Mathematical model analysis of a drug transmission with criminal law and its optimal control

by Fatmawati Fatmawati

Mathematical model analysis of a drug transmission with criminal law and its optimal control

Cite as: AIP Conference Proceedings **2192**, 060010 (2019); https://doi.org/10.1063/1.5139156
Published Online: 19 December 2019

Muhammad Hafiruddin, Fatmawati, and Miswanto

Mathematical Model Analysis of a Drug Transmission with Criminal Law and Its Optimal Control

Muhammad Hafiruddin[1,b)], Fatmawati[1,a)] and Miswanto[1,c)]

[1] *Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga, Surabaya 60115, Indonesia.*

[a)] Corresponding author: fatmawati@fst.unair.ac.id
[b)] muhammad.hafiruddin-2015@fst.unair.ac.id
[c)] miswanto@fst.unair.ac.id

Abstract. The Indonesian government has made various efforts in drug prevention policies through criminal law. However, law enforcement officials still contemplate the Drugs Law is oriented towards imprisonment, so drug abuse is considered as a criminal act. In fact, the government has declared 2014 as the year to save the victims of drug abuse through rehabilitation. In this paper, a drug transmission model with the criminal law aspect is presented and analyzed. The optimal control strategy is then applied in the form of rehabilitation efforts. Based on the model analysis, we found two equilibriums, namely the drugs-free equilibrium and the drug addiction equilibrium. The stability of the equilibriums depends on the basic reproduction number. The spread of drug addicts persists in the population if the basic reproduction number greater than unity. Based on the simulation, it can be seen that criminal law give a significant impact to decrease the number of mild drug addicts and also the number of heavy levels of drug addicts. Furthermore, the existence of the optimal control variable is determined through the Pontryagin's Maximum Principle method. The comparison of simulation results with and without control shows that rehabilitation efforts can reduce drug addicts transmission.

Keywords: criminal law, drugs model, optimal strategy, stability.

INTRODUCTION

In the modern era, drug abuse has become one of the main issues around the world. Drug abuse is often reported in electronic media and print media. The outbreak of drug misuse can have a negative impact on health, economy, and society. This such condition is very detrimental to individuals, society, and the state, especially the younger generation. The government has made various efforts through drug prevention policies, namely the repression of illicit drug trafficking and the prevention of drug abuse in society. Thus, the role of various parties is required to be able to fight drugs and overcome the increasing number of drugs for the future of the Indonesian nation free of drugs [1].

Indonesia is a country with the fourth most populous population in the world [2]. This is the target of drugs smuggling. In addition to the high demand for drug consumption, economic development in Indonesia is also relatively high, making it an attraction for drug syndicates. The rise of drugs smuggling to Indonesia is partly due to the vastness of Indonesian maritime with limited personnel [3].

In 2010 the Supreme Court issued Surat Edaran Mahkamah Agung (SEMA) No. 4 of 2010 concerning the placement of misuses, victims of misuse and drug addicts into rehabilitation institutions which become the reference for judges to impose rehabilitation decisions. However, so far law enforcement officials still see the Drugs Law is oriented towards imprisonment for drug users or addicts so that they are considered criminals. In fact, the government declared that 2014 is the year to save victims of drug misuse through rehabilitation [4]. Therefore, researchers are interested in examining the dynamics of the spread of drug addicts with the criminal law aspect.

The mathematical model plays an important role in understanding the behavior of epidemics and biology (see [5, 6, 7]). Drug abuse has almost identical characteristics with conventional epidemics. So far, various mathematical models describing the spread of drug abuse has been carried out (see [8, 9, 10, 11]). The author in [8] has studied the dynamic between homelessness and drug abuse. The dynamic of traditional drug users who switched into using

Proceedings of The 8th SEAMS-UGM International Conference on Mathematics and its Applications 2019
AIP Conf. Proc. 2192, 060010-1–060010-10; https://doi.org/10.1063/1.5139156
Published by AIP Publishing. 978-0-7354-1943-8/$30.00

synthesis drugs which caused social problems have been presented in [9]. The effect of media coverage on the dissemination and control of drug addicts has been developed in [10]. The authors in [11] analyzed the effects of providing family education and public health education on the number of drug addicts. Next, the optimal control strategies have been applied to reduce the spread of drug abuse. The influence of illegal drug use in the community by paying attention to interactions that occur between individuals who are vulnerable to illegal drug users and its optimal control analysis has been investigated in [12].

In this study, we consider a mathematical model that describes the dynamics of the spread of drugs addicts that derived by authors in [11] with adding criminal penalties aspect. We also present the application of optimal control strategies to reduce the number of drug addicts. We first analyze the model without control that is the stability of the equilibriums and supported by the results of a numerical approach. This is done to find out the dynamics of drug addicts spreading. From the analysis carried out, it is expected that the results can be more relevant to the dynamics of the drug addicts spreading in Indonesia. We then apply optimal control in the form of rehabilitation efforts to reduce the population of drug addicts.

MATHEMATICAL MODEL FORMULATION

In this section, we formulate a mathematical model of drug addicts with the criminal law aspects. The assumptions used for mathematical models of drugs addicts with criminal law aspect are as follows:

1. Populations that are vulnerable to drug addicts who have either received drug misuse education or have not received drug-addicted education can be mild level drug addicts.
2. The population of heavy drug addicts cannot be mild level drug addicts.
3. The population of light and heavy narcotics addicts will undergo criminal law if caught
4. The population of drug addicts who have undergone criminal law can return to being mild and severe drug addicts.
5. The population of drug addicts who stop using the drug cannot be mild or severe drug addicts.
6. The natural rate of birth and death of humans is considered the same
7. The population of narcotics addicts who can be exposed to criminal law is 14 years or older.

Description of variables and parameters used in the model can be seen in Table 1 and Table 2.

TABLE 1. Description of variables.

Variable	Description
$S(t)$	The number of individuals which susceptible to drug addicts at time t
$C(t)$	The number of individuals vulnerable to being a narcotic addict but has received drugs misuse education at time t
$L(t)$	The number of mild drugs addicts at time t
$H(t)$	The number of heavy level drugs addicts at time t
$W(t)$	The number of drugs addicts who undergo criminal law at time t
$R(t)$	The number of individuals who stopped using drugs at time t

We assume that all parameters are constant and non-negative. Based on the assumptions and definitions of variables and parameters, a transmission diagram can be shown in Figure 1. Based on Figure 1, the following system of ordinary differential equations is obtained:

$$\frac{dS}{dt} = (1-q)\,d\Lambda - \beta_1 S\,L - \beta_2 S\,H - (d+\mu)\,S \tag{1}$$

$$\frac{dC}{dt} = qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta)\,C \tag{2}$$

$$\frac{dL}{dt} = \beta_1 S\,L + \beta_2 S\,H + \beta_1 \xi CL + \beta_2 \xi CH - (\gamma + \pi + d + d_1)\,L + \omega W \tag{3}$$

TABLE 2. Description of parameters.

Parameter	Description
d	Natural human birth/death rate
Λ	Recruitment rate
q	Probability of human that receiving drugs misuse education
μ	Progression rate from S to C
β_1	The direct contact rate of S with L
β_2	The direct contact rate of S with H
ξ	Probability factors that resulting in direct contact levels decrease
d_1	The death rate of mild drugs addicts
d_2	The death rate of heavy drug addicts
γ	The transition rate from L to W
ω	The transition rate from W to L
π	Progression rate from L to H
δ	Progression rate from C to R
θ	The transition rate from H to W
σ	The transition rate from W to H
m	Progression rate from W to R

$$\frac{dH}{dt} = \pi L + \sigma W - (\theta + d + d_2) H \quad (4)$$

$$\frac{dW}{dt} = \theta H + \gamma L - (\omega + \sigma + m + d) W \quad (5)$$

$$\frac{dR}{dt} = mW + \delta \mathrm{C} - \mathrm{d}R. \quad (6)$$

To simplify the notation, we define

$$\begin{aligned} m_1 &= \gamma + \pi + d + d_1 \\ m_2 &= \theta + d + d_2 \\ m_3 &= \omega + \sigma + m + d. \end{aligned}$$

Because the R variable does not appear in another equation, we focuss on the following equation.

$$\begin{aligned} \frac{dS}{dt} &= (1-q)\, d\Lambda - \beta_1 S L - \beta_2 S H - (d+\mu) S \\ \frac{dC}{dt} &= qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta) C \\ \frac{dL}{dt} &= \beta_1 S L + \beta_2 S H + \beta_1 \xi CL + \beta_2 \xi CH - m_1 L + \omega W \\ \frac{dH}{dt} &= \pi L + \sigma W - m_2 H \\ \frac{dW}{dt} &= \theta H + \gamma L - m_3 W. \end{aligned} \quad (7)$$

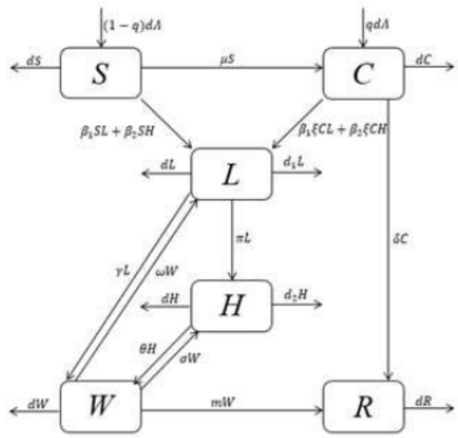


FIGURE 1. Transmission diagram of drugs addicts model with the criminal law aspect.

MODEL ANALYSIS

The mathematical model of drugs addicts with criminal law aspect has two equilibriums, namely a drugs-free (non-endemic) equilibrium and a drug addiction (endemic) equilibrium. The non-endemic equilibrium of the model is $E^0 = \left(S^0, C^0, L^0, H^0, W^0, R^0\right) = \left(\frac{(1-q)d\Lambda}{(d+\mu)}, \frac{(\mu+qd)d\Lambda}{(d+\mu)(d+\delta)}, 0, 0, 0\right)$ and endemic equilibrium $E^* = (S^*, C^*, L^*, H^*, W^*)$, where

$$
\begin{aligned}
L^* &= \frac{m_2 m_3 - \sigma\theta}{m_3\pi + \sigma\gamma} H^* \\
W^* &= \frac{\theta\pi + \gamma m_2}{m_3\pi + \sigma\gamma} H^* \\
S^* &= \frac{(1-q)(m_3\pi + \sigma\gamma)\, d\Lambda}{(\beta_1 (m_2 m_3 - \sigma\theta) + \beta_2 (m_3\pi + \sigma\gamma)) H^* + (d+\mu)(m_3\pi + \sigma\gamma)} \\
C^* &= \frac{(qd\Lambda + \mu S^*)(m_3\pi + \sigma\gamma)}{\beta_1 \xi (m_2 m_3 - \sigma\theta) + (\beta_2 \xi H^* + d + \delta)(m_3\pi + \sigma\gamma)}.
\end{aligned}
$$

The compartment L^*, S^*, C^* will exist if $m_2 m_3 > \sigma\theta$, while H^* is the root of the second degree polynomial as follows:

$$aH^{*2} + bH^* + c = 0. \tag{8}$$

where

$$
\begin{aligned}
a &= \xi(\beta_1 (x-y) + \beta_2)^2 (m_1 (x-y) - \omega z) \\
b &= (\beta_1 (x-y) + \beta_2)(m_1 (x-y) - \omega z)(d + \delta + \xi(d+\mu)) - d\Lambda\xi(\beta_1 (x-y) + \beta_2)^2, \\
c &= d^2\Lambda (\beta_1 (x-y) + \beta_2)(q - \xi q - 1) + d\Lambda (\beta_1 (x-y) + \beta_2)(q\delta - \delta - \xi\mu) \\
&\quad + (m_1(x-y) - \omega z)(d+\mu)(d+\delta), \\
x &= \frac{m_2 m_3}{m_3\pi + \sigma\gamma}, \\
y &= \frac{\sigma\theta}{m_3\pi + \sigma\gamma}, \\
z &= \frac{\pi\theta + \gamma m_2}{m_3\pi + \sigma\gamma}.
\end{aligned}
$$

Based on the above conditions, the terms $x > y$ are obtained. The Eq (8) have one positive solution if

(i) $a > 0$ and $c < 0$, or
(ii) $a < 0$ and $c > 0$.

The condition (ii) is not is not fulfilled because when $a < 0$, we have $c < 0$ as follows. Because $(q - \xi q - 1) < 0$ and $(q\delta - \delta - \xi\mu) < 0$, we get

$$c = d\Lambda\left(\beta_1 (x - y) + \beta_2\right)\left[d\left(q - \xi q - 1\right) + \left(q\delta - \delta - \xi\mu\right)\right] + \left(m_1(x - y) - \omega z\right)(d + \mu)(d + \delta) < 0.$$

Hence, the endemic equilibrium exist if

i). $m_2 m_3 > \sigma\theta$,
ii). $m_1 (x - y) > \omega z$,
iii). $d\Lambda\left(\beta_1 (x - y) + \beta_2\right)\left[d\left(1 - q(1 - \xi)\right) + \left(\xi\mu - \delta(q - 1)\right)\right] > \left(m_1(x - y) - \omega z\right)(d + \mu)(d + \delta)$.

Furthermore, it will be studied in the analysis of local stability from each equilibrium. This analysis can be used to determine the dynamics of system behavior in mathematical models of drug addicts with the criminal law aspect. In the first step, we determine the locally asymptotically stable of non-endemic equilibrium. Linearizing model (7) at the non-endemic equilibrium gives eigenvalues $\lambda_1 = -(d + \mu)$, $\lambda_2 = -(d + \delta)$ and the remainder are the roots of the following equation:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{9}$$

where

$$\begin{aligned}
a_1 &= m_1 + m_2 + m_3 - \beta_1 S^0 - \beta_1 \xi C^0 \\
a_2 &= -\beta_1 \xi C^0 m_3 - \beta_1 \xi C^0 m_2 - \gamma\omega - \pi\beta_2 \xi C^0 - \theta\sigma + m_1 m_2 - \beta_1 S^0 m_3 - \pi\beta_2 S^0 + m_1 m_3 - \beta_1 S^0 m_2 + m_2 m_3 \\
a_3 &= -m_2 m_3 \beta_1 S^0 + \theta\sigma\beta_1 S^0 - m_2 m_3 \beta_1 \xi C^0 + \theta\sigma\beta_1 \xi C^0 + m_1 m_2 m_3 - \theta\sigma m_1 - \pi m_3 \beta_2 S^0 - \pi m_3 \beta_2 \xi C^0 \\
&\quad -\pi\omega\theta - \gamma\sigma\beta_2 S^0 - \gamma\sigma\beta_2 \xi C^0 - \gamma\omega m_2.
\end{aligned}$$

The non-endemic equilibrium will be asymptotically stable if and only if Eq (9) has negative real parts roots. Using the Routh Hurwitz criteria, the cubic equation will has negative roots if $a_1, a_2, a_3 > 0$ and $a_1 a_2 - a_3 > 0$. Hence, the non-endemic equilibrium E^0 will be locally asymptotically stable if and only if $R_i < 1$, for i = 0,1,2,3, where

$$\begin{aligned}
R_0 &= \frac{\beta_1\left(S^0 + \xi C^0\right)}{m_1 + m_2 + m_3}, \\
R_1 &= \frac{R_0\left(\beta_1 (m_2 + m_3) + \pi\beta_2\right)(m_1 + m_2 + m_3) + \beta_1 (\gamma\omega + \theta\sigma)}{\beta_1 (m_1 m_2 + m_1 m_3 + m_2 m_3)}, \\
R_2 &= \frac{R_0 (m_1 + m_2 + m_3)\left(\beta_1 m_2 m_3 + \beta_2 (\pi m_3 + \gamma\sigma)\right) + \beta_1 (\theta\sigma m_1 + \pi\omega\theta + \gamma\omega m_2)}{\beta_1 \left(R_0 \theta\sigma (m_1 + m_2 + m_3) + m_1 m_2 m_3\right) F + m_3}, \\
R_3 &= \frac{a_3}{a_1 a_2}.
\end{aligned}$$

This represents that if all conditions are satisfied, there is no spread of drug addicts in the population.
Then, $R_0 = \frac{\beta_1(S^0 + \xi C^0)}{m_1 + m_2 + m_3}$ is referred to as the basic reproduction number or threshold parameter of a situation that indicates the condition of drug addicts is spreading or not.

Furthermore, the stability of the endemic equilibrium is difficult to solve analytically. Therefore, the stability of the endemic equilibrium will be analyzed through numerical simulations using phase fields. The parameter values used are presented in Table 3. The phase portrait of the model in the $L - W$ plane is given in Figure 2. Based on the three different initial values, it can be seen that the graphs tend to converge to the same point. In addition, the value of the basic reproduction ratio is $R_0 = 122,086 > 1$. Hence, the endemic equilibrium tends to be asymptotically stable if $R_0 > 1$. This shows that the population of drug addicts can transmit their consumptive behavior to an average of more than one new addict. Hence, drug addicts can spread or in other words the endemic occurrence of the drug addict.

In Figure 3, we display the effect of the criminal law aspect to the number of mild drug addicts (L) and the number of heavy level drugs addicts (H) for different values of γ and θ respectively. From Figure 3(a), it can be compared that as the parameter γ increase, the population of mild drugs addicts decreases significantly, while from Figure 3(b), it can be summarized that as the parameter θ increase, the population of heavy level drugs addicts tends to decrease.

TABLE 3. Parameters value of the model.

Parameter	Value	Unit	Source
d	0.02	$\frac{1}{year}$	[11]
Λ	1500	$\frac{\text{People}}{year}$	Assumed
q	0.8	$\frac{1}{year}$	[11]
μ	0.1	$\frac{1}{year}$	Assumed
β_1	0.7	$year$	[11]
β_2	0.8	$\frac{1}{year}$	[11]
ξ	0.9	$\frac{1}{year}$	[11]
d_1	0.2	$\frac{1}{year}$	[11]
d_2	0,.	$\frac{1}{year}$	[11]
γ	0.3	$\frac{1}{year}$	[11]
ω	0.001	$\frac{1}{year}$	Assumed
π	0.03	$\frac{1}{year}$	[11]
δ	0.01	$\frac{1}{\text{year}}$	[11]
θ	0.421	$\frac{1}{\text{year}}$	Assumed
σ	0.7	$\frac{1}{\text{year}}$	[11]
m	0.25	$\frac{1}{\text{year}}$	[11])

OPTIMAL CONTROL PROBLEM

In this study, an optimal strategy analysis will be carried out to control drug addicts on the model with the criminal law aspect. The control variable used is the rehabilitation effort (u). Rehabilitation efforts include medical and social rehabilitation. The control strategy is expected to minimize the population of drug addicts.

The mathematical models of drug addicts with criminal law accompanied by controlling strategies are as follows.

$$\begin{aligned}
\frac{dS}{dt} &= (1-q)\,d\Lambda - \beta_1 SL - \beta_2 SH - (d+\mu) \\
\frac{dC}{dt} &= qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta)\,C \\
\frac{dL}{dt} &= \beta_1 SL + \beta_2 SH + \beta_1 \xi CL + \beta_2 \xi CH - (\gamma+\pi+d+d_1)\,L + \omega W - uL \\
\frac{dH}{dt} &= \pi L + \sigma W - (\theta + d + d_2)\,H - uH \\
\frac{dW}{dt} &= \theta H + \gamma L - (\omega + \sigma + m + d)\,W - uW \\
\frac{dR}{dt} &= mW + \delta C - dR + uL + uH + uW.
\end{aligned} \tag{10}$$

The cost function of the drug addicts model with the criminal law aspect incorporates the control optimal strategy is formulated as follows:

$$Min\ J = \int_0^{t_f} \left(L + H + W + \frac{A}{2}u^2 \right) dt \tag{11}$$

where $u \in [0, 1]$, $t \in [0, t_f]$, and A denotes the positive constant in the form of rehabilitation costs.

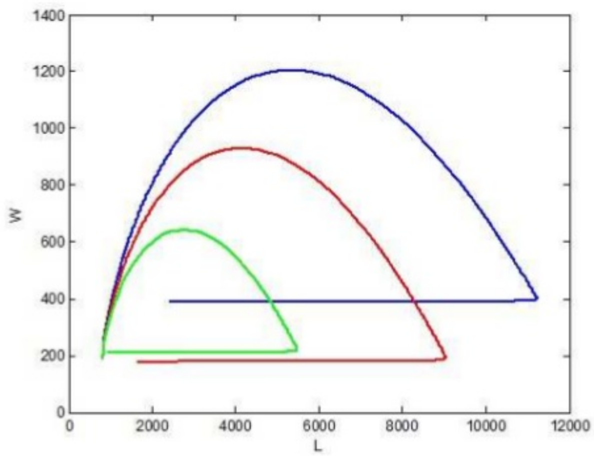
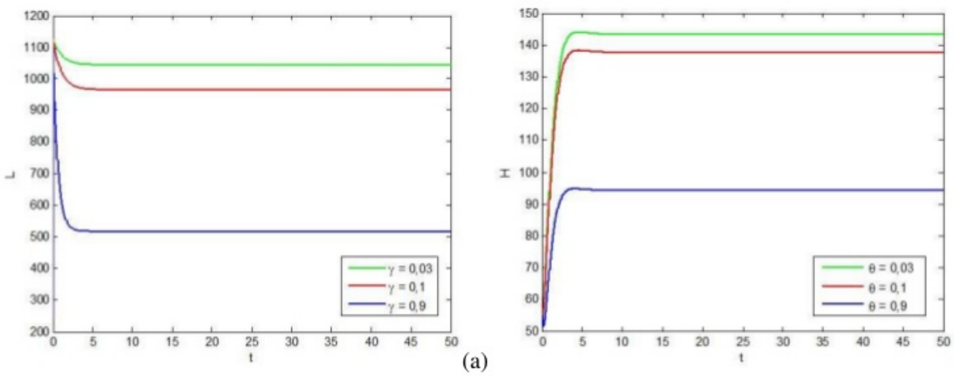


FIGURE 2. Phase portrait of the model in $L - W$ plane.



(a) (b)

FIGURE 3. (a) Comparison of population L for various γ, (b) Comparison of population H for various θ.

The problem of optimal control in the mathematical model of drug addicts is solved by applying the Pontryagin Maximum Principle [13]. Suppose the state variable in the model with the control strategy is as follows

$$x = (S, C, L, H, W, R)^T.$$

The initial values of all populations are assumed to be positive which are stated as follows

$$S(t_0), C(t_0), L(t_0), H(t_0), W(t_0), R(t_0) \geq 0.$$

The first step in the analysis of the problem of this control strategy is to form the Hamiltonian ($\mathcal{H}$) function.

$$\mathcal{H} = L + H + W + \frac{A}{2}u^2 + \sum_{i=1}^{6} \lambda_i^t f_i$$

where f_i is the right-hand side of the model (10) which is the $i-$th state variable equation. The variables λ_i denote the adjoint or co-state variables, for $i = 1, 2, ..., 6$.

The solution of the system is determined by taking the partial derivatives of the Hamiltonian function ($\mathcal{H}$) with respect to the associated state variables [14, 15].

Theorem 1 *Given optimal control u^* be the solution of the control systems (10)-(11). Then the adjoint variables are obtained by $\dot{\lambda}_i = -\frac{\partial \mathcal{H}}{\partial x}$, with transversality conditions $\lambda_i(t_f) = 0$, $i = 1, 2, \ldots, 6$, and*

$$u^* = min\left(1, max\left(0, \frac{(\lambda_5 - \lambda_6)\, W}{A_2}\right)\right). \tag{12}$$

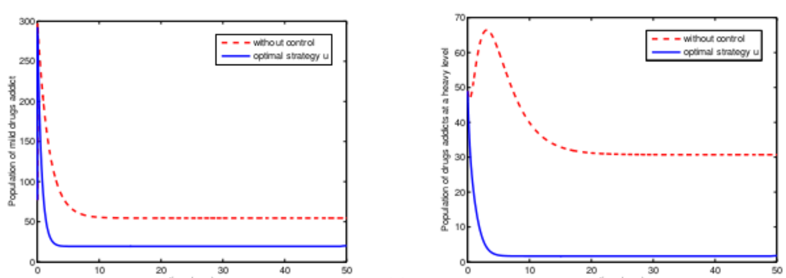
Proof. Using the condition $\dot{\lambda}_i = -\frac{\partial \mathcal{H}}{\partial x}$, we have the following adjoint system

$$\begin{aligned}
\dot{\lambda}_1 &= -\frac{\partial \mathcal{H}}{\partial S} = -\left[\lambda_1 \left(-\beta_1 L - \beta_2 H - (d+\mu)\right) + \lambda_2 \mu + \lambda_3 \left(\beta_1 L + \beta_2 H\right)\right] \\
\dot{\lambda}_2 &= -\frac{\partial \mathcal{H}}{\partial C} = -\left[\lambda_2 \left(-\beta_1 \xi L - \beta_2 \xi H\right) - (d+\delta) + \lambda_3 \left(\beta_1 \xi L + \beta_2 \xi H\right) + \lambda_6 \delta\right] \\
\dot{\lambda}_3 &= -\frac{\partial \mathcal{H}}{\partial L} = -\left[1 - \lambda_1 \beta_1 S - \lambda_2 \beta_1 \xi C + \lambda_3 \left(\beta_1 S + \beta_1 \xi C - m_1\right) + \lambda_4 \pi + \lambda_5 \gamma\right] \\
\dot{\lambda}_4 &= -\frac{\partial \mathcal{H}}{\partial H} = -\left[1 - \lambda_1 \beta_2 S - \lambda_2 \beta_2 \xi C + \lambda_3 \left(\beta_2 S + \beta_2 \xi C\right) + \lambda_4 m_2 + \lambda_5 \theta\right] \\
\dot{\lambda}_5 &= -\frac{\partial \mathcal{H}}{\partial W} = -\left[1 + \lambda_3 \omega + \lambda_4 \sigma - \lambda_5 \left(m_3 + u\right) + \lambda_6 \left(m + u\right)\right] \\
\dot{\lambda}_6 &= -\frac{\partial \mathcal{H}}{\partial R} = \lambda_6 d,
\end{aligned} \tag{13}$$

where $\lambda_i\left(t_f\right) = 0,\ i = 1, 2, 3, \ldots, 6$. Furthermore, in order to obtain optimal controls given in (12), we use the condition $\frac{\partial \mathcal{H}}{\partial u}$. □

NUMERICAL SIMULATION

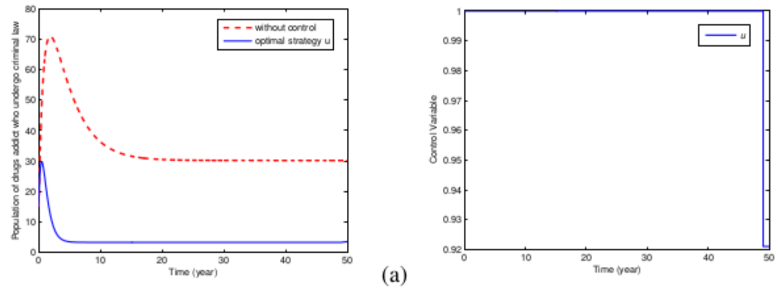
In this section, we discuss the numerical simulation of the model without and with control. The simulation is used to determine the effectiveness of rehabilitation efforts (u) as a form of strategy that can minimize the spread of drug addicts with the criminal law aspect. Simulation is done using the Runge-Kutta fourth-order scheme. The initial values used in this simulation are $(S(t_0), C(t_0), L(t_0), H(t_0), W(t_0), R(t_0)) = (135,\ 90,\ 77,\ 49,\ 15,\ 11)$. The time is taken in 50 years. The weighting constant for the cost of rehabilitation efforts is $A = 10$. The parameter values used to refer to Table 3.



(a) (b)

FIGURE 4. Comparison of population L (a) and H (b) with and without control

Figure 4 illustrates a comparison of the number of mild drug addicts and a heavy level of drug addicts populations before and after being given control. From Figure 4(a)-(b), it is seen that using the control strategy, the number of the population of mild-level drug addicts and also heavy-level drug addicts more decrease compared without the control. The strategy in the form of rehabilitation can reduce both populations until the end of the observation. More specifically, rehabilitation efforts can reduce mild drugs addicts and heavy drug addicts populations by 65% and 96% respectively.



(a) (b)

FIGURE 5. (a) Comparison of population W with and without control, (b) Control profile.

Figure 5(a) provides a comparison of the number of drug addicts who undergo criminal law population with and without the control strategy. The control strategy in the form of rehabilitation can reduce the population of drug addicts who undergo criminal law until the end of the observation. More specifically, rehabilitation efforts can reduce the population of drug addicts who undergo criminal law by 90%. Furthermore, the simulation results for the control profile is presented in Figure 5(b). From Figure 5(b) it can be seen that the rehabilitation effort should be given at the beginning of the observation until the 49th year is 100%, which means that the rehabilitation efforts carried out are fully intensive and then decreasing in the 50th year.

CONCLUSIONS

In this paper, we presented the mathematical model of drug addicts with criminal law aspect. The model has two equilibriums, namely a non-endemic equilibrium and an endemic equilibrium. Non-endemic equilibrium will be locally asymptotically stable if the basic reproduction number less than unity and complying several conditions, while the endemic equilibrium tends to be asymptotically stable if the basic reproduction number greater than unity. Based on the simulation, it is indicated that criminal law gives a significant effect to reduce the number of mild drug addicts and also the number of heavy levels drug addicts. Further, the model incorporates the control strategy in the form of rehabilitation efforts was analyzed. The optimal control problem is solved analytically by using the Pontryagin Maximum Principle. The numerical results of the model with and without control strategy show that the rehabilitation efforts can decrease the drug addicts in the population.

ACKNOWLEDGMENTS

Part of the research was financially supported by Universitas Airlangga 2019.

REFERENCES

[1] Badan Narkotika Nasional, *Pedoman Pencegahan Penyalahgunaan Narkotika Bagi Pecandu* (Badan Narkotika Nasional, Jakarta, 2004).

[2] United Nations, *World Population Prospects 2017* (2017) https://Population.un.org/wpp. [Accesed on 15th November, 2018].

[3] Tjazuli., *Indonesia (BUKAN) Surga Narkoba* (Edisi 159, Parlementeria, Jakarta, 2018).

[4] Badan Narkotika Nasional, *Pencanangan Tahun 2014 Sebagai Tahun Penyelamatan Pengguna Narkoba* (2014) www.bnn.go.id/read.pressrelease/11731/pencenangan-tahun-2014-sebagai-tahun-penyelamatan-pengguna-narkoba [Accesed on 15th November, 2018].

[5] Fatmawati and H. Tasman, Appl. Math. Sci. 7, 33793391 (2013).

[6] Fatmawati and H. Tasman, Math. Biosci. 262, 73-79 (2015).

[7] Fatmawati and H. Tasman, Int. J. Math. Math. Sci. 2016, Article ID 8261208 (2016).

[8] C. P. Bhunu, Commun. Nonlinear Sci. 19, 1908-1917 (2013).

[9] M. Ma, S. Liu, H. Xiang and J. Li, Physica A. 491, 641-649 (2017).
[10] M. Ma, S. Liu and J. Li, Commun. Nonlinear Sci. 50, 169-179 (2017).
[11] J. Li and M. Ma, Infectious Disease Modelling 3, 74-84 (2018).
[12] S. Mushayabasa and G. Tapedzesa, Comput Math Method M. 2015, Article ID 383154 (2015).
[13] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and E. F. Mishchenko, *The Mathematical Theory of Optimal Processes* (Wiley, New York, 1962).
[14] F. I. Lewis and V. L. Syrmos, *Optimal Control* (Willy Interscience, Canada, 2006).
[15] D. S. Naidu, *Optimal Control Systems* (CRC Press, New York, 2002)

Mathematical model analysis of a drug transmission with criminal law and its optimal control

ORIGINALITY REPORT

14%	5%	12%	0%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Jun Li, Mingju Ma. "The analysis of a drug transmission model with family education and public health education", Infectious Disease Modelling, 2018 Publication	2%
2	Moh Mashum Mujur Ihsanjaya, Nanang Susyanto. "Mathematical model of changes in smoking behavior which involves smokers who temporarily and permanently quit smoking", AIP Publishing, 2019 Publication	1%
3	Ken Prameswari, Fajar Adi-Kusumo. "A mathematical model for phase transition regulation on the cell cycle in early stage of breast cancer", AIP Publishing, 2019 Publication	1%
4	oaji.net Internet Source	1%
5	R. U. Diva Amalia, Fatmawati, Windarto, Didik	

Khusnul Arif. "Optimal control of predator-prey mathematical model with infection and harvesting on prey", Journal of Physics: Conference Series, 2018 — 1%
Publication

---

6 www.hindawi.com — 1%
Internet Source

---

7 Triyah Fatmawati, Badrus Zaman, Indah Werdiningsih. "Implementation of the common phrase index method on the phrase query for information retrieval", AIP Publishing, 2017 — 1%
Publication

---

8 www.m-hikari.com — 1%
Internet Source

---

9 Xu, R.. "Global dynamics of an SEIS epidemic model with saturation incidence and latent period", Applied Mathematics and Computation, 20120401 — 1%
Publication

---

10 J. Li, X. Zou. "Generalization of the Kermack-McKendrick SIR Model to a Patchy Environment for a Disease with Latency", Mathematical Modelling of Natural Phenomena, 2009 — 1%
Publication

---

11 Fatmawati, Hengki Tasman. "An optimal control strategy to reduce the spread of malaria resistance", Mathematical Biosciences, 2015 — 1%

Publication

12 Ahmadin, Fatmawati. "Mathematical modeling of drug resistance in tuberculosis transmission and optimal control treatment", Applied Mathematical Sciences, 2014 <1%

Publication

13 R. Xu. "Global dynamics of a vector disease model with saturation incidence and time delay", IMA Journal of Applied Mathematics, 12/01/2011 <1%

Publication

14 JINLIANG WANG, SHENGQIANG LIU, YASUHIRO TAKEUCHI. "THRESHOLD DYNAMICS IN A PERIODIC SVEIR EPIDEMIC MODEL", International Journal of Biomathematics, 2012 <1%

Publication

15 Wendi Wang. "Epidemic Models with Population Dispersal", Biological and Medical Physics Biomedical Engineering, 2006 <1%

Publication

16 Robert McCormack. "Multi-patch deterministic and stochastic models for wildlife diseases", Journal of Biological Dynamics, 2007 <1%

Publication

17 A. Fall, A. Iggidr, G. Sallet, J. J. Tewa. "Epidemiological Models and Lyapunov <1%

Functions", Mathematical Modelling of Natural Phenomena, 2008
Publication

18	link.springer.com Internet Source	<1%
19	Lawal Jibril, Sule Amiru. "On existence and sensitivity-index of a cholera carrier epidemic model", AIP Publishing, 2019 Publication	<1%
20	www.ircs.upenn.edu Internet Source	<1%
21	Dykema, K.J.. "Multilinear function series and transforms in free probability theory", Advances in Mathematics, 20070115 Publication	<1%
22	Timothy Robin Y. Teng, Elvira P. De Lara-Tuprio, Jay Michael R. Macalalag. "An HIV/AIDS epidemic model with media coverage, vertical transmission and time delays", AIP Publishing, 2019 Publication	<1%

Exclude quotes	Off	Exclude matches	Off
Exclude bibliography	On		

Mathematical model analysis of a drug transmission with criminal law and its optimal control

GRADEMARK REPORT

FINAL GRADE

/0

GENERAL COMMENTS

Instructor

PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9

PAGE 10

PAGE 11