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A Mathematical Model of Social Media Popularity with Standard Incidence Rate

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Abstract. Facebook, Instagram, Twitter are some popular social media. DeLegge and Wangler (2017) have developed a Susceptible-Infectious-Removed type mathematical model to describe social media popularity. The DeLegge and Wangler model was a bilinear incidence rate model. In this paper, we improve the DeLegge and Wangler model by considering standard incidence rate. The presented model takes the form of an ordinary differential equation system that describes dynamic of susceptible (population of who are not social media users), infectious population (population of social networks users) and removed population (population of who leave social media). The presented model has three equilibria namely the "no social media users" equilibrium, "very popular social media" equilibrium and "popular social media" equilibrium. We find that the three equilibria are conditionally asymptotically stable. We also perform some numerical simulations to verify the analytical results.

1. Introduction

The rapid development of science and technology provides positive benefits for human life, including the ease of communication between individuals. Communication costs in the past were expensive. However, the communication cost of the present is relatively cheap because of the existence of various social media. Social media is an online application that allows users to post information about their profile, including names, photos, and other material to be viewed by social media users. In addition, social media users can also communicate with each other in innovative ways [1].

One popular social media is Facebook [2]. Facebook began to exist in February 2004, Mark Zuckerberg, a student at Harvard University, launched this site [3]. Facebook is a friendship site from the United States. It was estimated 80% of internet users worldwide had a Facebook account in 2014, and 40% of them were active Facebook users or accessed Facebook once per month throughout the year. In October 2018, it was estimated 2.235 billion Facebook users were active throughout the world [4].

Facebook's popularity reminded social media users of the popularity of previous social media, namely Myspace. Myspace was launched in August 2003, but incomplete features on Myspace warned Facebook to develop better features. Facebook had better features than Myspace, where Facebook offered a choice of customized profile pages, sharing photos, sharing music and online games [5]. The incomplete feature caused the rapid decline of Myspace popularity. As a result, NewsCrop, owner of Myspace, sold Myspace. The fast decline of Myspace can also occur on other social media such as Facebook [6].

Mathematical modelling has a significant role in understanding many real problems, including the dynamics of social media users. Researchers constructed and analysed many mathematical models to describe the dynamics of social media users. Cannarella and Spechler used a SIR-like epidemiological model to describe user acceptance and user rejection of online social networks. From their model, Cannarella and Spechler predicted a fast decrease of Facebook activity in few years after 2014 [6]. Zhu et al. applied an epidemic mathematical model to explain adoption and leaving process in online social networks users. From their model, Zhu et al. predicted demographic evolution in online social networks users [7]. Tanaka et al. also applied a SIR-like epidemiological model to explain the growth and the decrease of a social networking services users. Tanaka et al. found that the growth of social networking services users could be quickened by invite of new service users [8]. Proskurnikov and Tempo discussed continuous and discrete dynamical models to describe the dynamics of social networks [9]. DeLegge and Wangler applied a SIR-like model to study dynamics of Facebook users. DeLegge and Wangler found that Facebook did not end yet at 2017 [10].

In their model, DeLegge and Wangler investigated the dynamics of susceptible populations (population of individuals are not currently social media users, but are open to join as social media users), infected population (population of social media users) and removed population (population of individuals are not currently members of the network and are not open to join). DeLegge and Wangler used bilinear incidence rate to model increasing rate of population of social media users. The bilinear incidence rate is only accurate in the early phases of an epidemic in a population of medium size ([11], [12]). In this paper, we improve the model from DeLegge and Wangler by consider standard incidence rate. In the next section, we present a mathematical model of social media users' dynamics with standard/fractional incidence rate. Then we discuss the linear stability of equilibria of the proposed model. We also perform some numerical simulations to illustrate analytical results of this study. Finally, the conclusions of this study are presented in the last section.

2. The proposed model

In this section, we proposed a mathematical model to describe dynamics of social media users. The proposed model is an improvement of the model from DeLegge and Wangler [10]. Here, we consider standard incidence rate to describe increasing rate of infected population. We constructed to proposed model under the following assumptions:

(1) The model contains three compartments, namely number of susceptible population (S), number of infected population (I) and number of removed population (R). Here S, I, R represent of individuals are not currently social media users but are open to join as social media users, number of social media user population, and number of individuals are not currently members of the network and are not open to join, respectively.
(2) Recruitment rate of susceptible population per time unit is constant.
(3) Natural mortality rate of susceptible, infected and removed population are constant.
(4) Rate at which removed individuals regain susceptibility is constant.
(5) The increasing rate of social media users due to social media promoting by social media users is constant.
(6) The decreasing rate of social media users due to social media leaving is constant.

Transmission diagram of the proposed model is presented in Figure 1.

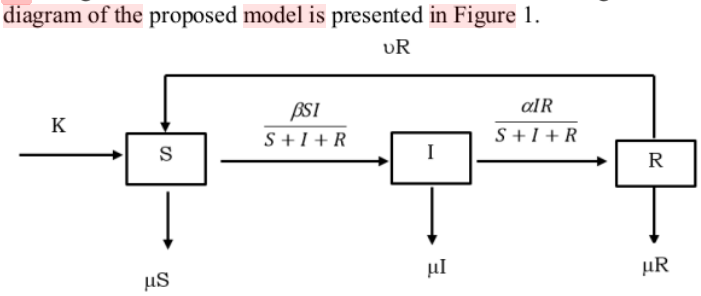

Figure 1. Transmission diagram of social media users mathematical model.

From the assumptions, the dynamics of social media users could be described by the following differential equation system:

$$\frac{dS}{dt} = K - \frac{\beta SI}{S+I+R} + \nu R - \mu S, \tag{1}$$

$$\frac{dI}{dt} = \frac{\beta SI}{S+I+R} - \frac{\alpha IR}{S+I+R} - \mu I, \tag{2}$$

$$\frac{dR}{dt} = \frac{\alpha IR}{S+I+R} - \nu R - \mu R. \tag{3}$$

The region of biological interest of the model in Eq. (1)-(3) is

$$\Omega \coloneqq \{(S, I, R) \in \mathbf{R}^3, S, I, R \geq 0, S + I + R > 0\}. \tag{4}$$

All parameters in the model in eq. $(1) - (3)$ are positive. When the population size is constant, the mathematical model with standard incidence rate in eq. (1)-(3) could be simplified into the model with bilinear incidence rate. Hence, the mathematical model with standard incidence rate could be considered as a generalization and an improvement of the model with bilinear incidence rate. Description of the parameters is presented in Table 1.

Table 1. Description of parameters of the proposed model

Parameter	Description
K	Recruitment rate of susceptible population per time unit
μ	Natural mortality rate of susceptible, infected and removed population are constant
ν	Reconsidering rate of social media using
β	Success rate of social media promoting
α	Success rate of social media leaving

The differential equations (1)-(3) describe the dynamics of susceptible, infected and removed population respectively. Susceptible population increases due to recruitment and removed individuals regain susceptibility. On the other hand, susceptible population decreases due to joining as social media users and natural mortality. Infected population increases because of susceptible population join as social media users. On the other hand, infected population decreases because of natural mortality and social media users leave the social media. Removed population increases due to social media users leave the social media. On the other hand, removed population decreases due to natural mortality and removed individuals regain susceptibility.

3. Analysis of the proposed model

The model in Eq. (1)-(3) has three equilibria, namely social media users-free equilibrium (E_1), "very popular social media" equilibrium (E_2) and "popular social media" equilibrium (E_3). The social media users-free equilibrium is given by:

$$E_1 \coloneqq (S_1, I_1, R_1) = \left(\frac{K}{\mu}, 0{,}0\right). \tag{5}$$

The “very popular social media" equilibrium (E_2) and “popular social media” equilibrium (E_3) is given by:

$$E_2 \coloneqq (S_2, I_2, R_2) = \left(\frac{K}{\beta}, \frac{K}{\beta}\left(\frac{\beta}{\mu} - 1\right), 0\right), \tag{6}$$

$$E_3 \coloneqq (S_3, I_3, R_3) = \left(\frac{K(\alpha-\nu)}{\mu(\beta+\alpha)}, \frac{K}{\alpha\mu}(\nu+\mu), \frac{K(\beta\alpha-\beta\mu-\beta\nu-\alpha\nu)}{\alpha\mu(\beta+\alpha)}\right), \tag{7}$$

respectively.

The social media users-free equilibrium always exists. The “very popular social media” equilibrium exists if $\beta > \mu$, while the “popular social media” equilibrium exist if $\beta\alpha > \beta(\mu + \nu) + \alpha\mu$.

The Jacobian matrix of the proposed model in Eq. (1)-(3) is given by:

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$$J = \begin{bmatrix} -\frac{\beta I(I+R)}{(S+I+R)^2} - \mu & -\frac{\beta S(S+R)}{(S+I+R)^2} & \frac{\beta SI}{(S+I+R)^2} + \nu \\ \frac{\beta I(I+R)}{(S+I+R)^2} + \frac{\alpha IR}{(S+I+R)^2} & \frac{\beta S(I+R)}{(S+I+R)^2} - \frac{\alpha R(S+R)}{(S+I+R)^2} - \mu & -\frac{\beta SI}{(S+I+R)^2} - \frac{\alpha I(S+I)}{(S+I+R)^2} \\ -\frac{\alpha IR}{(S+I+R)^2} & \frac{\alpha R(S+R)}{(S+I+R)^2} & \frac{\alpha IR}{(S+I+R)^2} - (\mu+\nu) \end{bmatrix}. \quad (8)$$

Theorem 1 gives the stability of the social media users-free equilibrium.

Theorem 1. *The social media users-free equilibrium* E_1 *is locally asymptotically stable if* $\beta < \mu$. *Moreover, the social media users-free equilibrium is unstable if* $\beta > \mu$.

Proof. The Jacobian matrix of the proposed model in eq. (1)-(3) evaluated at the social media users-free equilibrium E_1 is given by

$$J(E_1) = \begin{bmatrix} -\mu & -\beta & \nu \\ 0 & \beta-\mu & 0 \\ 0 & 0 & -(\mu+\nu) \end{bmatrix}. \quad (9)$$

Eigenvalues of $J(E_1)$ are obtained from the following characteristic polynomial

$$(\lambda+\mu)(\lambda-\beta+\mu)(\lambda+\mu+\nu) = 0. \quad (10)$$

Hence eigenvalues of $J(E_1)$ are $\lambda_1 = -\mu, \lambda_2 = \beta-\mu,$ and $\lambda_3 = -(\mu+\nu)$.

Hence all eigenvalues of $J(E_1)$ are negative if and only if $\beta < \mu$. Therefore, social media users-free equilibrium is locally asymptotically stable if $\beta < \mu$.

If $\beta > \mu$, then the characteristic polynomial in eq. (10) has one positive roots. Hence, the Jacobian matrix $J(E_1)$ has one positive eigenvalue. Consequently, the social media users-free equilibrium is unstable. This completes the proof. ■

Theorem 2 presents global stability of the social media users-free equilibrium.

Theorem 2. *If* $\beta \le \mu$, *then the social media users-free equilibrium* E_1 *is globally asymptotically stable.*

Proof. We define a Lyapunov function

$U: \{(S,I,R) \in \Omega : S > 0\} \to \mathbf{R}$ where $U(S,I,P) = I + R$.

U is a nonnegative function on the domain Ω. Moreover, U attains minimum value when $I = P = 0$.

The time derivative of U evaluated at the solution of mathematical model in eq. (1)-(3) is given by

$$\frac{dU}{dt} = \frac{dI}{dt} + \frac{dR}{dt} = -\frac{\beta I(I+R)}{S+I+R} - (\mu-\beta)I - (\mu+\nu)R \le 0.$$

In addition, we find $\frac{dU}{dt} = 0$ if and only if $I = R = 0$. Hence, $I(t) \to 0$ and $R(t) \to 0$ as $t \to \infty$. By using $I(t) = 0$ and $R(t) = 0$ in eq. (1), we obtain $S(t) \to \frac{K}{\mu}$ as $t \to \infty$. Consequently, by using LaSalle invariant principle, we find that every solution of the mathematical model in eq.(1)-(3) with initial value in Ω tends to the social media users-free equilibrium as $t \to \infty$ [13] ■.

Theorem 3 presents stability of "very popular social media" equilibrium.

Theorem 3. *The "very popular social media" equilibrium* E_2 *is locally asymptotically stable whenever* $\beta > \mu$ *and* $\alpha < (\mu+\nu)$.

Proof. The condition $\beta > \mu$ is the necessary and sufficient condition of existence of "very popular social media" equilibrium. Let $J(E_2)$ be the Jacobian matrix of the proposed model in eq. (1)-(3) evaluated at the "very popular social media" equilibrium E_2. Eigenvalues of $J(E_2)$ fulfils the following characteristic polynomial

$$\frac{1}{\beta}(\beta\lambda - \beta\alpha + \beta(\mu+\nu) + \alpha\mu)(\lambda^2 + \beta\lambda + \mu(\beta-\mu)) = 0.$$

Hence eigenvalues of $J(E_2)$ are $\lambda_1 = -\frac{1}{\beta}(\beta(\mu+\nu) + \alpha\mu - \beta\alpha)$ and the zeros of the following quadratic polynomial

$$(\lambda^2 + \beta\lambda + \mu(\beta-\mu)) = 0.$$

Since $\alpha < (\mu+\nu)$, *then* $\lambda_1 < 0$. *By using the Routh-Hurwitz theorem, all eigenvalues of* $J(E_2)$ are negative or complex eigenvalues with negative real parts if and only if $\alpha < (\mu+\nu)$ and $\beta > \mu$.

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Therefore, the "very popular social media" equilibrium is locally asymptotically stable if $\alpha < (\mu + \nu)$ and $\beta > \mu$. This completes the proof. ■

Theorem 4 presents stability of "popular social media" equilibrium.

Theorem 4. *If $\beta\alpha > \beta(\mu + \nu) + \alpha\mu$, then the "popular social media" equilibrium E_3 is locally asymptotically stable.*

Proof. Let $J(E_3)$ be the Jacobian matrix of the proposed model in eq. (1)-(3) evaluated at the "popular social media" equilibrium E_3. Eigenvalues of $J(E_3)$ are zeros of the following characteristic polynomial

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$$

where

$$a_1 = \frac{\beta(\mu + \nu) + \alpha\mu}{\alpha} = \frac{\beta(\mu + \nu)}{\alpha} + \mu,$$
$$a_2 = \frac{(\mu + \nu)[\beta\alpha - (\beta\nu + \alpha\mu)]}{\alpha},$$
$$a_3 = \frac{\mu(\mu + \nu)[\beta\alpha - (\beta\mu + \beta\nu + \alpha\mu)]}{\alpha}.$$

Since all parameters are positive, then $a_1 > 0$. It is clear that $a_2, a_3 > 0$ whenever $\beta\alpha - (\beta\mu + \beta\nu + \alpha\mu)$. We find that

$$a_1a_2 - a_3 = \frac{\beta(\mu + \nu)\big(\beta\alpha\mu + \beta\alpha\nu - \nu(\beta\mu + \beta\nu + \alpha\mu)\big)}{\alpha^2}$$
$$= \frac{\beta(\mu + \nu)\big[\beta\alpha\mu + \nu\big(\beta\alpha - (\beta\mu + \beta\nu + \alpha\mu)\big)\big]}{\alpha^2}.$$

By using the Routh-Hurwitz theorem, all eigenvalues of $J(E_3)$ are negative or complex eigenvalues with negative real parts if $\beta\alpha > (\beta\mu + \beta\nu + \alpha\mu)$. Consequently, the "popular social media" equilibrium is locally asymptotically stable if $\beta\alpha > (\beta\mu + \beta\nu + \alpha\mu)$. This completes the proof. ■

4. Numerical Simulations

In this section, we present some numerical simulations to describe solution of the proposed model at "social media users-free" equilibrium and very popular social media equilibrium. By choosing the following scaling

$$t^* = \mu t, s = \frac{S\mu}{K}, i = \frac{I\mu}{K}, r = \frac{R\mu}{K}, \tag{11}$$

We obtain the following dimensionless model

$$\frac{ds}{dt} = 1 - \frac{\beta si}{\mu(s+i+r)} + \frac{\nu}{\mu}r - s, \tag{12}$$
$$\frac{di}{dt} = \frac{\beta si}{\mu(s+i+r)} - \frac{\alpha ir}{\mu(s+i+r)} - i, \tag{13}$$
$$\frac{dr}{dt} = \frac{\alpha ir}{\mu(s+i+r)} - \frac{\nu}{\mu}r - r. \tag{14}$$

Here we omit the star sign for simplicity. The initial conditions are $s(0) = 0.6, i(0) = 0.2, r(0) = 0.2$. We simulate the proposed model from t = 0 until t = 10 dimensionless time unit. Parameter values used in the simulation are shown in Table 2.

From Table 2, we find that $\beta < \mu$. In this condition, success rate of social media promoting is lower than the removal rate of social media users due to natural mortality. This condition yields the social media users-free condition. Consequently, infectious population and removed population tend to zero for enough long time. This situation is illustrated in the Figure 2.

Table 2. Parameter values used in the simulation

Parameter	Value	Source
β	0.0005 / month	Assumption
μ	0.0013 / month	[10]
ν	0.0010 / month	[10]
α	0.0004 / month	[10]

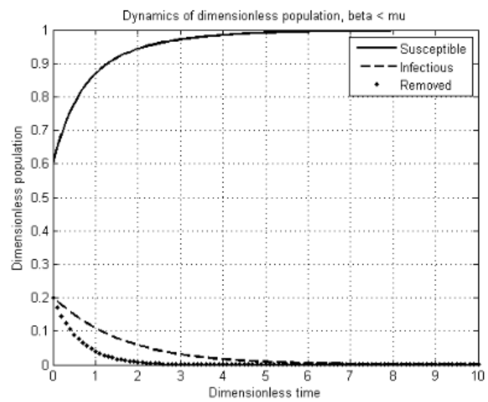

Figure 2. Dynamics of social media users for β < μ.

We also perform numerical simulation for the "very popular social media" condition. Parameter values for this simulation are shown in Table 2 except for parameter β, where β = 0.0045. In this condition, success rate of social media promoting is greater than the removal rate of social media users due to natural mortality. We also find that $\alpha < \mu + \nu$, where it represents the success rate of social media leaving is lower than the removal rate of the removed population due to reconsidering of social media using and natural mortality. This condition yields the very popular social media condition. Consequently, removed population tend to zero for enough long time. This situation is illustrated in the Figure 3.

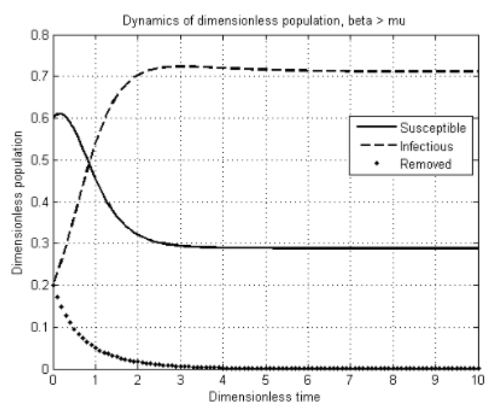

Figure 3. Dynamics of social media users for β > μ and (α < μ + ν).

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We also perform numerical simulation for the "popular social media" condition. Parameter values for this simulation are shown in Table 2 except for parameters β and α, where β = 0.02, α = 0.004. In this condition, success rate of social media promoting is greater than the removal rate of social media users due to natural mortality, and the success rate of social media leaving is greater than the removal rate of the removed population due to reconsidering of social media using and natural mortality. This condition yields the popular social media condition. Consequently, susceptible population, infectious population and removed population always exist and the values tend to the popular social media equilibrium for enough long time. This situation is illustrated in the Figure 4.

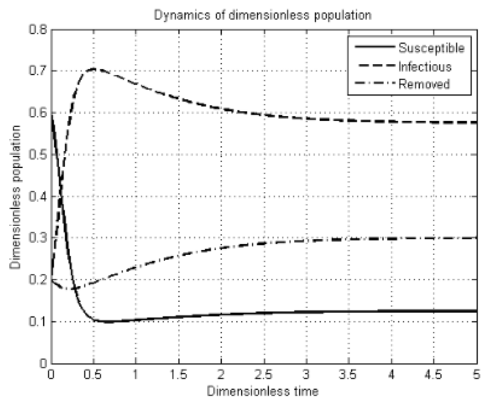

Figure 4. Dynamics of social media users for $\beta\alpha > (\beta\mu + \beta\nu + \alpha\mu)$.

5. Conclusion

In this paper, we have studied a mathematical model of social media popularity with standard incidence rate. The proposed model has three equilibria, namely the no social media user equilibrium, very popular social media equilibrium and popular social media equilibrium. We found that property of the proposed model was characterized by four parameters namely success rate of social media promoting, success rate of social media leaving, reconsidering rate of social media using and natural mortality of population.

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PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9