2019 International Conference on Electrical Engineering and Informatics (ICEEI)
July 2019, 9 - 10, Bandung, Indonesia

Centralized and decentralized H_∞ controller design for storey building systems using matrix inequality approach

1st Helisyah Nur Fadhilah
Department of Mathematics
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
helisyah07@gmail.com

2nd,* Didik Khusnul Arif
Department of Mathematics
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
didik@matematika.its.ac.id

3rd Fatmawati
Department of Mathematics
Universitas Airlangga
Surabaya, Indonesia
fatmawati@fst.unair.ac.id

4th Dieky Adzkiya
Department of Mathematics
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
dieky@matematika.its.ac.id

5th Guisheng Zhai
Department of Mathematical Sciences
Shibaura Institute of Technology
Saitama, Japan
zhai@shibaura-it.ac.jp

***Abstract*—Modern control methods have found their way into decentralized design of interconnected systems. Before we find decentralized controller, we first find the centralized controller. Centralized controller will be computed using Linear Matrix Inequality (LMI) approach and decentralized controller will be computed using Bilinear Matrix Inequality (BMI). The BMI will be solved using double LMI via homotopy method which can then be solved efficiently. Numerical example of centralized and decentralized controller of storey building system will be presented. The simulation results on centralized controller show that systems with centralized H_∞ controls have better performance than systems without control, and simulation results on decentralized controller show that $\| T_{zw}(s) \|_\infty < \gamma$.**

***Index Terms*—Centralized controller, Decentralized controller, Linear Matrix Inequality, Bilinear Matrix Inequality, Storey building systems**

I. Introduction

In the natural phenomenon, many systems such as communication networks, large-scale structures, power systems, and chemical processes can be modeled as interconnected systems with interacting subsystems [1]. The theory of large-scale systems is devoted to the problems that arise from above difficulties. The theory answers the fundamental questions of how to break down a given control problem into manageable sub problems which are only weakly related to each other and can be solved independently. As a result, the overall plant is no longer controlled by a single controller but by several independent controllers which all together represent a decentralized controller [2]. To design decentralized controllers robustness is required. Control is needed to design a controller that will stabilize the system. If the original system is unstable, the control design is needed to stabilize the system, but if the original system is stable, the control design is used to stabilize the system with other specification including desired convergence. Many scientists have developed a robust modern control theory of disruption and uncertainty of models such as H_∞ control and Linear Quadratic Gaussian (LQG) [3].

Based on [4], The control method is divided into two categories i.e. centralized and decentralized controller. The centralized controller is a single control system.If the system to be designed for control is a large scale system, the centralized control is less efficient. Because if the centralized controller is damaged, the controller is damaged and cannot control the whole system automatically. In recent years, modern control methods have found their way into decentralized design of interconnected systems leading to a wide variety of new concepts and results. Decentralizing means more self-organizing, more flexible to add and remove subsystems.

One of the modern and robust control is H_∞ control. In some references [5] [6], decentralized H_∞ controller is computed using several approaches. Many control problems that are normally intractable and require the solution to Bilinear Matrix Inequality (BMI) can be formulated as Linear Matrix Inequality (LMI) which can then be solved more eficiently [7]. In [8], the authors find decentralized H_∞ controller by using matrix inequality approach such as BMI.

To the best of our knowledge, there is no practical method to solve BMI directly [9] [10] [11], so in this paper we will solve the BMI using double LMI. For double LMI, we use the homotopy method to find the feasible solution. Basically homotopy method works from the easy one to difficult one gradually [12]. We first consider a centralized H_∞ controller based on LMI [13], then deform the centralized H_∞ controller to decentralized H_∞ controller.

*Corresponding author. `didik@matematika.its.ac.id`

In recent years, the technology of controlling the structure of storey building has attracted much attention. Homotopy method will be implemented to storey building system with 5 subsystems and we will find the decentralized H_∞ controller for each storey with specified dimension.

II. H_∞ CONTROL

The stable and continuous time systems are as follows [6]:

$$\begin{aligned} \dot{x} &= Ax + Bw \\ z &= Cx + Dw \end{aligned} \tag{1}$$

where w is the disturbance input and z is the performance output. The aim of this section is to give criteria for assuring upper bounds of the H_∞ norm from w to z for Linear Time Invariant (LTI) system [14], i.e. to show that

$$\| z \|_2 < \gamma \| w \|_2$$

or equivalently

$$\| z \|_2 < \gamma \| w \|_2 \Longleftrightarrow \int [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt < 0$$

For this problem, the following cost function can be used

$$g(x,w) = \| z \|^2 - \gamma^2 \| w \|^2 = z^T z - \gamma^2 w^T w, \tag{2}$$

and a quadratic Lyapunov function is chosen

$$V(x) = x^T P x \tag{3}$$

To assure internal stability of system, it is assumed that the Lyapunov matrix P is symmetric and positive definite ($P > 0$), that is $x^T P x > 0, \forall x \neq 0$. If $x(0) = 0$ the L_2-induced norm from w to z is less than γ if the Hamiltonian for (1) and (2) is negative for all x [14]:

$$\begin{aligned} H &= \dot{V} + g(x,w) \\ &= \dot{x}^T P x + x^T P \dot{x} + z^T z - \gamma^2 w^T w \\ &= x^T P(Ax + Bw) + (Ax + Bw)^T P x \\ &\quad + (Cx + Dw)^T (Cx + Dw) - \gamma^2 w^T w \end{aligned} \tag{4}$$

In order to assure that $\| z \|_2 < \gamma \| w \|_2$ then $H < 0$ must hold for all x and w.

III. LINEAR MATRIX INEQUALITY

Instead of completing the squares, the Hamiltonian (4) can be rewritten as:

$$\begin{bmatrix} x \\ w \end{bmatrix}^T \begin{bmatrix} PA + A^T P + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D - \gamma^2 I \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} < 0$$

which shall hold for all nonzero x, w. This implies that

$$\begin{bmatrix} PA + A^T P + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D - \gamma^2 I \end{bmatrix} < 0 \tag{5}$$

It can be further simplified using Schur lemma with multiplying by γ^{-1} and taking $P = \gamma^{-1} P$, we obtain

$$\begin{bmatrix} PA + A^T P & PB \\ B^T P & -\gamma I \end{bmatrix} + \begin{bmatrix} C^T \\ D^T \end{bmatrix} \gamma^{-1} I \begin{bmatrix} C & D \end{bmatrix} < 0$$

yields

$$\begin{bmatrix} PA + A^T P & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{6}$$

The last one of these inequalities in (6) is linear in (A, B, C, D) for a given P, from which we conclude that the set of system matrices satisfying the Riccati inequality or equivalently the LMI is convex [14]. The bounded real lemma states an extension of these results.

Lemma 3.1: The following statements are equivalent [8],

i $\| T_{zw}(s) \|_\infty < \gamma$ and A stable with $T_{zw}(s) = D + C(sI - A)^{-1} B$

ii there exists a solution $P > 0$ to the LMI

$$\begin{bmatrix} PA + A^T P & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{7}$$

IV. CENTRALIZED H_∞ CONTROLLER

The centralized H_∞ controller is also called H_∞ synthesis. In this section we will study H_∞ synthesis using LMI to get the centralized H_∞ controller. Suppose given a LTI system with state-space realization [8]

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned} \tag{8}$$

where $x \in R^{n_x}$. The input vector contains the disturbance signal, $w \in R^{n_w}$, and the control signal, $u \in R^{n_u}$. The output vector contains the measurement signal, $y \in R^{n_y}$, and the performance signal, $z \in R^{n_z}$. We assume that D_{22} is zero, i.e., the system is strictly proper from u to y. The matrices A,B_1,B_2,C_1,C_2,D_{11},D_{12}, and D_{21} are constant and of appropriate sizes.

The output feedback control problem consists of finding a dynamic controller with state space equations [8]

$$\begin{aligned} \dot{x}_F &= A_F x_F + B_F y \\ u &= C_F x_F + D_F y \end{aligned} \tag{9}$$

The centralized controller can be rewritten in single matrix as G_F as follows

$$G_F = \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix} \tag{10}$$

where dimension of x_F is the same as x in (8). The performance bound γ is defined as the H_∞ norm of the closed loop system from disturbance input signal, w, to the performance output, z. The closed loop system is obtained by applying the controller in (9) to system (8). We can write the closed loop system as follows.

$$\begin{array}{rcl} \dot{x} & = & (A + B_2 D_F C_2)x + B_2 C_F x_F \\ & & +(B_1 + B_2 D_F D_{21})w \\ \dot{x}_F & = & B_F C_2 x + A_F x_F + B_F D_{21} w \\ z & = & (C_1 + D_{12} D_F C_2)x + D_{12} C_F x_F \\ & & +(D_{11} + D_{12} D_F D_{21})w \end{array} \tag{11}$$

Introduce the notation of closed loop system as follows [14]

$$\left[\begin{array}{c|cc} A_C & B_{C1} & B_{C2} \\ \hline C_{C1} & D_{C11} & D_{C12} \\ C_{C2} & D_{C21} & \end{array}\right]$$

$$= \left[\begin{array}{cc|c|cc} A & 0_{n\times n} & B_1 & 0_{n\times n} & B_2 \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times r} & I_n & 0_{n\times n} \\ \hline C_1 & 0_{n\times n} & D_{11} & 0_{n\times n} & D_{12} \\ \hline 0_{n\times n} & I_n & 0_{n\times n} & & \\ C_2 & 0_{n\times n} & D_{21} & & \end{array}\right] \tag{12}$$

Using the notation (12) and (10) into (11), we have the closed loop system in a compact form as

$$\begin{array}{rcl} \dot{x}_C & = & (A_C + B_{C2} G_F C_{C2}) x_C \\ & & +(B_{C1} + B_{C2} G_F D_{C21})w \\ z & = & (C_{C1} + D_{C12} G_F C_{C2}) x_C \\ & & +(D_{C11} + D_{C12} G_F D_{C21})w \end{array} \tag{13}$$

$A_{Cl} = A_C + B_{C2} G_F C_{C2}$
$B_{Cl} = B_{C1} + B_{C2} G_F D_{C21}$
$C_{Cl} = C_{C1} + D_{C12} G_F C_{C2}$
$D_{Cl} = D_{C11} + D_{C12} G_F D_{C21}$

where $x_C = \begin{bmatrix} x^T & x_F^T \end{bmatrix}^T$. The closed loop system is internally stable and has an H_∞ norm of γ if there exists a symmetric $P = P^T > 0$ such that Lemma 3.1 holds or, equivalently

$$F(G_F, P) = \begin{bmatrix} PA_{Cl} + A_{Cl}^T P & PB_{Cl} & C_{Cl}^T \\ B_{Cl}^T P & -\gamma I & D_{Cl}^T \\ C_{Cl} & D_{Cl} & -\gamma I \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\gamma I & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} + \left[\begin{bmatrix} PA_C & PB_{C1} & 0 \\ 0 & 0 & 0 \\ C_{C1} & D_{C11} & 0 \end{bmatrix}\right.$$

$$\left. + \begin{bmatrix} PB_{C2} & PB_{C2} & 0 \\ 0 & 0 & 0 \\ D_{C12} & D_{C12} & 0 \end{bmatrix} G_F \begin{bmatrix} C_{C2} & 0 & 0 \\ 0 & D_{C21} & 0 \\ 0 & 0 & 0 \end{bmatrix}\right]$$

$$+ \left[\begin{bmatrix} PA_C & PB_{C1} & 0 \\ 0 & 0 & 0 \\ C_{C1} & D_{C11} & 0 \end{bmatrix} + \begin{bmatrix} PB_{C2} & PB_{C2} & 0 \\ 0 & 0 & 0 \\ D_{C12} & D_{C12} & 0 \end{bmatrix}\right.$$

$$\left. G_F \begin{bmatrix} C_{C2} & 0 & 0 \\ 0 & D_{C21} & 0 \\ 0 & 0 & 0 \end{bmatrix}\right]^T < 0$$

Centralized H_∞ controller problem can be obtained using LMI approach such as [8]

$$F(G_F, P) < 0 \tag{14}$$

Inequality (14) is an BMI with respect to P and G_F, with G_F has been obtained via an existing method [15] [13].

V. Decentralized H_∞ controller

The N-channel LTI system described by [12] is as follows

$$\begin{array}{rcl} \dot{x} & = & Ax + B_1 w + \sum_{i=1}^{N} B_{2i} u_i \\ z & = & C_1 x + D_{11} w + \sum_{i=1}^{N} D_{12i} u_i \\ y_i & = & C_{2i} x + D_{21i} w \end{array} \tag{15}$$

where x ,w,u_i,y_i and z have the same meaning as in section IV where $i = 1, 2, ..., N$ represents the total channel. The matrices A,B_1,B_{2i},C_1,C_{2i},D_{11},D_{12i}, and D_{21i} are constant and of appropriate sizes. In this case, we assumed matrix $D_{22} = 0$. The decentralized output-feedback control problem for (15) consists of finding a dynamic controller with state space equations as follows [12]

$$\begin{array}{rl} \dot{\hat{x}}_i & = \hat{A}_i \hat{x}_i + \hat{B}_i y_i \\ u_i & = \hat{C}_i \hat{x}_i + \hat{D}_i y_i \end{array} \tag{16}$$

where $\hat{x}_i \in R_{\hat{n}_i}$ is the state of the i-controller,$\hat{n}_i$ is a specifed dimension of local controller, and $\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i, i = 1, 2, ..., N$ are constant matrices to be determined.

We collect the controller state $\hat{x}_i$ and coefficient matrices $\hat{A}_i$,$\hat{B}_i$,$\hat{C}_i$,and $\hat{D}_i$ as follows

$$\begin{array}{rcl} \hat{x} & = & \begin{bmatrix} \hat{x}_1^T & \hat{x}_2^T & \cdots & \hat{x}_N^T \end{bmatrix}^T \\ \hat{A}_D & = & diag\{\hat{A}_1, \hat{A}_2, \cdots, \hat{A}_N\} \\ \hat{B}_D & = & diag\{\hat{B}_1, \hat{B}_2, \cdots, \hat{B}_N\} \\ \hat{C}_D & = & diag\{\hat{C}_1, \hat{C}_2, \cdots, \hat{C}_N\} \\ \hat{D}_D & = & diag\{\hat{D}_1, \hat{D}_2, \cdots, \hat{D}_N\} \end{array}$$

and

$$\begin{array}{rcl} B_2 & = & \begin{bmatrix} B_{21} & B_{22} & \cdots & B_{2N} \end{bmatrix} \\ C_2 & = & \begin{bmatrix} C_{21}^T & C_{22}^T & \cdots & C_{2N}^T \end{bmatrix}^T \\ D_{12} & = & \begin{bmatrix} D_{121} & D_{122} & \cdots & D_{12N} \end{bmatrix} \\ D_{21} & = & \begin{bmatrix} D_{211}^T & D_{212}^T & \cdots & D_{21N}^T \end{bmatrix}^T \end{array} \tag{17}$$

The decentralized controller can be rewritten in single matrix

$$G_D = \begin{bmatrix} \hat{A}_D & \hat{B}_D \\ \hat{C}_D & \hat{D}_D \end{bmatrix} \tag{18}$$

For a specified disturbance attenuation level $\gamma > 0$, design a decentralized controller in equation (16) for system (15) so the resultant closed loop system is stable and $\| T_{zw}(s) \|_\infty < \gamma$.

The closed loop system is obtained by applying the decentralized controller in (16) to system (15) as follows.

$$\begin{array}{rcl} \dot{x} & = & (A + B_2\hat{D}_D C_2)x + B_2\hat{C}_D\hat{x} \\ & & +(B_1 + B_2\hat{D}_D D_{21})w \\ \dot{\hat{x}} & = & \hat{B}_D C_2 x + \hat{A}_D\hat{x} + \hat{B}_D D_{21} w \\ z & = & (C_1 + D_{12}\hat{D}_D C_2)x + D_{12}\hat{C}_D\hat{x} \\ & & +(D_{11} + D_{12}\hat{D}_D D_{21})w \end{array} \tag{19}$$

Introduce the notation of closed loop system of decentralized H_∞ controller as follows [12]

$$\left[\begin{array}{c|cc} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \end{array}\right]$$

$$= \left[\begin{array}{cc|c|cc} A & 0_{n\times\hat{n}} & B_1 & 0_{n\times\hat{n}} & B_2 \\ 0_{\hat{n}\times n} & 0_{\hat{n}\times\hat{n}} & 0_{\hat{n}\times r} & I_{\hat{n}} & 0_{\hat{n}\times m} \\ \hline C_1 & 0_{p\times\hat{n}} & D_{11} & 0_{p\times\hat{n}} & D_{12} \\ \hline 0_{\hat{n}\times n} & I_{\hat{n}} & 0_{\hat{n}\times r} & & \\ C_2 & 0_{q\times\hat{n}} & D_{21} & & \end{array}\right] \tag{20}$$

using the notation (20) and (18) into (19), we have the closed loop system in a compact form as

$$\begin{array}{rcl} \dot{\tilde{x}} & = & (\tilde{A} + \tilde{B}_2 G_D\tilde{C}_2)\tilde{x} + (\tilde{B}_1 + \tilde{B}_2 G_D\tilde{D}_{21})w \\ z & = & (\tilde{C}_1 + \tilde{D}_{12}G_D\tilde{C}_2)\tilde{x} + (\tilde{D}_{11} + \tilde{D}_{12}G_D\tilde{D}_{21})w \end{array} \tag{21}$$

$$\begin{array}{l} A_{Cld} = \tilde{A} + \tilde{B}_2 G_D\tilde{C}_2 \\ B_{Cld} = \tilde{B}_1 + \tilde{B}_2 G_D\tilde{D}_{21} \\ C_{Cld} = \tilde{C}_1 + \tilde{D}_{12}G_D\tilde{C}_2 \\ D_{Cld} = \tilde{D}_{11} + \tilde{D}_{12}G_D\tilde{D}_{21} \end{array}$$

where $\tilde{x} = \left[\begin{array}{cc} x^T & \hat{x}^T \end{array}\right]^T$. The closed loop system is internally stable and has an H_∞ norm of γ if there exists a symmetric $\tilde{P} = \tilde{P}^T > 0$ such that Lemma 3.1 holds or, equivalently

$$F(G_D,\tilde{P}) = \left[\begin{array}{ccc} \tilde{P}A_{Cld} + A_{Cld}^T\tilde{P} & \tilde{P}B_{Cld} & C_{Cld}^T \\ B_{Cld}^T P & -\gamma I & D_{Cld}^T \\ C_{Cld} & D_{Cld} & -\gamma I \end{array}\right]$$

$$= \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & -\gamma I & 0 \\ 0 & 0 & -\gamma I \end{array}\right] + \left[\left[\begin{array}{ccc} \tilde{P}\tilde{A} & \tilde{P}\tilde{B}_1 & 0 \\ 0 & 0 & 0 \\ \tilde{C}_1 & \tilde{D}_{11} & 0 \end{array}\right]\right.$$

$$\left. + \left[\begin{array}{ccc} \tilde{P}\tilde{B}_2 & \tilde{P}\tilde{B}_2 & 0 \\ 0 & 0 & 0 \\ \tilde{D}_{12} & \tilde{D}_{12} & 0 \end{array}\right] G_D \left[\begin{array}{ccc} \tilde{C}_2 & 0 & 0 \\ 0 & \tilde{D}_{21} & 0 \\ 0 & 0 & 0 \end{array}\right]\right]$$

$$+ \left[\left[\begin{array}{ccc} \tilde{P}\tilde{A} & \tilde{P}\tilde{B}_1 & 0 \\ 0 & 0 & 0 \\ \tilde{C}_1 & \tilde{D}_{11} & 0 \end{array}\right]\right.$$

$$\left. + \left[\begin{array}{ccc} \tilde{P}\tilde{B}_2 & \tilde{P}\tilde{B}_2 & 0 \\ 0 & 0 & 0 \\ \tilde{D}_{12} & \tilde{D}_{12} & 0 \end{array}\right] G_D \left[\begin{array}{ccc} \tilde{C}_2 & 0 & 0 \\ 0 & \tilde{D}_{21} & 0 \\ 0 & 0 & 0 \end{array}\right]\right]^T < 0$$

The existence condition for decentralized H_∞ controller is a BMI with respect to variables G_D and $\tilde{P}$ due to the term $\tilde{P}\tilde{B}_2 G_D \left[\begin{array}{ccc} \tilde{C}_2 & \tilde{D}_{21} & 0 \end{array}\right]$. Decentralized H_∞ controller problem such as BMI problems is as follows [12]

Theorem 5.1: System (15) is stabilizable with the disturbance attenuation level γ via a decentralized controller (16) composed of $\hat{n}_i$-dimensional local controllers if and only if there exist a matrix G_D of (18) and a positive definite matrix $\tilde{P}$ such that

$$F(G_D,\tilde{P}) < 0 \tag{22}$$

VI. ALGORITHM OF HOMOTOPY METHOD

The existence condition to find the decentralized H_∞ controller in (22) is a BMI. In BMI we will find the solution of two variables, in this case G_D and $\tilde{P}$. Currently, there is no practical method to solve BMI directly [9] [10] [11], so in this paper we will use double LMI in [12] to find the feasible solution of G_D and $\tilde{P}$. A BMI is an LMI in G_D for fixed $\tilde{P}$ and an LMI in $\tilde{P}$ for fixed G_D. The idea for a decentralized H_∞ controller using homotopy method is to transform the full order centralized H_∞ controller at each step, we solve BMI as an LMI obtained by suitably fixing one of the two variables, in this case G_D and $\tilde{P}$. To use the homotopy method, we consider a matrix function as follows [12]

$$H_o(G_D,\tilde{P},\eta) = F((1-\eta)G_F + \eta G_D,\tilde{P}) \tag{23}$$

where $\eta \in [0,1]$ is a real number. G_F is centralized H_∞ controller as in equation (10). The term $(1-\eta)G_F + \eta G_D$ in (23) is defined as homotopy interpolating a centralized H_∞ controller and a desired decentralized H_∞ controller. Then we can partition the matrix function in (23) into two (double) LMI based on the η values as follows

$$H_o(G_D,\tilde{P},\eta) = \left\{\begin{array}{ll} F(G_F,\tilde{P}), & \eta = 0 \\ F(G_D,\tilde{P}), & \eta = 1 \end{array}\right.$$

For $\eta = 0$, it is an LMI in $\tilde{P}$, and by using the result of $\tilde{P}$ in $\eta = 0$, next we have an LMI in G_D for $\eta = 1$. So the problem to find decentralized controller in (22) that can be rewritten as follows

$$H_o(G_D,\tilde{P},\eta) < 0, \eta \in [0,1] \tag{24}$$

We need to find the solution of $G_D,\tilde{P}$ at $\eta = 0$, which we denote by $(G_{D_0},\tilde{P}_0)$.

First, we need to build a homotopy path and initial value of G_{D_0} and $\tilde{P}_0$ to solve the BMI. Let K be a positive integer and the points of homotopy path is $(K+1)$, with $r = 0,1,2,...,K$, we have η_r/K in interval $\eta \in [0,1]$. Then, set $G_{D_0} = 0$ and find the solution of $\tilde{P}_0$ from solution of centralized H_∞ controller $F(G_F,\tilde{P}) < 0$. If feasible, we can go to next η_r and find the G_{D_r} by using $\tilde{P}_r := \tilde{P}_0$ and so on one by one finding the feasible G_{D_r} and $\tilde{P}_r$ until $r := K$.

To connect the matrix function in (24) to solve the BMI we use the algorithm of the homotopy method for decentralized H_∞ controller described by [12].

VII. Simulation of Storey Building System

In recent years, the technology of controlling the structure of storey buildings has attracted much attention. This is done to reduce structural response, such as speed, displacement, acceleration, and force under disturbances that can be in the form of earthquakes, strong winds, and other disasters. The system to be used in the proposed research is a system of building structures illustrated in Figure 1 [16]

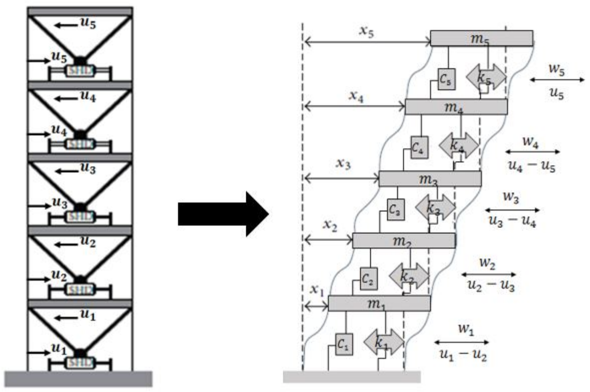

Fig. 1. Illustration of a Storey Building System with $n = 5$

In Figure 1, semi-active hydraulic dampers allocated between every two neighboring floors. The motion equation of storey building in Figure 1 is presented in the form of a second order differential equation as follows

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Ew(t) + B_0u(t) \tag{25}$$

To find the state space equation for the previous linear continuous dynamical system we let:

$$x_1 = x \qquad \text{and} \qquad x_2 = \dot{x}$$

which presents the interstory drifts $x(t)$ and interstory $\dot{x}(t)$ velocities in increasing order. By substituting the above equations into Equation (25), we get the following system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \ddot{x}_1 &= -M^{-1}Kx_1 - M^{-1}Cx_2 + M^{-1}Ew + M^{-1}B_0u \end{aligned}$$

and in matrix form we have:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix} w + \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix} u \tag{26}$$

Based on Equation (26), let matrices A,B_1,and B_2:

$$A = \begin{bmatrix} 0_{5\times5} & I_{5\times5} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0_{5\times1} \\ M^{-1}E \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0_{5\times5} \\ M^{-1}B_0 \end{bmatrix}$$

where M, C, and K are matrices of the mass, dampers and spring coefficients of the system respectively. The variable x is a displacement vector that depends on time t, u is a force vector that is controlled and depends on time t, and w is an external vector of interference (for example in this case generated from earthquakes and strong winds) which depends on time t. B_0 and E are control force place and external disturbance place matrices, respectively, M, C, K, B_0 and E are defined as follows [17]:

$$M = 10^3 \times \begin{bmatrix} 215.2 & 0 & 0 & 0 & 0 \\ 0 & 209.2 & 0 & 0 & 0 \\ 0 & 0 & 207.0 & 0 & 0 \\ 0 & 0 & 0 & 204.8 & 0 \\ 0 & 0 & 0 & 0 & 266.1 \end{bmatrix}$$

$$C = 10^3 \times \begin{bmatrix} 650.4 & 231.1 & 0 & 0 & 0 \\ 231.1 & 548.9 & 202.5 & 0 & 0 \\ 0 & 202.5 & 498.6 & 182.0 & 0 \\ 0 & 0 & 182.0 & 466.7 & 171.8 \\ 0 & 0 & 0 & 171.8 & 318.5 \end{bmatrix}$$

$$K = 10^6 \times \begin{bmatrix} 260 & 113 & 0 & 0 & 0 \\ 113 & 212 & 99 & 0 & 0 \\ 0 & 99 & 188 & 89 & 0 \\ 0 & 0 & 89 & 173 & 84 \\ 0 & 0 & 0 & 84 & 84 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = -M \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

The original system is asymptotically stable, so we will design controller that will stabilize the system faster.

We employ the **hinflmi** Control Toolbox of MATLAB, the disturbance attenuation level γ by centralized H_∞ controller is 3.11587. In this simulation, the disturbance is sinusoidal signal $w = 10^2e^{-t}sin(\pi t)$. Based on the simulation result, we can summarise the velocity of motion of five-storey building in Figure 2 as follows.

Considering Figure 2, it is clear that the proposed control algorithm was able to prevent excessive vibration of the structure and this reduces the internal forces in the structure. It also reduces the damages to structures during the period of

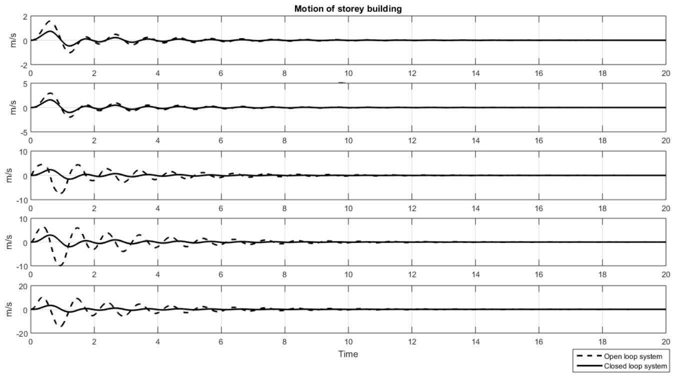

Fig. 2. Simulation result of five-storey building system

earthquake. Reduction of displacement in floors and convergence toward zero occurred for all of the floors as error of state variable. Figures 2 display the displacements of all five floors together when both under controlled and uncontrolled forms are considered.

After we get centralized H_∞ controller, next we will find decentralized H_∞ controller using homotopy method. We set the value of $\gamma = 4$ which is larger than γ of centralized H_∞ controller. Algorithm of homotopy method converges with $K \leq 64$. We find the γ value in several cases on storey building system as in Table I.

Tabel I
THE γ VALUE OF EACH CASES

H_∞ control	Order of controller	γ value
Centralized	$n_1 = n_2 = n_3 = n_4 = n_5 = 2$	3.11587
Decentralized	$n_1 = n_3 = n_4 = n_5 = 2$,$n_2 = 1$	3.17894
	$n_1 = n_2 = n_3 = n_4 = 2$, $n_5 = 1$	3.17892
	$n_1 = n_5 = 2$ and $n_2 = n_3 = n_4 = 1$	3.19405

Based on Table I, we know that decentralized H_∞ controller disturbance attenuation level γ of all cases are less than 4, it means that decentralized H_∞ controllers have quite good performance and the algorithm runs well.

We know that decentralized H_∞ controllers cannot achieve better performances than the best of such centralized controllers but The γ values show that the output-feedback controllers computed with the methodology presented in Section VI have a pretty good disturbance attenuation level when compared with the centralized output-feedback controller. This result on Table I is reasonable, because the total order or the information of decentralized H_∞ controllers is less than centralized H_∞ controllers. For this reason, decentralized controller is more efficient in implementation for large-scale systems.

VIII. CONCLUSION

To get a decentralized H_∞ controller, it is necessary to have a centralized controller first. A centralized H_∞ controller has been computed using LMI approach and decentralized H_∞ controller is computed using BMI approach via the homotopy method. Based on the simulation results on the storey building system for centralized H_∞ controller, closed loop system has better performance than open loop system. We have found decentralized H_∞ controller with the initial value of γ which is larger than γ of centralized H_∞ controller. Based on the result, decentralized H_∞ controller have a pretty good disturbance attenuation level compared with the centralized output-feedback controller. The methodology has been successfully applied to design centralized and decentralized H_∞ controllers for the vibrational response of a five-storey building under excitation.

REFERENCES

[1] S. Sojoudi and A. G. Aghdam, "Overlapping control systems with optimal information exchange," *Automatica*, vol. 45, no. 5, pp. 1176–1181, 2009.

[2] L. Bakule, "Decentralized control: An overview," *IFAC Proceedings Volumes*, vol. 40, no. 9, pp. 39 – 48, 2007. 11th IFAC Symposium on Large Scale Complex Systems Theory and Applications.

[3] Fatmawati, R. Saragih, B. Riyanto, and Y. Soeharyadi, "Model and controller order reduction for infinite dimensional systems," *Journal of Engineering and Technological Sciences*, vol. 42, no. 1, pp. 1–16, 2013.

[4] R. Zaeim and M. A. Nekoui, "Centralized and decentralized controller design for FGS using linear and nonlinear model," in *2009 Second International Conference on Computer and Electrical Engineering*, vol. 1, pp. 56–60, 2009.

[5] R. A. Date and J. H. Chow, "A parametrization approach to optimal H_2 and H_∞ decentralized control problems," *Automatica*, vol. 29, no. 2, pp. 457 – 463, 1993.

[6] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard H_2 and H_∞ control problems," *IEEE Transactions on Automatic Control*, vol. 34, pp. 831–847, Aug 1989.

[7] A. Hassibi, J. How, and S. Boyd, "A path-following method for solving BMI problems in control," in *Proceedings of the 1999 American Control Conference (Cat. No. 99CH36251)*, vol. 2, pp. 1385–1389 vol.2, June 1999.

[8] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *International Journal of Robust and Nonlinear Control*, vol. 4, no. 4, pp. 421–448, 1994.

[9] K. . Goh, M. G. Safonov, and G. P. Papavassilopoulos, "A global optimization approach for the BMI problem," in *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, vol. 3, pp. 2009–2014 vol.3, Dec 1994.

[10] S.-M. Liu and G. Papavassilopoulos, "Numerical experience with parallel algorithms for solving the BMI problem," *IFAC Proceedings Volumes*, vol. 29, no. 1, pp. 1827 – 1832, 1996. 13th World Congress of IFAC, 1996, San Francisco USA, 30 June - 5 July.

[11] H. Fujioka and K. Hoshijima, "Bounds for the BMI eigenvalue problem," *Transactions of the Society of Instrument and Control Engineers*, vol. 33, no. 1, pp. 616–621, 1997.

[12] G. Zhai, M. Ikeda, and Y. Fujisaki, "Decentralized H_∞ controller design: a matrix inequality approach using a homotopy method," *Automatica*, vol. 37, no. 4, pp. 565 – 572, 2001.

[13] P. Gahinet, A. Nemirovskii, A. J. Laub, and M. Chilali, "The LMI control toolbox," in *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, vol. 3, pp. 2038–2041 vol.3, Dec 1994.

[14] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. SIAM studies in applied mathematics, Society for Industrial and Applied Mathematics, 1994.

[15] T. Iwasaki and R. Skelton, "All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas," *Automatica*, vol. 30, no. 8, pp. 1307 – 1317, 1994.

[16] J. Rubi-Masseg, F. Palacios-Quionero, and J. M. Rossell, "Decentralized static output-feedback H_∞ controller design for buildings under seismic excitation," *Earthquake Engineering & Structural Dynamics*, vol. 41, no. 7, pp. 1199–1205, 2012.

[17] J. Mesbahi and A. Malek, "Synchronized control for five-story building under earthquake loads," *Journal of applied mathematics*, vol. 2015, pp. 1–8, 2015.