

Extension of Wolfe Method for Solving Quadratic Programming with Interval Coefficients

Syaripuddin, Herry Suprajitno, and Fatmawati

<https://doi.org/10.1155/2017/9037857>

Volume 2017



also developed by scimago:



SCIMAGO INSTITUTIONS RANKINGS

SJR

Scimago Journal & Country Rank

Enter Journal Title, ISSN or Publisher Name

[Home](#)[Journal Rankings](#)[Country Rankings](#)[Viz Tools](#)[Help](#)[About Us](#)

Journal of Applied Mathematics

Country [Egypt](#) - [SJR Ranking of Egypt](#)**Subject Area and Category** [Mathematics](#)
[Applied Mathematics](#)**Publisher** [Hindawi Publishing Corporation](#)**Publication type** Journals**ISSN** 16870042, 1110757X**Coverage** 2001-ongoing**Scope** Journal of Applied Mathematics is a refereed journal devoted to the publication of original research papers and review articles in all areas of applied, computational, and industrial mathematics.[Homepage](#)[How to publish in this journal](#)[Contact](#)[Join the conversation about this journal](#)

33

H Index

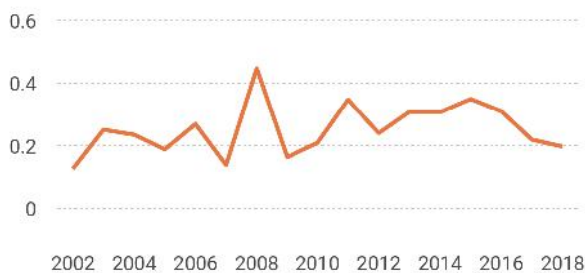
Quartiles



Applied Mathematics



SJR



Citations per document



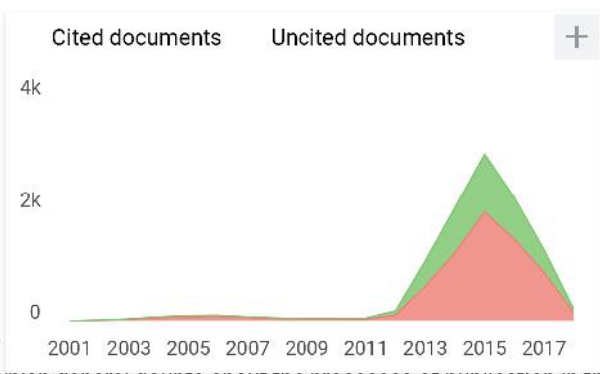
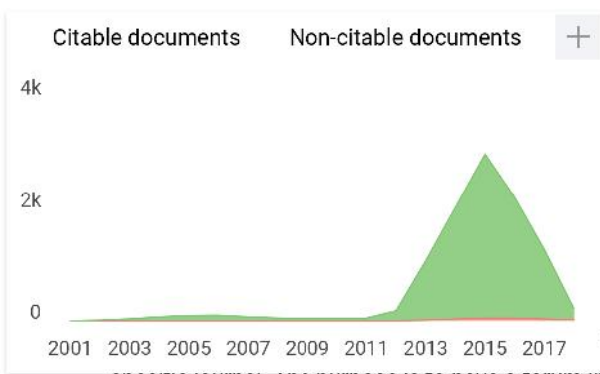
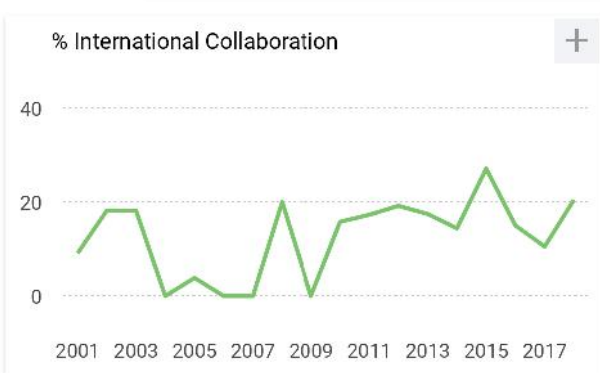
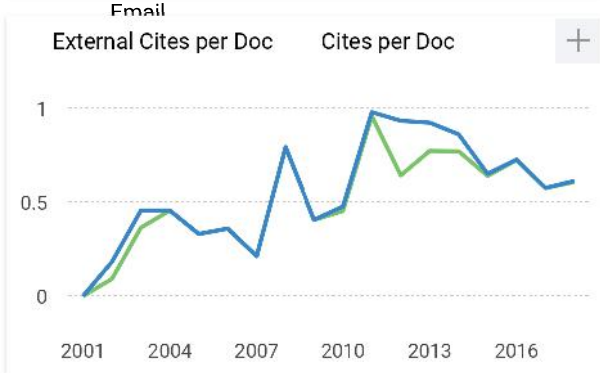
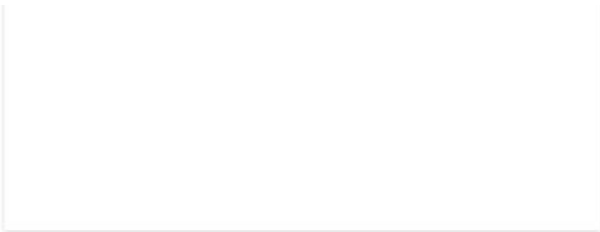
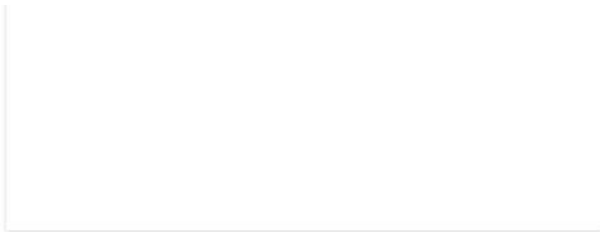
Total Cites

Self-Cites



1.8k





specific journal. The purpose is to have a forum in which general doubts about the processes of publication in the channels with your editor.

Journal of Applied Mathematics

Q4 Applied Mathematics best quartile

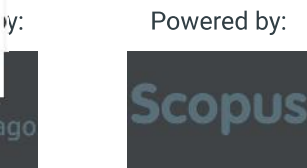
SJR 2018 0.2

powered by scimagojr.com

← Show this widget in your own website

Just copy the code below and paste within your HTML code:

`<a href="https://www.scimagojr.com/journalsearch.php?q=144917&tip=sid&cl..."`



Follow us on @ScimagoJR

Scimago Lab, Copyright 2007-2019. Data Source: Scopus®





Hindawi

Journal of Applied Mathematics

Journal Menu

[About this Journal](#)
[Abstracting and Indexing](#)
[Aims and Scope](#)
[Article Processing Charges](#)
[Bibliographic Information](#)
[Editorial Board](#)
[Editorial Workflow](#)
[Publication Ethics](#)
[Reviewer Resources](#)
[Subscription Information](#)
[Table of Contents](#)

Special Issues Menu

[Annual Issues](#)
[Open Special Issues](#)
[Published Special Issues](#)
[Special Issue Resources](#)

[Subscribe to
Table of Contents Alerts](#)

Abstracting and Indexing

The following is a list of the Abstracting and Indexing databases that cover Journal of Applied Mathematics published by Hindawi. For more information on how Hindawi makes your published work discoverable click [here](#).

- ▶ Academic OneFile
- ▶ Academic Search Alumni Edition
- ▶ Academic Search Complete
- ▶ Advanced Technologies Database with Aerospace
- ▶ Aerospace Database
- ▶ Aluminium Industry Abstracts
- ▶ ANTE: Abstracts in New Technologies and Engineering
- ▶ Cabell's Directories
- ▶ Civil Engineering Abstracts
- ▶ CNKI Scholar
- ▶ Computer and Information Systems Abstracts
- ▶ Corrosion Abstracts
- ▶ Current Abstracts
- ▶ Current Index to Statistics (CIS)
- ▶ DBLP Computer Science Bibliography
- ▶ Directory of Open Access Journals (DOAJ)
- ▶ Earthquake Engineering Abstracts
- ▶ EBSCO Engineering Source
- ▶ EBSCO MainFile
- ▶ EBSCOhost Connection
- ▶ EBSCOhost Research Databases
- ▶ Electronics and Communications Abstracts
- ▶ Engineering Research Database
- ▶ Euclid Prime
- ▶ Google Scholar
- ▶ HighBeam Research
- ▶ InfoTrac Custom journals
- ▶ INIST's Catalog of Articles and Monographs
- ▶ INSPEC
- ▶ J-Gate Portal
- ▶ Mathematical Reviews (MathSciNet)
- ▶ Mechanical and Transportation Engineering Abstracts
- ▶ Open Access Journals Integrated Service System Project (GoOA)
- ▶ Primo Central Index
- ▶ ProQuest Advanced Technologies and Aerospace Collection
- ▶ ProQuest Computer Science Journals
- ▶ ProQuest Engineering Collection
- ▶ ProQuest SciTech Premium Collection
- ▶ ProQuest Technology Collection
- ▶ Referativnyi Zhurnal (VINITI)
- ▶ Scopus
- ▶ Statistical Theory and Method Abstracts (STMA-Z)
- ▶ Technology Research Database
- ▶ The Electronic Library of Mathematics (EMIS ELibM)
- ▶ The Summon Service
- ▶ WorldCat Discovery Services
- ▶ Zentralblatt MATH Database (zbMATH)

All of Hindawi's content is archived in Portico, which provides permanent archiving for electronic scholarly journals, as well as via the LOCKSS initiative.





About Hindawi

Meet the Team
Contact Us
Blog
Jobs

Publish with Us

Submit Manuscript
Browse Journals
For Authors

Work with Us

Publishers
Editors

Legal

Terms of Service
Privacy Policy
Copyright





Hindawi

Journal of Applied Mathematics

Journal Menu

[About this Journal](#)
[Abstracting and Indexing](#)
[Aims and Scope](#)
[Article Processing Charges](#)
[Bibliographic Information](#)
[Editorial Board](#)
[Editorial Workflow](#)
[Publication Ethics](#)
[Reviewer Resources](#)
[Subscription Information](#)
[Table of Contents](#)

Special Issues Menu

[Annual Issues](#)
[Open Special Issues](#)
[Published Special Issues](#)
[Special Issue Resources](#)

[Subscribe to
Table of Contents Alerts](#)

Aims and Scope

Journal of Applied Mathematics is a refereed journal devoted to the publication of original research papers and review articles in all areas of applied, computational, and industrial mathematics.



About Hindawi

[Meet the Team](#)
[Contact Us](#)
[Blog](#)
[Jobs](#)

Publish with Us

[Submit Manuscript](#)
[Browse Journals](#)
[For Authors](#)

Work with Us

[Publishers](#)
[Editors](#)

Legal

[Terms of Service](#)
[Privacy Policy](#)
[Copyright](#)





Hindawi

Journal of Applied Mathematics

Journal Menu

[About this Journal](#)
[Abstracting and Indexing](#)
[Aims and Scope](#)
[Article Processing Charges](#)
[Bibliographic Information](#)
[Editorial Board](#)
[Editorial Workflow](#)
[Publication Ethics](#)
[Reviewer Resources](#)
[Subscription Information](#)
[Table of Contents](#)

Special Issues Menu

[Annual Issues](#)
[Open Special Issues](#)
[Published Special Issues](#)
[Special Issue Resources](#)

[Subscribe to
Table of Contents Alerts](#)

Editorial Board

Academic Editors

- ▶ Saeid Abbasbandy, Khomeini International University, Qazvin, Iran
- ▶ M. Montaz Ali, University of the Witwatersrand, South Africa
- ▶ Igor Andrianov, RWTH Aachen University, Germany
- ▶ Ali R. Ashrafi, Kashan University, Iran
- ▶ Jafar Biazar, University of Guilan, Iran
- ▶ Bruno Carpentieri, University of Groningen, The Netherlands
- ▶ Song Cen, Tsinghua University, China
- ▶ Han H. Choi, Dongguk University, Republic of Korea
- ▶ Sazzad Hossien Chowdhury, International Islamic University Malaysia, Malaysia
- ▶ Carlos Concha, Universidad de Chile, Chile
- ▶ Kai Diethelm, University of Applied Sciences Würzburg-Schweinfurt, Germany
- ▶ Meng Fan, Northeast Normal University, China
- ▶ Elisa Francomarino, Università degli Studi di Palermo, Italy
- ▶ Huijun Gao, Harbin Institute of Technology, China
- ▶ Bernard J. Geurts, University of Twente, The Netherlands
- ▶ Keshlan S. Govinder, University of KwaZulu-Natal, South Africa
- ▶ Ferenc Hartung, University of Pannonia, Hungary
- ▶ Wei-Chiang Hoing, Jiangsu Normal University, China
- ▶ Ying Hu, Université Rennes 1, France
- ▶ Dan Huang, Hohai University, China
- ▶ Mustafa Ilic, Firat University, Turkey
- ▶ Lucas Jodar, Universitat Politècnica de Valencia, Spain
- ▶ Waqar A. Khan, National University of Sciences and Technology, Pakistan
- ▶ Jong Hae Kim, Sun Moon University, Republic of Korea
- ▶ Kannan Krithivasan, SASTRA University, India
- ▶ Mirosław Lachowicz, University of Warsaw, Poland
- ▶ Hak-Keung Lam, King's College London, United Kingdom
- ▶ Peter G. L. Leach, University of Cyprus, Cyprus
- ▶ Chong Liu, Qingdao University, China
- ▶ Yansheng Liu, Shandong Normal University, China
- ▶ Chongxin Liu, Xi'an Jiaotong University, China
- ▶ Nazim I. Mahmudov, Eastern Mediterranean University, Turkey
- ▶ Oluwole D. Makinde, Stellenbosch University, South Africa
- ▶ Francisco J. Marcellán, Universidad Carlos III de Madrid, Spain
- ▶ Panayotis Takis Mathiopoulos, National Observatory of Athens, Greece
- ▶ Michael McAleer, Erasmus University Rotterdam, The Netherlands
- ▶ Alain Miranville, Université de Poitiers, France
- ▶ Ram N. Mohapatra, University of Central Florida, USA
- ▶ Donal O'Regan, National University of Ireland, Ireland
- ▶ Turgut Özlü, Ege University, Turkey
- ▶ Juan Manuel Peña, Universidad de Zaragoza, Spain
- ▶ Hector Pomares, University of Granada, Spain
- ▶ Mehmet Sezer, Celal Bayar University, Turkey
- ▶ N. Shahzad, King Abdul Aziz University, Saudi Arabia
- ▶ Hui-Shen Shen, Shanghai Jiao Tong University, China
- ▶ Fernando Simões, Instituto Superior Técnico, Portugal
- ▶ Theodore E. Simos, King Saud University, Saudi Arabia
- ▶ Abdel-Maksoud A. Soliman, Suez Canal University, Egypt
- ▶ Qiankun Song, Chongqing Jiaotong University, China
- ▶ Nasser-Eddine Tatar, King Fahd University of Petroleum and Minerals, Saudi Arabia
- ▶ Mariano Torrisi, Università degli Studi di Catania, Italy



- ▶ Qing-Wen Wang , Shanghai University, China
- ▶ Frank Werner , Otto von Guericke University of Magdeburg, Germany
- ▶ Man Leung Wong, Lingnan University, Hong Kong
- ▶ Xiaohui Yuan , Huazhong University of Science & Technology, China
- ▶ Sheng Zhang , Bohai University, China
- ▶ Zhihua Zhang , Shandong University, China
- ▶ Yun-Bo Zhao , Zhejiang University of Technology, China
- ▶ Jian G. Zhou , Manchester Metropolitan University, United Kingdom



About Hindawi

- Meet the Team
- Contact Us
- Blog
- Jobs

Publish with Us

- Submit Manuscript
- Browse Journals
- For Authors

Work with Us

- Publishers
- Editors

Legal

- Terms of Service
- Privacy Policy
- Copyright





Hindawi

Journal of Applied Mathematics

Journal Menu

[About this Journal](#)
[Abstracting and Indexing](#)
[Aims and Scope](#)
[Article Processing Charges](#)
[Bibliographic Information](#)
[Editorial Board](#)
[Editorial Workflow](#)
[Publication Ethics](#)
[Reviewer Resources](#)
[Subscription Information](#)
[Table of Contents](#)

Special Issues Menu

[Annual Issues](#)
[Open Special Issues](#)
[Published Special Issues](#)
[Special Issue Resources](#)

[Subscribe to
Table of Contents Alerts](#)

Table of Contents [76–100 of 3,338 articles]

- ▶ [An Optimal Investment Strategy and Multiperiod Deposit Insurance Pricing Model for Commercial Banks](#), Grant E. Muller
Research Article (10 pages), Article ID 8942050, Volume 2018 (2018)
- ▶ [Applied Artificial Bee Colony Optimization Algorithm in Fire Evacuation Routing System](#), Chen Wang, Lincoln C. Wood, Heng Li, Zhenye Aw, and Abolfazl Keshavarzsaleh
Research Article (17 pages), Article ID 7962952, Volume 2018 (2018)
- ▶ [The Equivalent Linearization Method with a Weighted Averaging for Solving Undamped Nonlinear Oscillators](#), D. V. Hieu, N. Q. Hai, and D. T. Hung
Research Article (15 pages), Article ID 7487851, Volume 2018 (2018)
- ▶ [A Comparative Study on Stabilized Finite Element Methods for the Convection-Diffusion-Reaction Problems](#), Ali Sendur
Research Article (16 pages), Article ID 4259634, Volume 2018 (2018)
- ▶ [A Mathematical Model of Treatment and Vaccination Interventions of Pneumococcal Pneumonia Infection Dynamics](#), Mohammed Kizito and Julius Tumwiine
Research Article (16 pages), Article ID 2539465, Volume 2018 (2018)
- ▶ [On Minimizing the Ultimate Ruin Probability of an Insurer by Reinsurance](#), Christian Kasumo, Juma Kasozi, and Dmitry Kuznetsov
Research Article (11 pages), Article ID 9180780, Volume 2018 (2018)
- ▶ [A Stochastic Model for Malaria Transmission Dynamics](#), Rachel Waema Mbogo, Livingstone S. Luboobi, and John W. Odhiambo
Research Article (13 pages), Article ID 2439520, Volume 2018 (2018)
- ▶ [Bridging the Gap between Economic Modelling and Simulation: A Simple Dynamic Aggregate Demand-Aggregate Supply Model with Matlab](#), José M. Gaspar
Research Article (13 pages), Article ID 3193068, Volume 2018 (2018)
- ▶ [Understanding Dengue Control for Short- and Long-Term Intervention with a Mathematical Model Approach](#), A. Bustamam, D. Aldila, and A. Yuwanda
Research Article (13 pages), Article ID 9674138, Volume 2018 (2018)
- ▶ [Table of Contents for Year 2017](#)
- ▶ [Corrigendum to “Improved Combinatorial Benders Decomposition for a Scheduling Problem with Unrelated Parallel Machines”](#), Francisco Regis Abreu Gomes and Geraldo Robson Mateus
Corrigendum (2 pages), Article ID 2465891, Volume 2017 (2017)
- ▶ [An Analysis of a Semelparous Population Model with Density-Dependent Fecundity and Density-Dependent Survival Probabilities](#), Arild Wikan
Research Article (14 pages), Article ID 8934295, Volume 2017 (2017)
- ▶ [Periodic Travelling Wave Solutions of Discrete Nonlinear Schrödinger Equations](#), Dirk Hennig
Research Article (5 pages), Article ID 3694103, Volume 2017 (2017)
- ▶ [Hybrid Algorithm of Particle Swarm Optimization and Grey Wolf Optimizer for Improving Convergence Performance](#), Narinder Singh and S. B. Singh
Research Article (15 pages), Article ID 2030489, Volume 2017 (2017)
- ▶ [New Integrals Arising in the Samara-Valencia Heat Transfer Model in Grinding](#), J. L. González-Santande
Research Article (5 pages), Article ID 3591713, Volume 2017 (2017)
- ▶ [Generalized Hybrid One-Step Block Method Involving Fifth Derivative for Solving Fourth-Order Ordinary Differential Equation Directly](#), Mohammad Alkasasbeh and Zurni Omar
Research Article (14 pages), Article ID 7637651, Volume 2017 (2017)
- ▶ [Analysis of Economic Burden of Seasonal Influenza: An Actuarial Based Conceptual Model](#), S. S. N. Perera
Research Article (6 pages), Article ID 4264737, Volume 2017 (2017)

with Linear Canonical Transform, Mawardi Bahri and Muh. Saleh Arif Fatimah
Research Article (10 pages), Article ID 3247364, Volume 2017 (2017)

- ▶ Some New Volterra-Fredholm-Type Nonlinear Discrete Inequalities with Two Variables Involving Iterate Sums and Their Applications, Run Xu
Research Article (14 pages), Article ID 1474052, Volume 2017 (2017)
- ▶ A Greedy Clustering Algorithm Based on Interval Pattern Concepts and the Problem of Optimal Box Positioning, Stepan A. Nersisyan, Vera V. Pankratieva, Vladimir M. Staroverov, and Vladimir E. Podolski
Research Article (9 pages), Article ID 4323590, Volume 2017 (2017)
- ▶ Extension of Wolfe Method for Solving Quadratic Programming with Interval Coefficients, Syaripuddin, Herry Suprajitno, and Fatmawati
Research Article (6 pages), Article ID 9037857, Volume 2017 (2017)
- ▶ Analysis of a Heroin Epidemic Model with Saturated Treatment Function, Isaac Mwangi Wangari and Lewi Stone
Research Article (21 pages), Article ID 1953036, Volume 2017 (2017)
- ▶ Simulation of Wellbore Stability during Underbalanced Drilling Operation, Reda Abdel Azim
Research Article (12 pages), Article ID 2412397, Volume 2017 (2017)
- ▶ Gutman Index and Detour Gutman Index of Pseudo-Regular Graphs, S. Kavithaa and V. Kaladevi
Research Article (8 pages), Article ID 4180650, Volume 2017 (2017)
- ▶ On the Solution of the Eigenvalue Assignment Problem for Discrete-Time Systems, El-Sayed M. E. Mostafa, Abdallah W. Aboutahoun, and Fatma F. S. Omar
Research Article (12 pages), Article ID 7256769, Volume 2017 (2017)
- ▶ A Guide on Spectral Methods Applied to Discrete Data in One Dimension, Martin Seilmayer and Matthias Ratajczak
Review Article (27 pages), Article ID 5108946, Volume 2017 (2017)

« previous 25 articles

next 25 articles »



About Hindawi

Meet the Team
Contact Us
Blog
Jobs

Publish with Us

Submit Manuscript
Browse Journals
For Authors

Work with Us

Publishers
Editors

Legal

Terms of Service
Privacy Policy
Copyright

Research Article

Extension of Wolfe Method for Solving Quadratic Programming with Interval Coefficients

Syaripuddin,¹ Herry Suprajitno,² and Fatmawati²

¹*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Mulawarman University, Kampus Gunung Kelua, Jl. Barong Tongkok, Samarinda 1068, Indonesia*

²*Department of Mathematics, Faculty of Science and Technology, Airlangga University, Kampus C Unair, Jl. Mulyorejo, Surabaya 60115, Indonesia*

Correspondence should be addressed to Fatmawati; fatma47unair@gmail.com

Received 10 April 2017; Accepted 1 August 2017; Published 14 September 2017

Academic Editor: Frank Werner

Copyright © 2017 Syaripuddin et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Quadratic programming with interval coefficients developed to overcome cases in classic quadratic programming where the coefficient value is unknown and must be estimated. This paper discusses the extension of Wolfe method. The extended Wolfe method can be used to solve quadratic programming with interval coefficients. The extension process of Wolfe method involves the transformation of the quadratic programming with interval coefficients model into linear programming with interval coefficients model. The next step is transforming linear programming with interval coefficients model into two classic linear programming models with special characteristics, namely, the optimum best and the worst optimum problem.

1. Introduction

Quadratic programming is a special form of nonlinear programming which has special characteristics; that is, the objective function is in quadratic forms and constraint functions are linear form [1]. Although the quadratic programming is part of nonlinear programming, the completion is still adopting some linear programming problem solving methods, one of which is the Wolfe method. This method transforms the quadratic programming problem into a linear programming problem. Wolfe [2] modified the simplex method to solve quadratic programming problem by adding conditions of the Karush-Kuhn-Tucker (KKT) and changing the objective function of quadratic forms into a linear form.

Interval quadratic programming is a development of the classic quadratic programming that utilizes interval analysis theory developed by Moore [3]. This development aim is to accommodate cases which contain the uncertainty, that is, when the data value is unknown for certain, but the data lies within an interval where the values of the upper limit and lower limit are known. The special characteristics of the interval quadratic programming problem are the coefficients

of the objective function and constraint functions are in the interval form.

Research on quadratic programming with interval coefficients has been conducted by Liu and Wang [4]. However, the coefficients of quadratic forms in the objective function or the developed model are not in the interval form yet. Furthermore, Li and Tian [5] generalized the model in [4] by assuming that the quadratic coefficients of the objective function are in the interval form. References [4, 5] used the duality theory to create a method of solving the quadratic programming with interval coefficients. Quadratic programming model with interval coefficients is transformed into two classic quadratic programming models with the special characteristics, called the best optimum and worst optimum problem. A completion method was developed based on the method of solving the linear programming with interval coefficients that have been discussed by some researchers [6–10].

This paper will discuss the extension of Wolfe method to solve quadratic programming with interval coefficients. Therefore, this article will focus on how to transform the quadratic programming with interval coefficients into linear

programming with interval coefficients. Furthermore, the linear programming with interval coefficients which has been obtained from the transformation will be solved by using the method in [8].

This paper is organized as follows. Section 2 discusses interval arithmetic operations. In Section 3, a general form of linear programming with interval coefficients is stated. In Section 4, a general form of quadratic programming with interval coefficients is stated. Extension of Wolfe method as one method of solving the quadratic programming with interval coefficients is discussed in Section 5, whereas Section 6 discusses numerical examples, and Section 7 provides some concluding remarks.

2. Interval Arithmetic

The basic definition and properties of interval number and interval arithmetic can be seen at Moore [3], Alefeld and Herzberg [11], and Hansen [12].

Definition 1. A closed real interval $\underline{x} = [x_I, x_S]$ denoted by \underline{x} is a real interval number which can be defined completely by

$$\underline{x} = [x_I, x_S] = \{x \in \mathfrak{R} \mid x_I \leq x \leq x_S; x_I, x_S \in \mathfrak{R}\}, \quad (1)$$

where x_I and x_S are called infimum and supremum, respectively.

Definition 2. A real interval number $\underline{x} = [x_I, x_S]$ is called degenerate, if $x_I = x_S$.

Definition 3. Let $\underline{x} = [x_I, x_S]$ and $\underline{y} = [y_I, y_S]$; then

1. $\underline{x} + \underline{y} = [x_I + y_I, x_S + y_S]$ (addition),
2. $\underline{x} - \underline{y} = [x_I, x_S] - [y_I, y_S] = [x_I, x_S] + [-y_S, -y_I] = [x_I - y_S, x_S - y_I]$ (subtraction),
3. $\underline{x} \cdot \underline{y} = [\min\{x_I y_I, x_I y_S, x_S y_I, x_S y_S\}, \max\{x_I y_I, x_I y_S, x_S y_I, x_S y_S\}]$ (multiplication),
4. $\underline{x}/\underline{y} = \underline{x}(1/\underline{y}) = [x_I, x_S][1/y_S, 1/y_I], 0 \notin \underline{y}$ (division).

3. Linear Programming with Interval Coefficients

The general form of linear programming with interval coefficients is defined as follows:

$$\text{Maximize } \underline{z} = \sum_{j=1}^n [c_{jI}, c_{jS}] x_j \quad (2a)$$

subject to

$$\sum_{j=1}^n [a_{ijI}, a_{ijS}] x_j \leq [b_{iI}, b_{iS}], \quad i = 1, 2, \dots, m \quad (2b)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n, \quad (2c)$$

where $x_j \in \mathfrak{R}, [c_{jI}, c_{jS}], [b_{iI}, b_{iS}],$ and $[a_{ijI}, a_{ijS}] \in I(\mathfrak{R}).$

The model in (2a)–(2c) is solved by means of transforming the linear programming with interval coefficients into two classic linear programming models with the special characteristics, namely, the best optimum and the worst optimum problems. The best optimum problem has properties of best version on the objective function and maximum feasible area on the constraint function. On the other hand, the worst optimum problem has a characteristic that it is the worst version of the objective function and the minimum feasible area of the constraint function.

Chinneck and Ramadan [8] provide a rule to determine the best and worst optimum problem in a linear programming problem with interval coefficients. The constraints of linear programming with interval coefficients which have an inequality sign (\leq) in (2b) have characteristics of the maximum feasible area and the minimum feasible area which is given by the following theorem.

Theorem 4 (Chinneck and Ramadan [8]). *Suppose that we have an interval inequality given $\sum_{j=1}^n [a_{jI}, a_{jS}] x_j \leq [b_I, b_S]$, where $x_j \geq 0$. Then, $\sum_{j=1}^n a_{jI} x_j \leq b_S$ is maximum feasible area and $\sum_{j=1}^n a_{jS} x_j \leq b_I$ is minimum feasible area.*

The objective function of linear programming with interval coefficients for the case of maximizing (2a) has characteristics of the best version and the worst version of the objective function is expressed in the following theorem.

Theorem 5 (Chinneck and Ramadan [8]). *If $\underline{z} = \sum_{j=1}^n [c_{jI}, c_{jS}] x_j$ is the objective function for $x_j \geq 0$, then $\sum_{j=1}^n c_{jS} x_j \geq \sum_{j=1}^n c_{jI} x_j$, where $\sum_{j=1}^n c_{jS} x_j$ is the best version of the objective function and $\sum_{j=1}^n c_{jI} x_j$ is the worst version of the objective function.*

4. Quadratic Programming with Interval Coefficients

The general form of quadratic programming with interval coefficients introduced by Li and Tian [5] is defined as follows:

$$\begin{aligned} \text{Maximize } \underline{z} &= \sum_{j=1}^n [c_{jI}, c_{jS}] x_j \\ &+ \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_j x_k \end{aligned} \quad (3a)$$

subject to

$$\sum_{j=1}^n [a_{ijI}, a_{ijS}] x_j \leq [b_{iI}, b_{iS}], \quad i = 1, 2, \dots, m \quad (3b)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n, \quad (3c)$$

where $x_j \in \mathfrak{R}$, $[c_{jI}, c_{jS}]$, $[q_{ijkI}, q_{ijkS}]$, $[b_{iI}, b_{iS}]$, $[a_{ijI}, a_{ijS}] \in I(\mathfrak{R})$, $\sum_{j=1}^n \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_j x_k$ is negative semidefinite, and $I(\mathfrak{R})$ are the set of all interval numbers in \mathfrak{R} .

The model as shown in (3a)–(3c) is a generalization of the model in [4]. The coefficient of the objective function and constraints of the quadratic programming with interval coefficients model in (3a)–(3c) have an interval form. The idea to solve the model is the extension of Wolfe method. This method focuses on how to transform the quadratic programming with interval coefficients in (3a)–(3c) into linear programming with interval coefficients in (2a)–(2c). Furthermore, linear programming with interval coefficients obtained from the transformation is done using the method in [8].

Extension of Wolfe method is the main result in this paper. The fundamental difference between the extensions of Wolfe method and the method in [5] is, on the extension of Wolfe method, quadratic programming model with interval coefficients is transformed into linear programming with interval coefficients, while, in [5], the model of quadratic programming with interval coefficients is maintained.

5. Extension of Wolfe Method

Wolfe method is one method for solving quadratic programming problems by means of transforming the quadratic programming problems into a linear programming problem. Wolfe [2] modified the simplex method to solve quadratic programming problems by adding a requirement Karush-Kuhn-Tucker (KKT) and changing the quadratic objective function into a linear objective function.

The extension of Wolfe method is used to solve quadratic programming problem with interval coefficients. Steps of extension of Wolfe method are declared as follows.

Form of Lagrange function for the problem in (3a)–(3c) is

$$\begin{aligned}
 L(x, y, r, \lambda, \mu) &= \sum_{j=1}^n [c_{jI}, c_{jS}] x_j + \frac{1}{2} \\
 &\cdot \sum_{j=1}^n \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_j x_k \\
 &- \sum_{i=1}^m \lambda_i \left(\sum_{j=1}^n ([a_{ijI}, a_{ijS}] x_j - [b_{iI}, b_{iS}] + y_i^2) \right) \\
 &- \sum_{j=1}^n \mu_j (-x_j + r_j^2),
 \end{aligned} \tag{4}$$

where $\lambda_i, i = 1, 2, \dots, m, \mu_j, j = 1, 2, \dots, n$, are Lagrange multipliers and $L(x, y, r, \lambda, \mu)$ is Lagrange function with interval coefficients.

Local minimum points of the function L were obtained by the first partial derivatives of the function L with respect

to the variables and equating to zero (KKT necessary conditions) (see [13, 14]).

$$\begin{aligned}
 \frac{\partial L}{\partial x_i} &= [c_{jI}, c_{jS}] + \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_k \\
 &- \sum_{i=1}^m [a_{ijI}, a_{ijS}] \lambda_i + \mu_j = 0, \\
 & \qquad \qquad \qquad j = 1, 2, \dots, n,
 \end{aligned} \tag{5a}$$

$$\frac{\partial L}{\partial y_i} = -2\lambda_i y_i = 0, \quad i = 1, 2, \dots, m, \tag{5b}$$

$$\frac{\partial L}{\partial r_i} = -2\mu_j r_j = 0, \quad j = 1, 2, \dots, n, \tag{5c}$$

$$\begin{aligned}
 \frac{\partial L}{\partial \lambda_i} &= \sum_{j=1}^n [a_{ijI}, a_{ijS}] x_j + y_i^2 - [b_{iI}, b_{iS}] \\
 &= 0, \quad i = 1, 2, \dots, m,
 \end{aligned} \tag{5d}$$

$$\frac{\partial L}{\partial \mu_i} = x_j - r_j^2 = 0, \quad j = 1, 2, \dots, n, \tag{5e}$$

$$x_j, \lambda_i, \mu_j, y_i, r_j \geq 0, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n. \tag{5f}$$

Results simplification of (5a)–(5f) is

$$\begin{aligned}
 &- \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_k + \sum_{i=1}^m [a_{ijI}, a_{ijS}] \lambda_i - \mu_j \\
 &= [c_{jI}, c_{jS}], \quad j = 1, 2, \dots, n,
 \end{aligned} \tag{6a}$$

$$\sum_{j=1}^n [a_{ijI}, a_{ijS}] x_j + s_i = [b_{iI}, b_{iS}], \quad i = 1, 2, \dots, m, \tag{6b}$$

$$x_j, \lambda_i, \mu_j, s_i \geq 0, \quad j = 1, 2, \dots, n, \quad i = 1, 2, \dots, m \tag{6c}$$

and satisfies the complementary conditions,

$$\mu_j x_j = 0, \quad j = 1, 2, \dots, n, \quad \lambda_i s_i = 0, \quad i = 1, 2, \dots, m. \tag{6d}$$

Add artificial variables $v_j, j = 1, 2, \dots, n$, in (6a) for an initial basis, as follows:

$$\begin{aligned}
 &- \sum_{k=1}^n [q_{jkI}, q_{jkS}] x_k + \sum_{i=1}^m [a_{ijI}, a_{ijS}] \lambda_i - \mu_j + v_j \\
 &= [c_{jI}, c_{jS}].
 \end{aligned} \tag{7}$$

Furthermore, create a linear programming with interval coefficients, where the objective function is to minimize the number of artificial variables $v_j, j = 1, 2, \dots, n$, and constraint is (7), (6b), (6c), and (6d) obtained from necessary conditions of KKT.

$$\text{Minimize } z = v_1 + v_2 + \dots + v_n \tag{8a}$$

subject to,

$$-\sum_{k=1}^n [q_{jkI}, q_{jkS}] x_k + \sum_{i=1}^m [a_{ijI}, a_{ijS}] \lambda_i - \mu_j + \nu_j \quad (8b)$$

$$= [c_{jI}, c_{jS}], \quad j = 1, 2, \dots, n$$

$$\sum_{j=1}^n [a_{ijI}, a_{ijS}] x_j + s_i = [b_{iI}, b_{iS}], \quad i = 1, 2, \dots, m \quad (8c)$$

$$x_j, \lambda_i, \mu_j, s_i, \nu_j \geq 0, \quad j = 1, 2, \dots, n, \quad i = 1, 2, \dots, m \quad (8d)$$

satisfying the complementary conditions,

$$\begin{aligned} \mu_j x_j &= 0, \quad j = 1, 2, \dots, n, \\ \lambda_i s_i &= 0, \quad i = 1, 2, \dots, m, \end{aligned} \quad (8e)$$

where $\nu_j \geq 0$ is artificial variable.

The model as shown in (8a)–(8e) is linear programming with interval coefficients which is added by complementary conditions. This model is the result of the transformation from the quadratic programming with interval coefficients model by extension of Wolfe method.

The next step, linear programming with interval coefficients model in (8a)–(8e), was solved by transforming into two linear programming cases with the special characteristics, namely, the best and the worst optimum problem. The transformation process can be written in Algorithm 6 as follows.

Algorithm 6.

- (1) Given a quadratic programming problem with interval coefficients in (3a)–(3c), Extension of Wolfe method is based on (3a)–(3c) equivalent to the linear programming with interval coefficients in (8a)–(8e).
- (2) Use Theorems 4 and 5 for transforming the linear programming with interval coefficients in (8a)–(8e) into two classic linear programming models with special characteristics; namely,

(a) the best optimum problem is

$$\text{Minimize } z_S = \nu_1 + \nu_2 + \dots + \nu_n \quad (9a)$$

subject to,

$$-\sum_{k=1}^n q_{jkS} x_k + \sum_{i=1}^m a_{ijI} \lambda_i - \mu_j + \nu_j = c_{jS}, \quad (9b)$$

$$j = 1, 2, \dots, n$$

$$\sum_{j=1}^n a_{ijI} x_j + s_i = b_{iS}, \quad (9c)$$

$$i = 1, 2, \dots, m$$

$$x_j, \lambda_i, \mu_j, s_i, \nu_j \geq 0, \quad (9d)$$

$$j = 1, 2, \dots, n, \quad i = 1, 2, \dots, m$$

satisfying the complementary conditions,

$$\mu_j x_j = 0, \quad j = 1, 2, \dots, n, \quad (9e)$$

$$\lambda_i s_i = 0, \quad i = 1, 2, \dots, m,$$

(b) the worst optimum problem is

$$\text{Minimize } z_I = \nu_1 + \nu_2 + \dots + \nu_n \quad (10a)$$

subject to,

$$-\sum_{k=1}^n q_{jkI} x_k + \sum_{i=1}^m a_{ijS} \lambda_i - \mu_j + \nu_j = c_{jI}, \quad (10b)$$

$$j = 1, 2, \dots, n$$

$$\sum_{j=1}^n a_{ijS} x_j + s_i = b_{iI}, \quad (10c)$$

$$i = 1, 2, \dots, m$$

$$x_j, \lambda_i, \mu_j, s_i, \nu_j \geq 0, \quad (10d)$$

$$j = 1, 2, \dots, n, \quad i = 1, 2, \dots, m$$

satisfying the complementary conditions,

$$\mu_j x_j = 0, \quad j = 1, 2, \dots, n, \quad (10e)$$

$$\lambda_i s_i = 0, \quad i = 1, 2, \dots, m.$$

- (3) The optimum value of the quadratic programming with interval coefficients is obtained by combining the optimum value from the worst and the best optimum problem; that is, $z = [z_I, z_S]$.

Algorithm 6 shows that the best and the worst optimum problem are linear programming models added by complementary conditions. Thus, both problems can be solved by simplex method.

6. Numerical Example

Consider the following example of quadratic programming with interval coefficients in the journal Li and Tian [5].

$$\begin{aligned} \text{Minimize } z & \\ &= [-10, -6] x_1 + [2, 3] x_2 + [-1, 1] x_1 x_2 \quad (11a) \\ &+ [4, 10] x_1^2 + [10, 20] x_2^2 \end{aligned}$$

subject to

$$[1, 2] x_1 + 3x_2 \leq [1, 10] \quad (11b)$$

$$[-2, 8] x_1 + [4, 6] x_2 \leq [4, 6] \quad (11c)$$

$$x_1, x_2 \geq 0. \quad (11d)$$

TABLE I: Two classic linear programming models with special characteristics.

The best optimum problem	The worst optimum problem
(1) Classic linear programming model	(2) Classic linear programming model
Minimize $z_S = v_1 + v_2$	Minimize $z_I = v_1 + v_2$
subject to	subject to
$8x_1 - x_2 + \lambda_1 - 2\lambda_2 - \mu_1 + v_1 = 10$	$20x_1 + x_2 + 2\lambda_1 + 8\lambda_2 - \mu_1 + v_1 = 6$
$x_1 - 20x_2 - 3\lambda_1 - 4\lambda_2 + \mu_2 + v_2 = 2$	$-x_1 - 40x_2 - 3\lambda_1 - 6\lambda_2 + \mu_2 + v_2 = 3$
$x_1 + 3x_2 + s_1 = 10$	$2x_1 + 3x_2 + s_1 = 1$
$-2x_1 + 4x_2 + s_2 = 6$	$8x_1 + 6x_2 + s_2 = 4$
$x_i, \lambda_i, \mu_i, s_i, v_i \geq 0, i = 1, 2$	$x_i, \lambda_i, \mu_i, s_i, v_i \geq 0, i = 1, 2$
satisfying complementary conditions:	satisfying complementary conditions:
$\lambda_1 s_1 = 0, \lambda_2 s_2 = 0, \text{ and } \mu_1 x_1 = 0, \mu_2 x_2 = 0$	$\lambda_1 s_1 = 0, \lambda_2 s_2 = 0, \text{ and } \mu_1 x_1 = 0, \mu_2 x_2 = 0$
<i>Solution:</i> $z_S = 6.25, x_1 = 1, 25, \text{ and } x_2 = 0.$	<i>Solution:</i> $z_I = 0.9, x_1 = 0.3, \text{ and } x_2 = 0$

According to Li and Tian [5], for the solution of the model in (11a)–(11c), the best optimum problem is $z_S = -0.9, x_1 = 0.3,$ and $x_2 = 0,$ the worst optimum problem is $z_I = -6.25, x_1 = 1, 25,$ and $x_2 = 0,$ and the optimum value is $\underline{z} = [z_I, z_S] = [-6.25, -0.9].$

This paper presents only the maximization problem so that any minimization problem will be converted into maximization problem, the simple procedure to convert a minimization problem to a maximization problem and vice versa. Simply multiply the objective function of a minimization problem by -1 converting it into a maximization problem and vice versa.

$$\begin{aligned}
& \text{Maximize } z \\
& = [6, 10] x_1 + [-3, -2] x_2 \\
& \quad + [-1, 1] x_1 x_2 + [-10, -4] x_1^2 \\
& \quad + [-20, -10] x_2^2
\end{aligned} \tag{12a}$$

subject to

$$[1, 2] x_1 + 3x_2 \leq [1, 10] \tag{12b}$$

$$[-2, 8] x_1 + [4, 6] x_2 \leq [4, 6] \tag{12c}$$

$$x_1, x_2 \geq 0. \tag{12d}$$

We apply the extension of Wolfe method for transforming quadratic programming with interval coefficients model in ((12a)–(12d)) into linear programming with interval coefficients model. We have

$$\text{Minimize } z = v_1 + v_2 \tag{13a}$$

subject to

$$\begin{aligned}
& [8, 20] x_1 + [-1, 1] x_2 + [1, 2] \lambda_1 + [-2, 8] \lambda_2 - \mu_1 \\
& \quad + v_1 = [6, 10]
\end{aligned} \tag{13b}$$

$$\begin{aligned}
& [-1, 1] x_1 + [20, 40] x_2 + [3, 3] \lambda_1 + [4, 6] \lambda_2 - \mu_2 \\
& \quad + v_2 = [-3, -2]
\end{aligned} \tag{13c}$$

$$[1, 2] x_1 + 3x_2 \leq [1, 10] \tag{13d}$$

$$[-2, 8] x_1 + [4, 6] x_2 \leq [4, 6] \tag{13e}$$

$$x_1, x_2 \geq 0. \tag{13f}$$

We apply Algorithm 6 for transforming linear programming with interval coefficients model in ((13a)–(13f)) into two classic linear programming models with special characteristics, namely, the best optimum and the worst optimum problem. The result of the transformation is shown in Table 1.

So, the optimum value of the quadratic programming with interval coefficients is obtained by combining the optimum value from the worst and the best optimum problem; that is, $\underline{z} = [z_I, z_S] = [0.9, 6.25].$ This solution gives the same value as obtained by Li and Tian [5].

7. Conclusion

This paper presents an extension of Wolfe method. The extension of Wolfe method performed by transforming the quadratic programming with interval coefficients model into linear programming with interval coefficients model. Furthermore, linear programming with interval coefficients model is transformed into two classic linear programming models using Algorithm 6. The extension of Wolfe method has a particular benefit: the final model is linear programming. Hence, it can be solved by the simplex method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

- [1] F. S. Hillier and G. J. Lieberman, *Introduction to operations research*, Holden-Day, Inc., Oakland, Calif., Third edition, 1980.

- [2] P. Wolfe, "The simplex method for quadratic programming," *Econometrica*, vol. 27, pp. 382–398, 1959.
- [3] R. E. Moore, *Interval Analysis*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1966.
- [4] S.-T. Liu and R.-T. Wang, "A numerical solution method to interval quadratic programming," *Applied Mathematics and Computation*, vol. 189, no. 2, pp. 1274–1281, 2007.
- [5] W. Li and X. Tian, "Numerical solution method for general interval quadratic programming," *Applied Mathematics and Computation*, vol. 202, no. 2, pp. 589–595, 2008.
- [6] S. C. Tong, "Interval number and fuzzy number linear programmings," *Fuzzy Sets and Systems. An International Journal in Information Science and Engineering*, vol. 66, no. 3, pp. 301–306, 1994.
- [7] K. Ramadan, *Linear Programming with Interval [Msc. thesis]*, Carleton University, Ottawa, Ontario, 1997.
- [8] J. W. Chinneck and K. Ramadan, "Linear programming with interval coefficients," *Journal of the Operational Research Society*, vol. 51, no. 2, pp. 209–220, 2000.
- [9] D. Kuchta, "A modification of a solution concept of the linear programming problem with interval coefficients in the constraints," *Central European Journal of Operations Research (CEJOR)*, vol. 16, no. 3, pp. 307–316, 2008.
- [10] H. Suprajitno and I. B. Mohd, "Interval linear programming," in *Proceedings of ICOMS-3*, Bogor, Indonesia, 2008.
- [11] G. Alefeld and J. Herzberger, *Introduction to Interval Computations*, Academic Press, New York, NY, USA, 1983.
- [12] E. Hansen, *Global optimization using interval analysis*, vol. 165 of *Monographs and Textbooks in Pure and Applied Mathematics*, Marcel Dekker, Inc., New York, 1992.
- [13] J. Zhang, "Optimality condition and wolfe duality for invex interval-valued nonlinear programming problems," *Journal of Applied Mathematics*, vol. 2013, Article ID 641345, 2013.
- [14] H.-C. Wu, "The Karush-KUHn-Tucker optimality conditions in an optimization problem with interval-valued objective function," *European Journal of Operational Research*, vol. 176, no. 1, pp. 46–59, 2007.