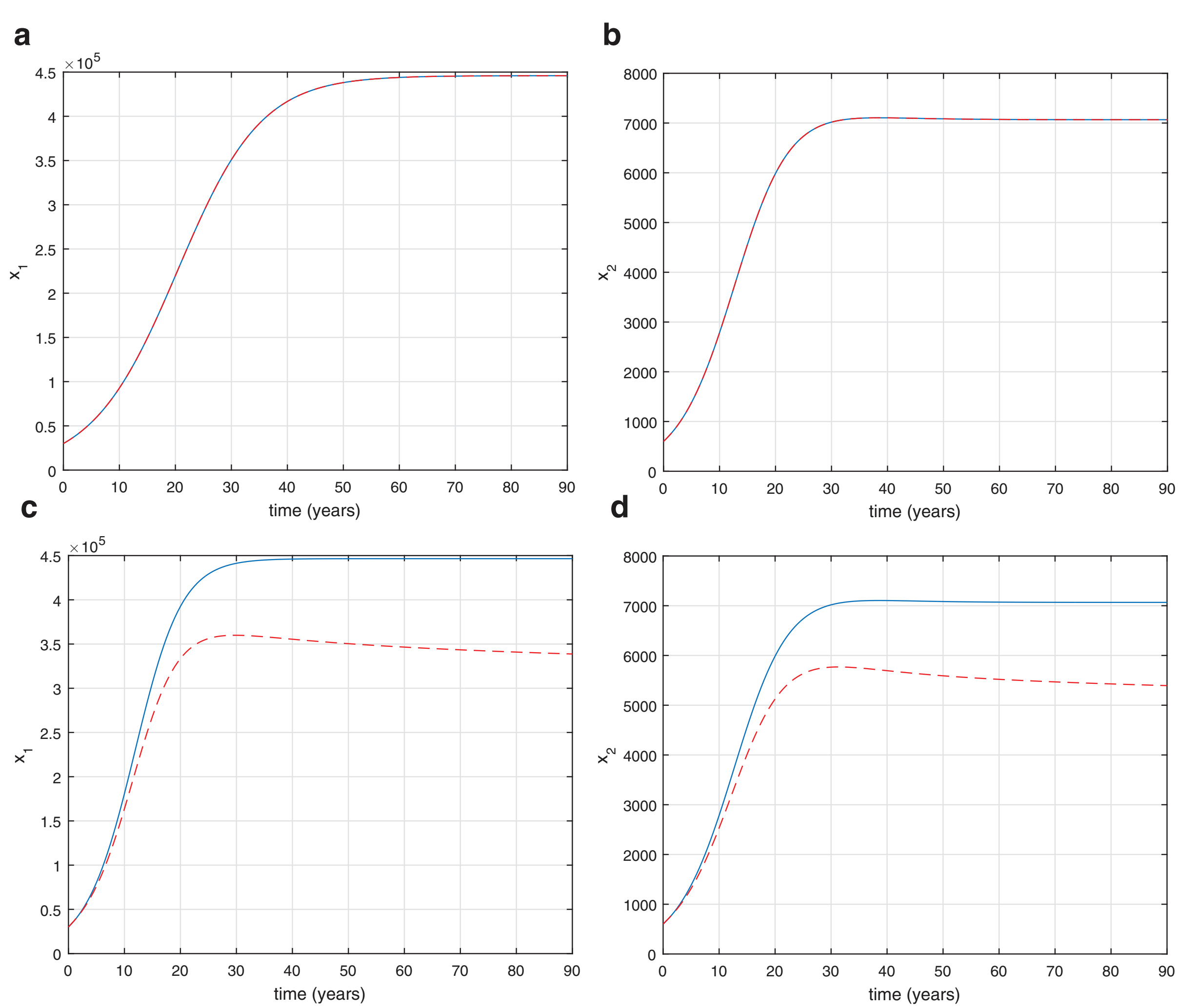

Fig. 9. The behavior of x_1 and x_2 for $\alpha = 1, 0.95$ when time is up to 90 years.

probability of crossing is selected as 0.5. Because the value of *xrate* corresponds to the probability of crossing, the parameter value *xrate* is used as 0.5. While the probability of mutation is selected by using five kinds of values, namely 0.025, 0.05, 0.125, 0.25 and 0.5 in order to obtain optimum results. Each value of the five mutation probability values will be carried out seven times. In this study, the number of generations is 100.

The parameter estimation result used is the median of seven attempts at each mutation probability. This is because the median value of the parameter estimation results is not affected by

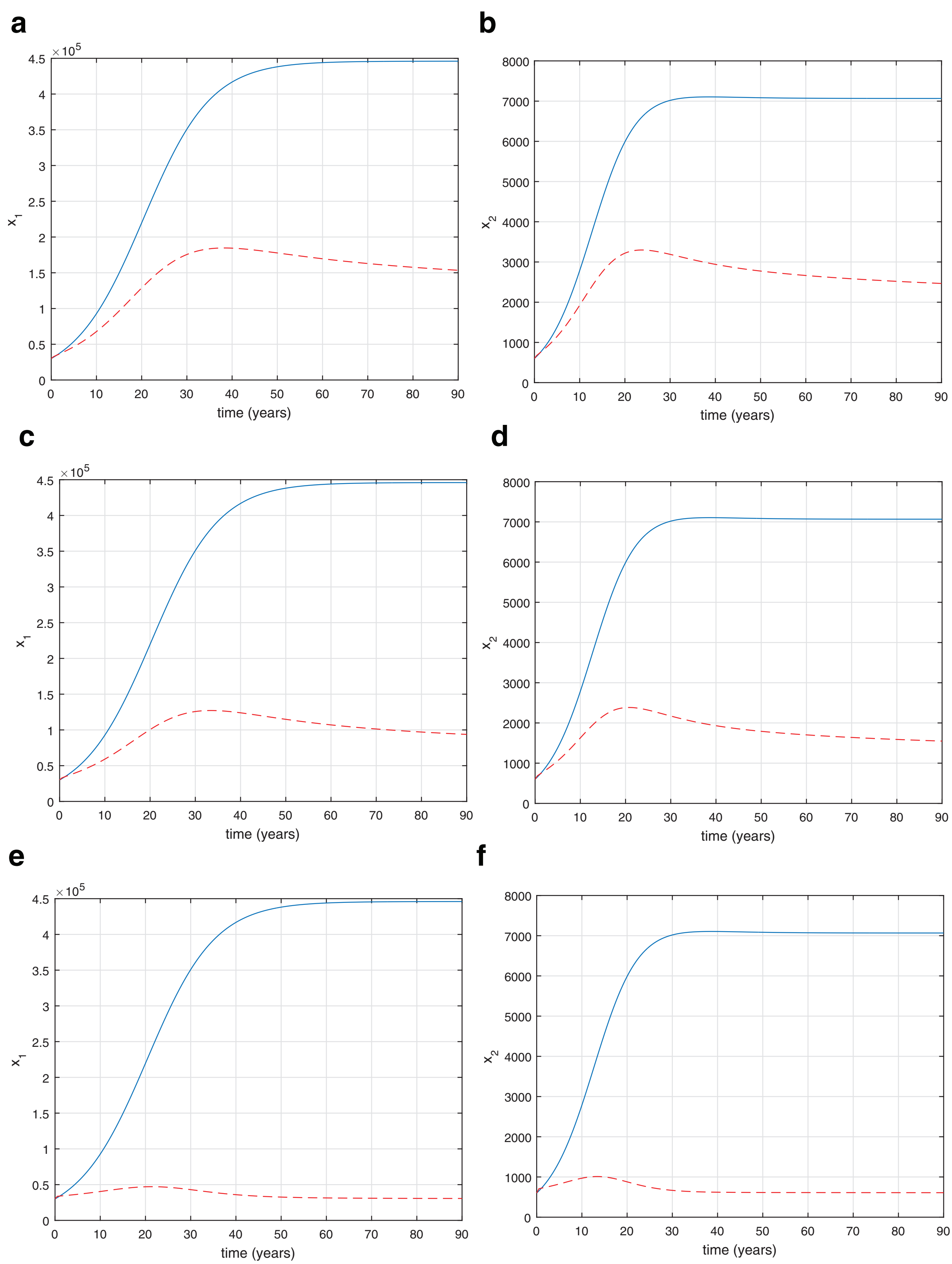

Fig. 10. The behavior of x_1 and x_2 for $\alpha = 0.8, 0.7, 0.2$ when time is up to 90 years.

extreme values. The median parameter estimation results are presented in Table 1.

Based on the Table 1, it can be seen that for the probability of mutation 0.025 and 0.05 it has a small error value compared to the error value at the probability of mutations of 0.125, 0.25 and 0.5. This shows that when the mutation probability is less than or equal to 0.05, the parameter estimation results have good results because they have a small error. Whereas if the mutation probability is too large, it can damage a good chromosome. This is seen when the mutation probability value is more than 0.05, the error is greater.

As shown in the Table 1, the smallest error is 0.1847 with the parameter values as follows $R_1 = 0.13243, \alpha_1 = 5.6190 \times 10^{-8}, K_1 = 447318.198, R_2 = 0.19901, \alpha_2 = 2804 \times 10^{-8}$ and $K_2 = 7540.6219$. This value is achieved when the population is 100 and the probability of mutation is 0.05.

Fig. 2 displays the estimated profit of commercial bank to the data. From Fig. 2, the real data and the estimated profit is reported rather a difference. This difference is caused by economic factors such as the increase in oil prices that occurred from 2005 to 2008 as well as other economic factors that could affect the amount of commercial bank profits. However, the results of the simulations show that the profit of the commercial bank has increased over time. This result is not much different from the increase in commercial bank profits in real data.

Fig. 3 presents the estimated profit of rural bank to the data. Looking at Fig. 3, it is apparent that the profit of rural bank between estimation results and real data is not much different. The

Table 1
Simulation result of seven experiments with different P_m.

P_m	R_1	α_1	K_1	R_2	α_2	K_2	Error
0.025	0.1401	6.9476×10^{-8}	719986.6246	0.19682	4.55×10^{-8}	6581.2022	0.18475
0.05	0.13243	5.6190×10^{-8}	447318.198	0.19901	2.804×10^{-8}	7540.6219	0.1847
0.125	0.1317	6.0993×10^{-8}	432504.4277	0.2099	6.8338×10^{-8}	4759.2183	0.1945
0.25	0.0971	1.4554×10^{-8}	221746.734	0.16717	2.4071×10^{-8}	8611.3981	0.20925
0.5	0.21981	2.2437×10^{-8}	168244.5208	0.17407	1.2171×10^{-8}	6080.5399	0.22968

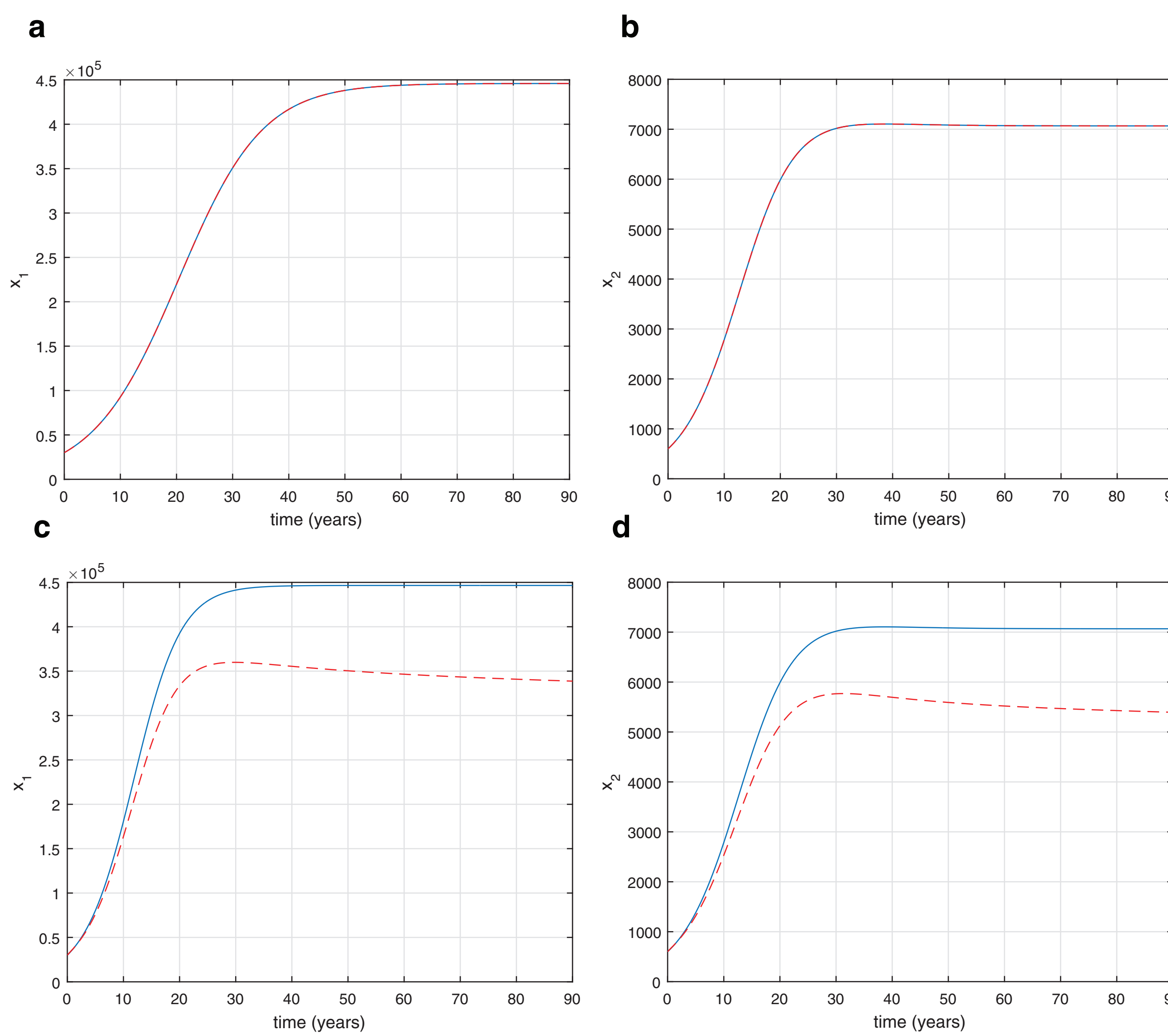

Fig. 11. The behavior of x_1 and x_2 for $\alpha = 1, 0.95$ when time is up to 90 years and the parameters $R_1 = 0.23243$ and $R_2 = 0.29901$.

estimation results show that rural bank profit has increased every year which is not much different from the real data.

5. Numerical results

In this section we find the numerical results of the model (7) by the technique given in [29,37] which is efficiently applied to many real life problems that is modeled in the sense of Atangana–Baleanu fractional integral operator. To construct the desired numerical scheme for the fractional model (7) with new derivative we proceed as below.

For simplicity, we express the model (7) given by the form

$$\begin{aligned} {}_0^{AB}D_t^{\alpha} x_1 &= \mathcal{K}_1(t, x_1, x_2), \\ {}_0^{AB}D_t^{\alpha} x_2 &= \mathcal{K}_2(t, x_1, x_2). \end{aligned} \tag{10}$$

System (7) can be converted in the following form using fundamental theorem of integration.

$$\begin{aligned} x_1(t) - x_1(0) &= \frac{(1-\alpha)}{AB(\alpha)}\mathcal{K}_1(t, x_1) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)}\int_0^t \mathcal{K}_1(\xi, x_1)(t-\xi)^{\alpha-1}d\xi, \\ x_2(t) - x_2(0) &= \frac{(1-\alpha)}{AB(\alpha)}\mathcal{K}_2(t, x_2) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)}\int_0^t \mathcal{K}_1(\xi, x_2)(t-\xi)^{\alpha-1}d\xi. \end{aligned} \tag{11}$$

For $t = t_{n+1},\ n = 0, 1, 2, \ldots,$ we obtain from Eq. (11)

$$\begin{aligned} x_1(t_{n+1}) - x_1(t_0) &= \frac{1-\alpha}{AB(\alpha)}\mathcal{K}_1(t_n, x_1) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}} \mathcal{K}_1(\tau, x_1)(t_{n+1}-\tau)^{\alpha-1}d\tau, \\ x_2(t_{n+1}) - x_2(t_0) &= \frac{1-\alpha}{AB(\alpha)}\mathcal{K}_2(t_n, x_2) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}} \mathcal{K}_2(\tau, x_2)(t_{n+1}-\tau)^{\alpha-1}d\tau. \end{aligned} \tag{12}$$

Approximating the integral in Eq. (12), using two point interpolation polynomial and then simplifying we finally get the following iterative solution for the model given (7). In similar way for the rest of equations of system (7), we obtained the recursive

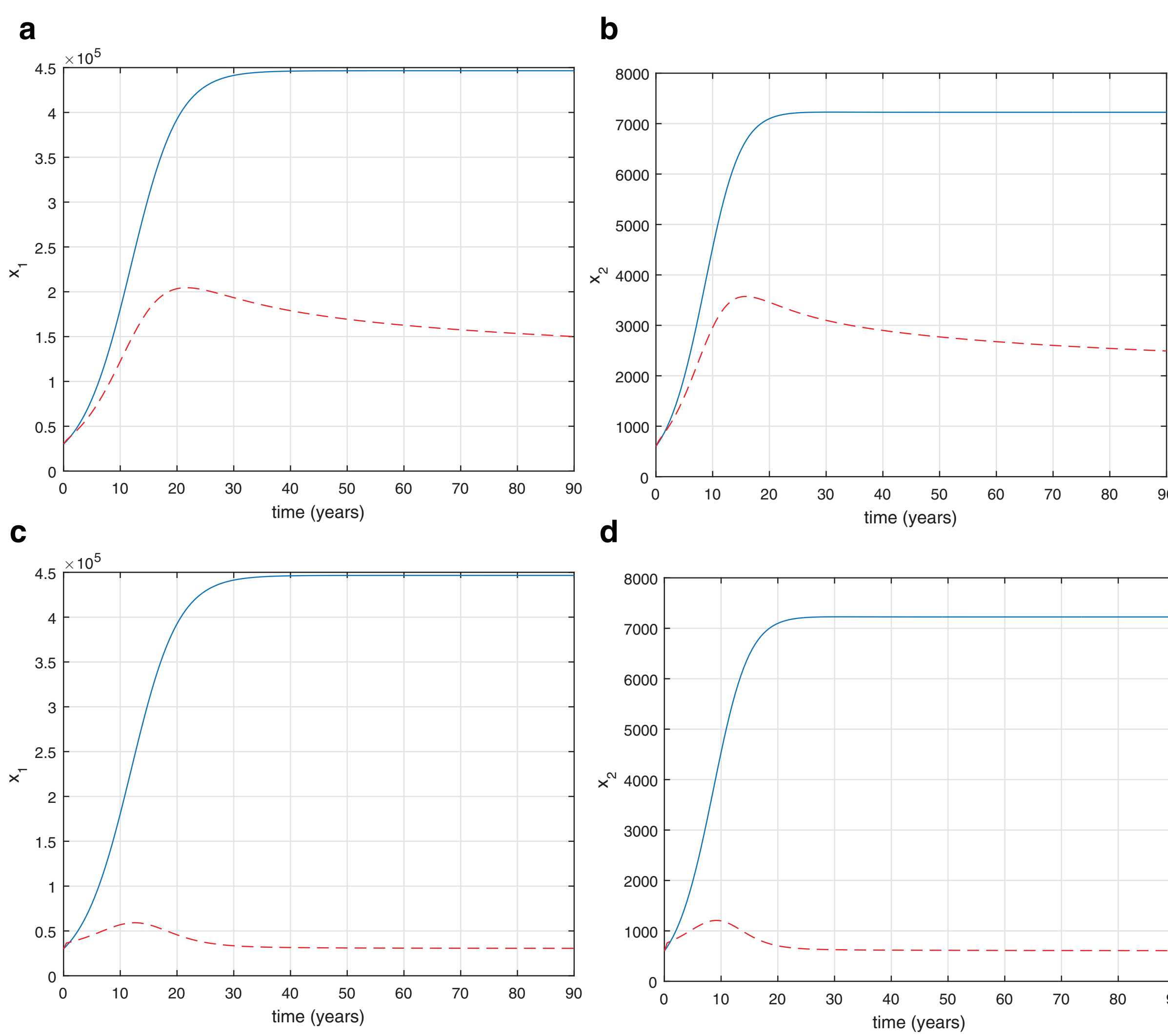

Fig. 12. The behavior of x_1 and x_2 for $\alpha = 0.7, 0.2$ when time is up to 90 years and the parameters $R_1 = 0.23243$ and $R_2 = 0.29901$.

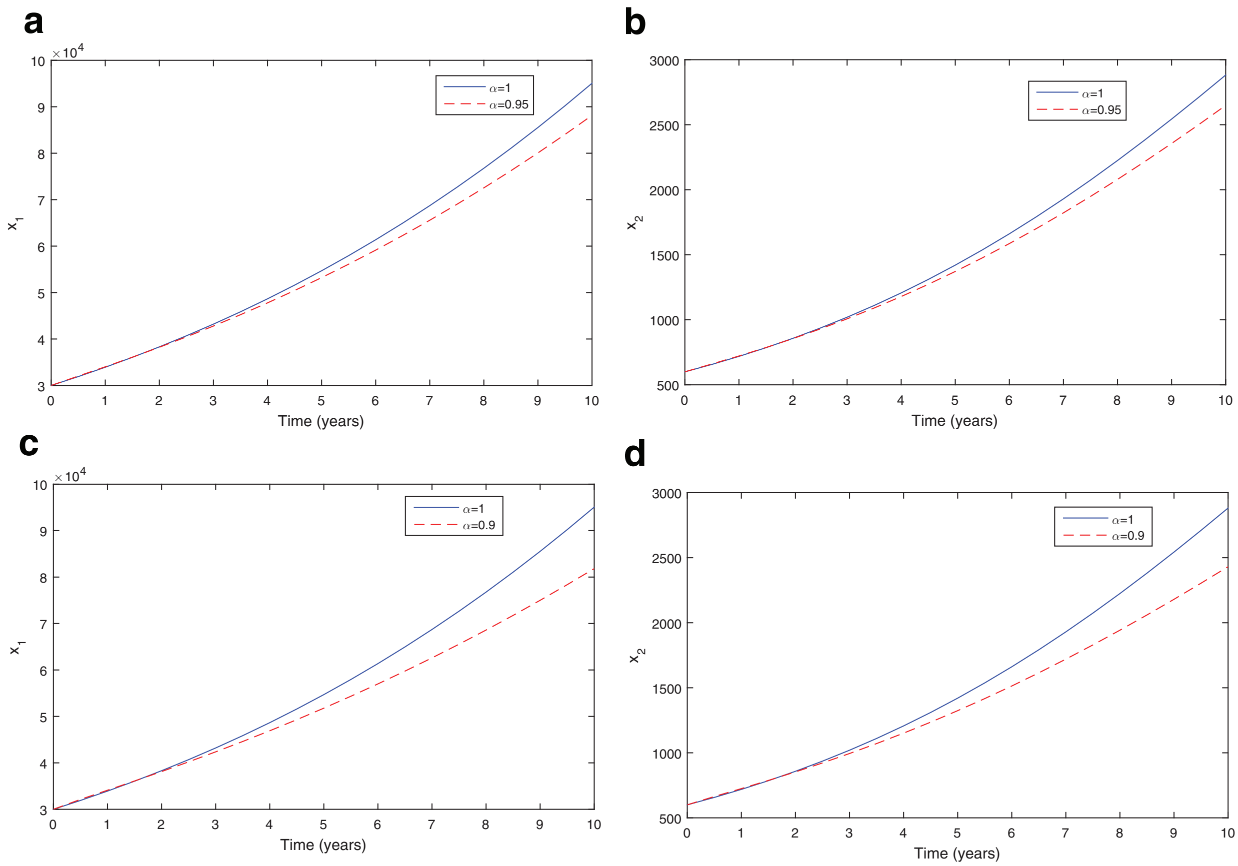

Fig. 13. Graphical results for the Caputo model when $\alpha = 1, 0.95, 0.9,$ for x_1 and x_2.

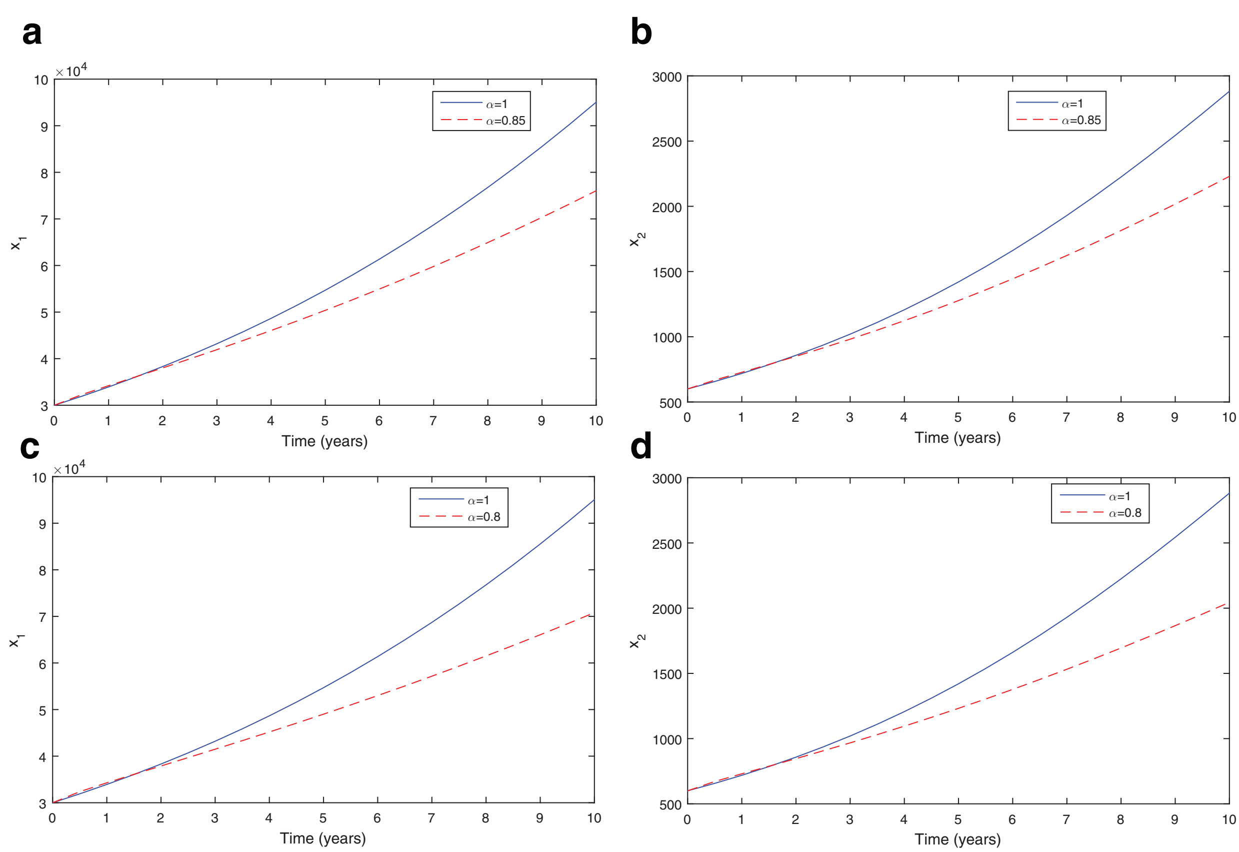

Fig. 14. Graphical results for the Caputo model when $\alpha = 1, 0.85, 0.8,$ for x_1 and x_2.

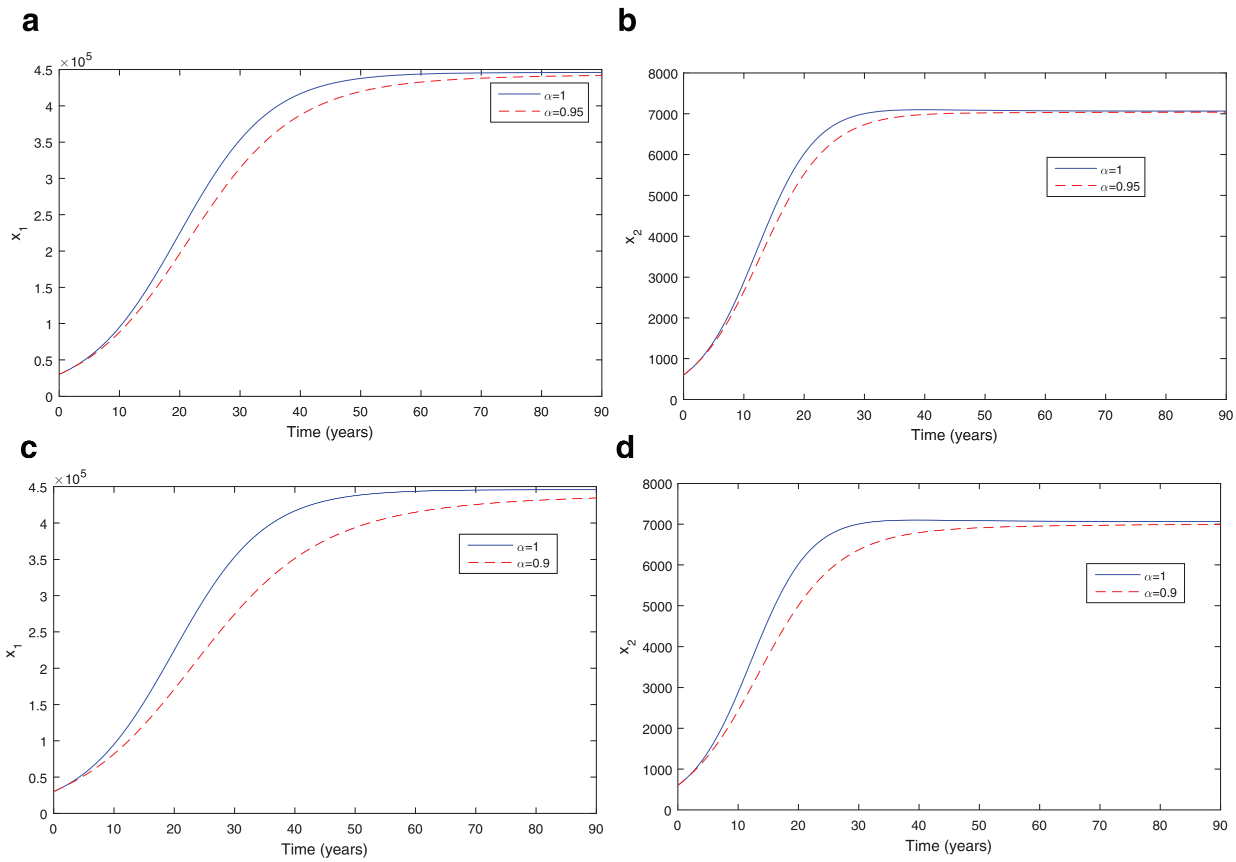

Fig. 15. Graphical results for the Caputo model for long time when $\alpha = 1, 0.95, 0.9$, for x_1 and x_2.

formulae as below

$$\begin{aligned} x_1(t_{n+1}) &= x_1(t_0) + \frac{1-\alpha}{AB(\alpha)}\mathcal{K}_1(t_n, x_1) + \frac{\alpha}{AB(\alpha)} \\ &\times \sum_{k=0}^{n}\Bigg(\frac{h^\alpha \mathcal{K}_1(t_k, x_1)}{\Gamma(\alpha+2)}((n+1-k)^\alpha(n-k+2+\alpha) \\ &- (n-k)^\alpha(n-k+2+2\alpha)) \\ &- \frac{h^\alpha \mathcal{K}_1(t_{k-1}, x_1)}{\Gamma(\alpha+2)}((n+1-k)^{\alpha+1} \\ &- (n-k)^\alpha(n-k+1+\alpha))\Bigg), \end{aligned} \tag{13}$$

$$\begin{aligned} x_2(t_{n+1}) &= x_2(t_0) + \frac{1-\alpha}{AB(\alpha)}\mathcal{K}_2(t_n, x_2) + \frac{\alpha}{AB(\alpha)} \\ &\times \sum_{k=0}^{n}\Bigg(\frac{h^\alpha \mathcal{K}_2(t_k, x_2)}{\Gamma(\alpha+2)}((n+1-k)^\alpha(n-k+2+\alpha) \\ &- (n-k)^\alpha(n-k+2+2\alpha)) \\ &- \frac{h^\alpha \mathcal{K}_2(t_{k-1}, x_2)}{\Gamma(\alpha+2)}((n+1-k)^{\alpha+1} \\ &- (n-k)^\alpha(n-k+1+\alpha))\Bigg). \end{aligned} \tag{14}$$

After the successful of the numerical scheme on the fractional model (7) as explained above, we find the graphical results of the model (7), by considering and assigning values to the fractional parameter $\alpha \in [0, 1]$, and model relevant parameters. The parameters used in numerical simulations are estimated from real data and are $\alpha_1 = 5.6190 \times 10^{-8}$, $\alpha_2 = 2.8040 \times 10^{-8}$, $K_1 = 447318.198$, $K_2 = 7540.6219$, $R_1 = 0.13243$, $R_2 = 0.19901$ and initial values are $x_1 = 3 \times 10^4$ and $x_2 = 600$.

The graphical results are obtained by using different values assigned to the fractional order parameter α, see Figs. 4–8 and their subgraphs. In each figure, the blue solid curve represents the graph of x_1 and x_2 for $\alpha = 1$ and then the dotted red curve represent the behavior of x_1 and x_2 for $\alpha = 0.95, 0.90, 0.85, 0.80$ respectively. Figs. 9 and 10 is the simulation curve of both the variables for a long time. One can see in Figs. 9 and 10 that both the commercial and rural bank can reach to maximum level by the chosen maximum profit rates parameters K_1 an K_2. The graphical results given in Figs. 9 and 10 initially plotted for the integer case, then for $\alpha = 0.95, 0.8, 0.7, 0.2$. One can see upon reaching by obtaining the maximal profit the fractional order parameter efficiently reduce the curve for both the banks. Further Figs. 11 and 12 show the behavior of the bank profit for the long term level and the early profit behavior for the long term level by choosing $R_1 = 0.23243$ and $R_2 = 0.29901$. The values of $\alpha = 1, 0.95$ is plotted in Fig. 11 and the values of $\alpha = 0.7, 0.2$ is given in Fig. 12.

The graphical results for the long period of time with different profit rate is shown and the early profit with fractional order parameter is also presented. It is upon the bank that it may decide different policy to achieve the maximal profit in the target years. This model is very suitable and could be very useful for the early prediction of the maximal profit required by the bank in any country.

Moreover, we proposed the model in Caputo derivative given by (8) and obtained the graphical results are for various values of the fractional parameter α depicted in Figs. 13–16 while the comparison of Caputo and Atangana–Baleanu derivative models are depicted in Figs. 17 and 18. In Fig. 13–16 we considered the fractional order parameter $\alpha = 1, 0.95, 0.9, 0.85, 0.8$ and presented graphical results for the model parameters for small and long time. The comparison of both the operators for a long time is presented by

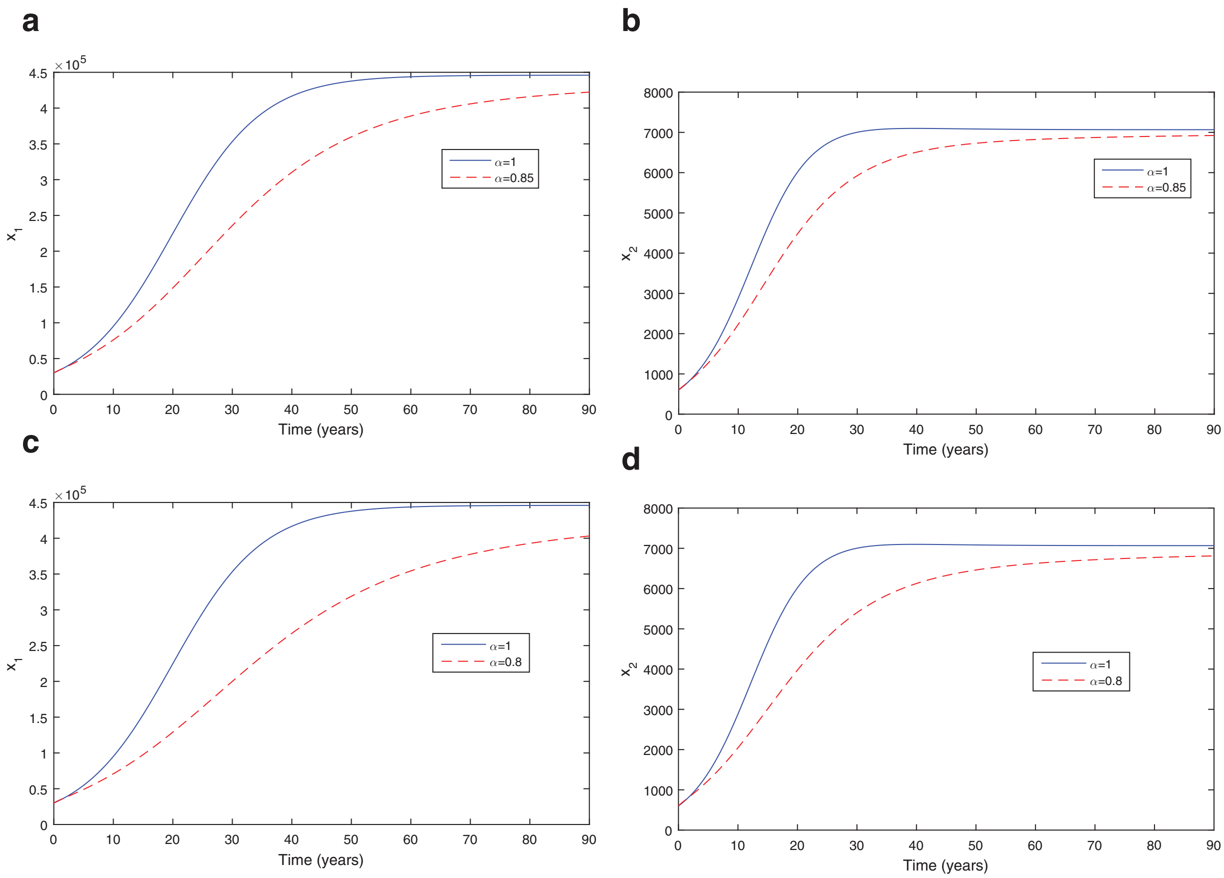

Fig. 16. Graphical results for the Caputo model for long time when $\alpha = 1, 0.85, 0.8,$ for x_1 and x_2.

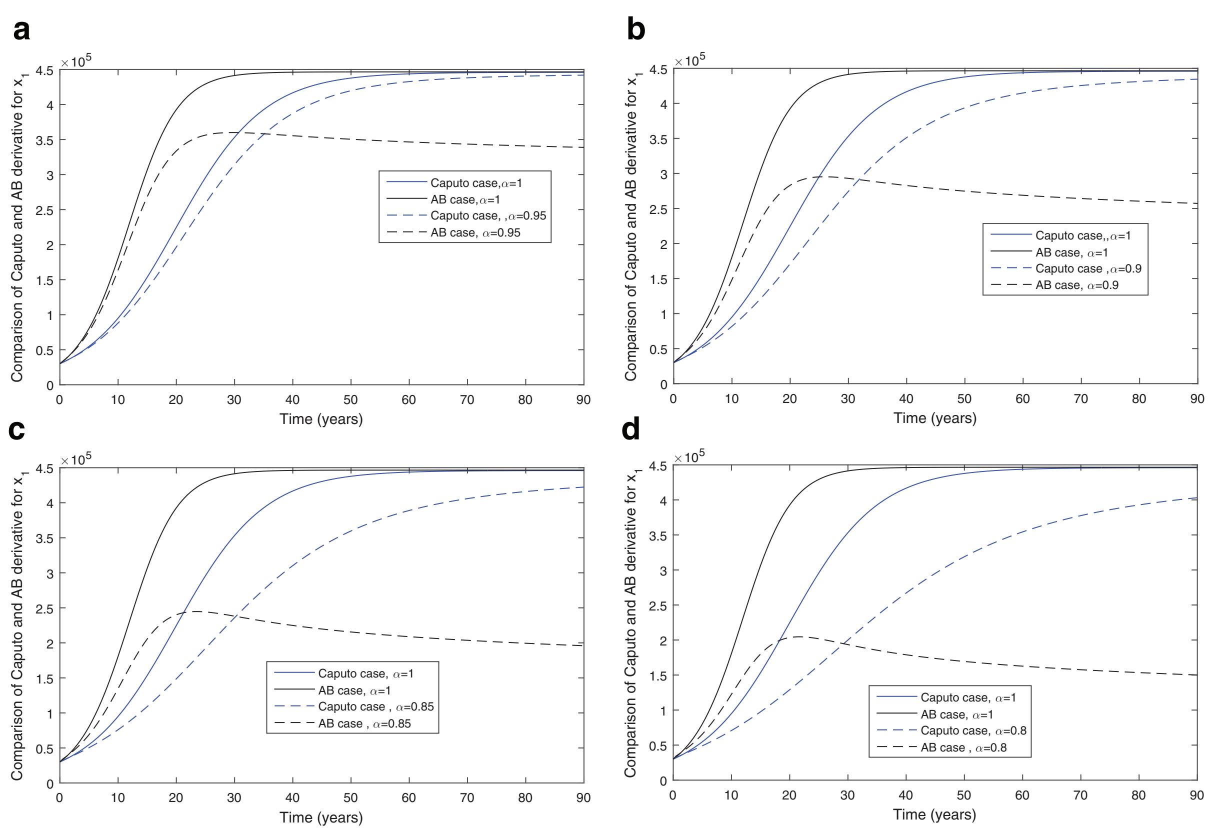

Fig. 17. Comparison results of Caputo and AB derivative for x_1 when $\alpha = 1, 0.95, 0.9, 0.85, 0.8$.

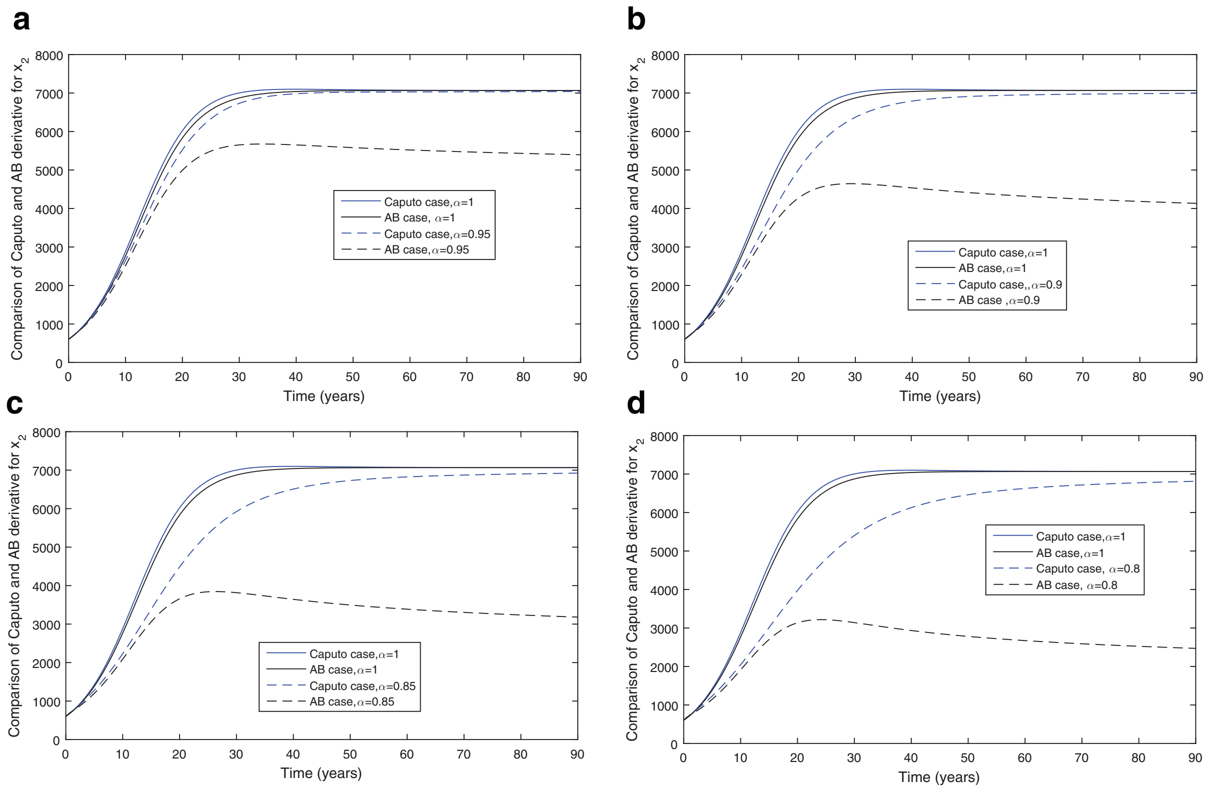

Fig. 18. Comparison results of Caputo and AB derivative for x_2 when $\alpha = 1, 0.95, 0.9, 0.85, 0.8$.

considered the fractional parameter $\alpha = 1, 0.95, 0.9, 0.85, 0.8$. These two operators used in the numerical simulations provided useful results for proposed model.

6. Conclusion

We presented a mathematical model for the dynamics of competition between commercial and rural banks in Indonesia. The parameter estimation of the Lotka–Volterra competition model was performed using the genetic algorithm method. The corresponding parameters were estimated based on annual profit data of commercial and rural banks in Indonesia. The estimation results capable to predict the profit of commercial and rural banks every year which is not much different from the real data. Thus, the competition model between commercial and rural banks in Indonesia was investigated in the fractional sense. The fractional model was explored trough the Atangana and Baleanu and Caputo fractional derivative. The numerical simulation of the fractional model was conducted using the Adams–Bashforth scheme while the Caputo derivative numerical results are obtained by using the predictor corrector method. We performed many simulations results for the obtained estimated parameters which are the ballistics and give realistic information about the bank current and future maximal profit. Comparison of Atangana–Baleanu and Caputo derivative in the form of graphical results for various values of the fractional order parameters are presented. Both the rural and commercial banks modeled equations for the fitted parameters are plotted and their effect on fractional parameter was observed. Then, for any bank it is the desire to make its profit more and more which could help to reduce the poverty and inflation in the market. Therefore, our simulation results shows the long term behavior up to 90 years the maximal profit can be achieved by both the banks. Further, we explored numerical results for the early maximal profit for both the banks that he can achieve if some proper arrangement and policy are determined. The long term behavior of the model with this new Atangana–Baleanu derivative could be more useful that it gives information for the maximal profit at any instant of the long term level. It is observed that both the operators used for the model is useful and provides useful information at each point of interest of fractional order parameter α.

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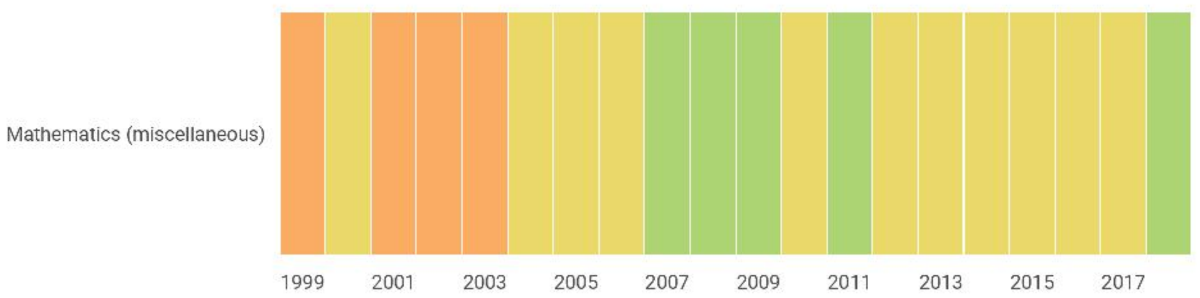

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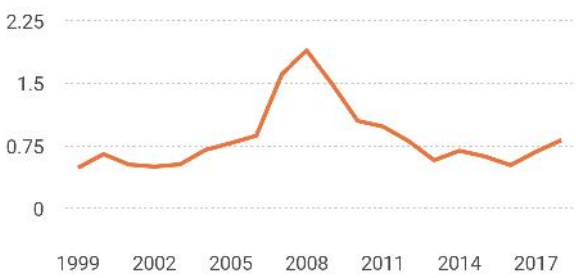

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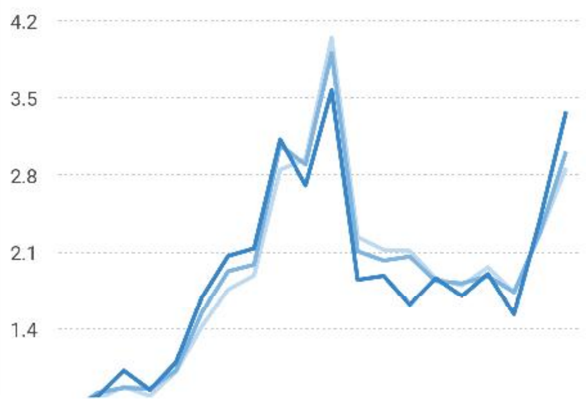

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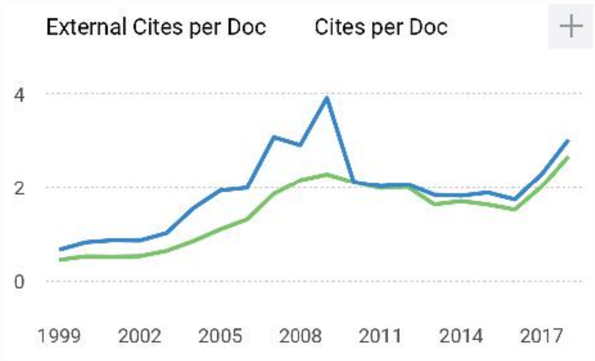

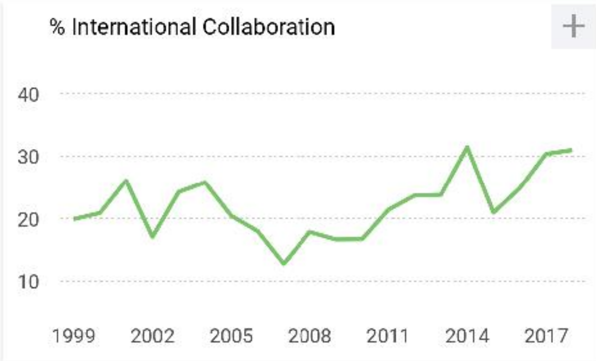

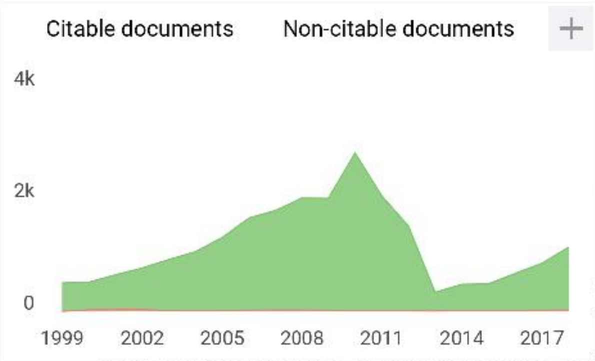

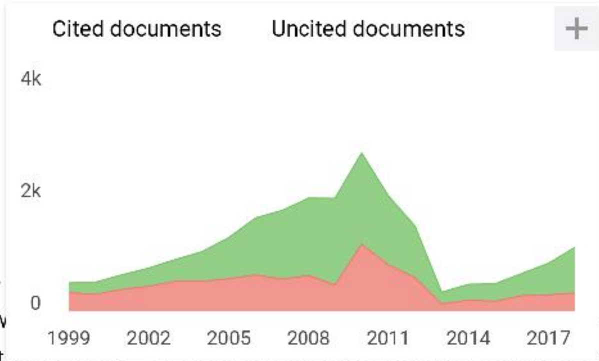

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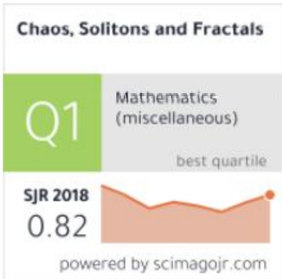

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