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Fatmawati, Windarto, Lathifah Hanif

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## Application of optimal control strategies to HIV-malaria co-infection dynamics

Fatmawati<sup>1</sup>, Windarto<sup>1</sup> and Lathifah Hanif<sup>1</sup>

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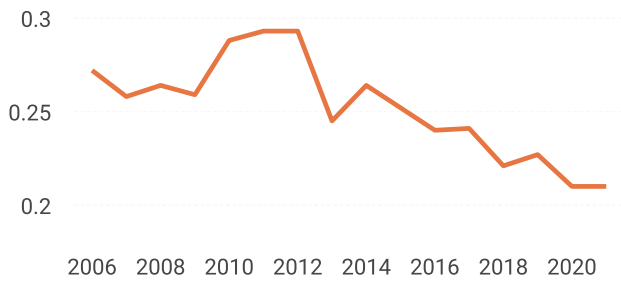
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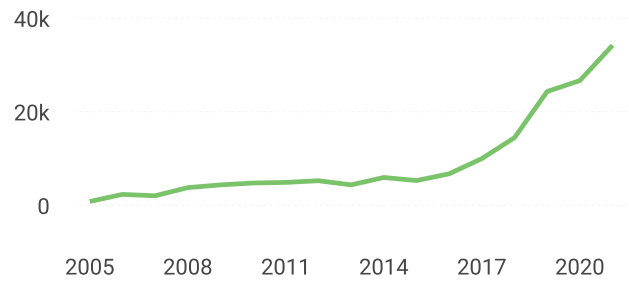
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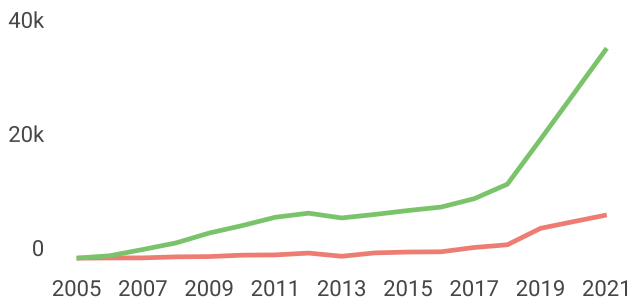
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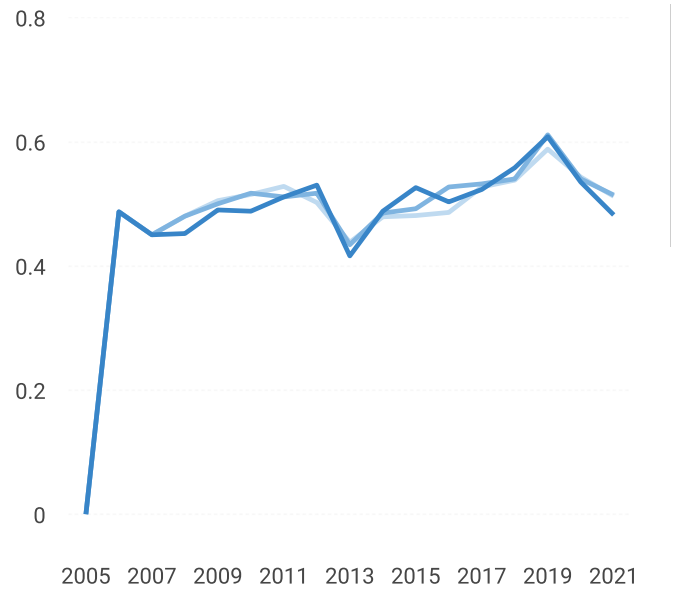
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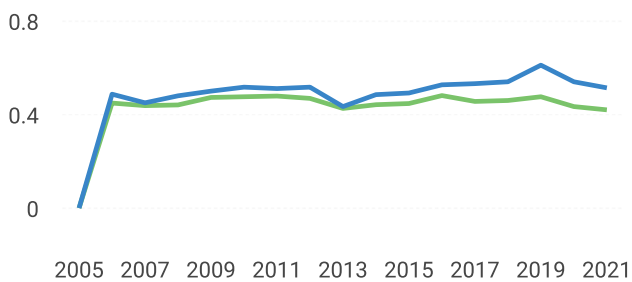
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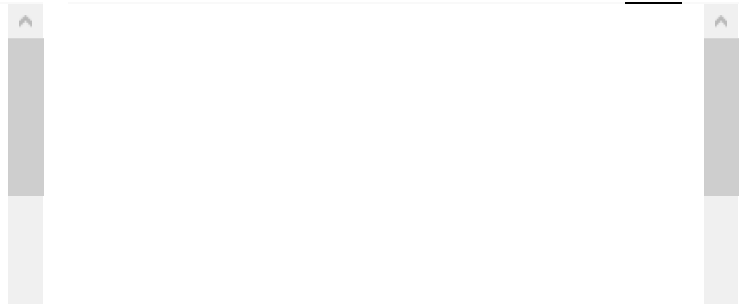


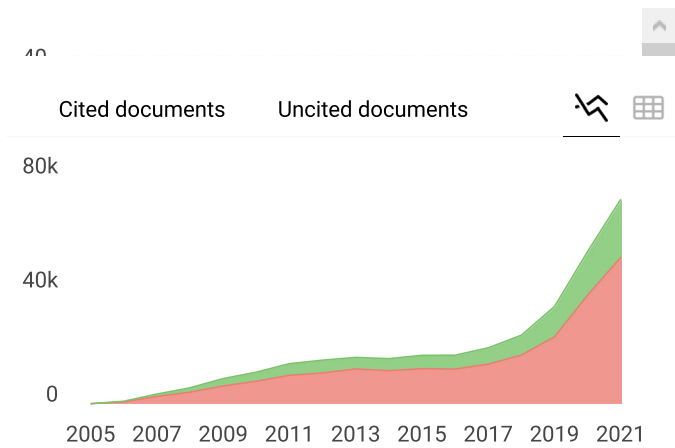
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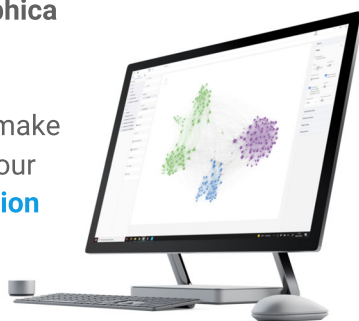
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## Welcome Message from the Conference Chair

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The honorable

Rector of ITS,

Keynote Speaker, Director of Research and Community Service,

Invited Speakers,

Dean Of Faculty Of Mathematics, Computing, and Data Science

Head of Mathematics Department

Ladies and Gentlemen,

Assalamu'alaikum warahmatullahi wabarokatuh

On behalf of the ICoMPAC 2017 organizing committee, I am honored and delighted to welcome you to the third International Conference on Mathematics; Pure, Applied and Computation (ICoMPAC 2017) at Wyndham Hotel, Surabaya.

At this year, we are so pleased to accept many papers from Indonesia, Malaysia, Japan, Taiwan, Germany, Thailand, Taiwan, USA, Australia and UK. It is a great pleasure to have 1 keynote speaker and 3 invited speakers with us in this conference to share their knowledge.

This year's conference is themed "Mathematics for Supporting Society Welfare" with the hope that mathematics can take an active role in improving society welfare. The aim of this conference is to provide a forum for researchers, educators, students and industries to exchange ideas, to communicate and discuss research findings and new advancement in mathematics, and to explore possible avenues to foster academic and student exchange, as well as scientific activities. The conference will be a venue to communicate and discuss how mathematics can give contributions to improving society welfare, and give solutions to problems faced by industries.

As a conference chair of ICoMPAC 2017, I realized that the success of this conference depends ultimately on the many people who have worked with us in planning and organizing this conference, in particular for the review process and preparing the technical programs. Recognition should go to the Local Organizing Committee members who



have all worked extremely hard for the detail of important aspects of the conference programs.

Last but not least, I would like to thank Institute of Physics (IOP), for the cooperation for publishing papers presented in this conference to their proceedings. I hope this conference will be proven to be an inspiring experience for you. Enjoy your participation in the ICoMPAC 2017 and we hope that you have a memorable time visiting Surabaya. We also hope you return for the next ICoMPAC with even more colleagues.

Thank You,

Wassalamu'alaikum Wr. Wb.

Mardlijah

Conference Chair

## **The Speech from The Dean of Faculty of Mathematics, Computing, and Data Sciences**

Bismillahirrahmanirrahim,

Assalamualaikum warahmatullahi wabarakatuh.

Allahumma Sholli ala saydina Muhammad, wa'ala ali saydina Muhammad, robbis rohli sodri, wayassirli amri, wahlul ukhdatan minlisani yapkohu koili, amma ba'du,

The Honorable Rector of ITS (Prof. Ir. Joni Hermana, M.Sc.ES. Ph.D.), the distinguished speakers, the invited speakers, the participants and the committee of ICoMPAC 2017. We sincerely welcome and thank you for your coming to this event. This event has been going on for the third time. And for this time take the theme of mathematics for supporting the welfare of society.

The increasing number of people is a big problem for the state country in the world, especially developing countries including Indonesia. Indonesia is the country with the fourth largest population in the world after China, India and the United States. Rapid population growth can create complex problems for a country, such as economic, social, educational, cultural and criminal issues.

Economist Alfred Marshall (1842-1924) professor of political science Cambridge University says that poverty is a matter of concern and should be eliminated. For that there needs to be a great effort on the welfare of this society, namely by combining the ability to think logically, thoroughly and mastery of mathematics. This can be done one of them through enhancing the application of basic and applied research results in the field of mathematics. In line with this, it is necessary to work hard and smart in improving the quality and quantity of research in the field of mathematical modeling, operations research, stochastic, biomathematics, actuarial, statistics and finance which focuses on issues in local, national and international scope, either as scientific development or to contribute on problem solving in social life.

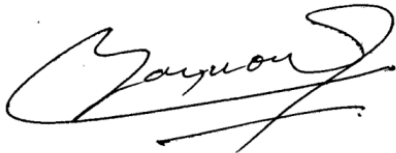
ICOMPAC 3 is the right moment to convey the idea to improve the welfare of society based on mathematics. Therefore, in this conference we hope there are efforts to implement the research results of mathematics, statistics and actuary in realizing a society that can meet all needs, whether that is basic need, psychological social or development need that can provide tangible and sustainable contribution for community.

We as the Dean of the Faculty of Mathematics, Computing, and Science Data congratulate the conference, hopefully produce a brilliant idea in the prosperity of society, success and smooth.

We do apologize if there is anything less pleased.  
Thank you for attention.

Wabillahi Taufiq Walhidayah,  
Wasalamualaikum Warahmatullahi Wabarakatuh.

Dean,

A handwritten signature in black ink, appearing to read 'Basuki Widodo', with a stylized flourish at the end.

Prof. Dr. Basuki Widodo, MSc.

## **The Speech from The Rector of Institut Teknologi Sepuluh Nopember**

I would like to convey my sincere congratulation to all involved parties for the successful organization of the third International Conference on Mathematics: Pure, Applied and Computation ICoMPAC 2017, organized by the Department of Mathematics Institut Teknologi Sepuluh Nopember (ITS) Surabaya. The ICoMPAC 2017 is held as part of our 57<sup>th</sup> ITS Anniversary.

It is a great pleasure and honor for me to welcome to all participants and thank to the keynote speakers and all invited speakers for the worthy time to share your experiences and expertise to all conference participants. I do believe that your participation to this conference is a highlight and give a significant insight to all of us. I expect that your patronage and support towards the advancement of knowledge through this event, will contribute to the future development of Mathematics.

As we know that the role of Mathematics is vital in many aspects of life. There are many problems arise in social, business, economic, environment, and many others that could be solved by Mathematics. I am sure that, ICoMPAC will be the flagship conference for researchers, students, and professionals in the area of Mathematics and its applications to disseminate their research advancements and discoveries, to network and exchange ideas in order to solve more problems.

Last but not least, I wish all participants have a very interesting and learning experiences during the conference. Moreover, I do hope that new collaborations among participants could be established. To our foreign guests, I wish you a memorable stay in Surabaya. We welcome you anytime to visit our university, Institut Teknologi Sepuluh Nopember, Surabaya.

Prof. Ir. Joni Hermana M.Sc.ES. Ph.D

Rector of the Institut Teknologi Sepuluh Nopember (ITS) Indonesia

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Amirul Hakam, Basuki Widodo, Tri Yogi Yuwono and Chairul Imron

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# Application of optimal control strategies to HIV-malaria co-infection dynamics

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**Abstract.** This paper presents a mathematical model of HIV and malaria co-infection transmission dynamics. Optimal control strategies such as malaria preventive, anti-malaria and antiretroviral (ARV) treatments are considered into the model to reduce the co-infection. First, we studied the existence and stability of equilibria of the presented model without control variables. The model has four equilibria, namely the disease-free equilibrium, the HIV endemic equilibrium, the malaria endemic equilibrium, and the co-infection equilibrium. We also obtain two basic reproduction ratios corresponding to the diseases. It was found that the disease-free equilibrium is locally asymptotically stable whenever their respective basic reproduction numbers are less than one. We also conducted a sensitivity analysis to determine the dominant factor controlling the transmission. sic reproduction numbers are less than one. We also conducted a sensitivity analysis to determine the dominant factor controlling the transmission. Then, the optimal control theory for the model was derived analytically by using Pontryagin Maximum Principle. Numerical simulations of the optimal control strategies are also performed to illustrate the results. From the numerical results, we conclude that the best strategy is to combine the malaria prevention and ARV treatments in order to reduce malaria and HIV co-infection populations.

## 1. Introduction

HIV (Human Immunodeficiency Virus) is the infectious diseases that can develop into AIDS (Acquired Immune Deficiency Syndrome). HIV infects the immune system and weakens human defence system againsts infections and some types of cancer. A weakened immune in patients with HIV/AIDS will lead to a variety of bacteria/viruses are easy to infect the body (opportunistic infections). Until now, HIV/AIDS is untreatable disease, but the effective antiretroviral (ARV) drugs could control the disease. Moreover, a HIV patient with antiretroviral therapy could can enjoy healthy, long and productive lives [1].

Malaria is a contagious disease that is still a major health problem in the world. The disease is caused by parasites that are transmitted to people through the bites of infected female Anopheles mosquitoes. The female mosquito Anopheles require human blood for their reproductive process. In 2015, WHO reported that there are 91 countries and areas had ongoing malaria transmission. Malaria is preventable and curable, and increased efforts are dramatically reducing the malaria burden in many places [2].

Until now, malaria has not been regarded as an opportunistic infection of HIV disease. However, the weakening of the immune system due to HIV infection affects an increase in



malaria infections infected people. Malaria and HIV co-infection has caused more than 2 million deaths every year [3]. In areas highly endemic malaria, HIV-infected individuals more at risk of developing severe malaria. Some reports also recommend that antimalarial therapy failure might be more common in HIV-infected adults with low CD4-cell counts compared to those not infected with HIV. People who are stricken with acute malaria increases viral replication so that the increase in HIV viral load in patients with HIV [4].

Mathematical models are useful to analyze the transmission dynamics of HIV and malaria co-infection. The dynamics of HIV and malaria co-infection spread have been conducted in the literature [5, 6, 7]. For instance, the authors in [5] constructed and analyzed a simple mathematical model to study the co-infection of HIV and malaria. Mukandavire et al. [6] formulated and analyzed a realistic mathematical model for HIV-malaria co-infection, which combines the key epidemiological and biological features of each of the two diseases. Nyabadza et al. [7] developed a mathematical model to describe population dynamics of HIV/AIDS and malaria co-infections.

The optimal control strategies have been employed in the study of epidemiological models such as HIV, TB, Malaria, HCV, co-infection TB-HIV and Malaria-Cholera diseases dynamics [8, 9, 10, 11, 12, 13, 14]. Very little studies have been applied the optimal control to HIV and malaria co-infection model that conducted researchers. Recently, the authors in [15] have used optimal control strategies for a nonlinear dynamical system to describe the dynamics and effects of HIV-malaria co-infection in a workplace. In this paper, we developed a mathematical model of the HIV and malaria co-infection transmission dynamics of incorporate malaria preventive, anti-malaria treatment and ARV treatment for HIV as control optimal strategies.

The remainder of this paper is organized as follows. In Section 2, we propose a mathematical model of HIV and malaria co-infection transmission with controls on malaria prevention, anti-malaria and ARV treatments. In Section 3, we analyze the model without controls and perform sensitivity analysis of the basic reproduction numbers. In the Section 4, we present the optimal control analysis. In Section 5, we perform some numerical simulations to illustrate the purpose of the treatments. The conclusion of is presented in the last section.

## 2. Model formulation

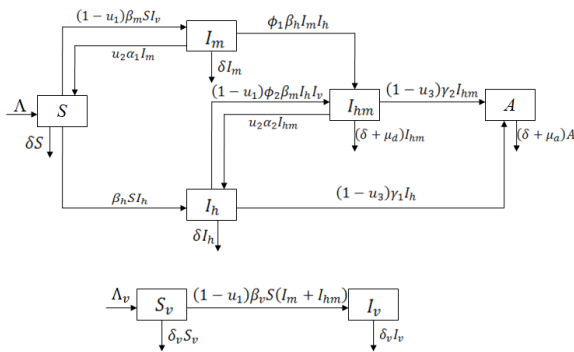
In general, the population is classified into two classes namely human (host) population and the mosquito (vector) population. The human population was assumed to be homogeneous and closed. The total human population at time  $t$ , denoted by  $N(t)$ , is classified into five classes, namely, the class of susceptible population ( $S(t)$ ), the class of the infected with malaria only ( $I_m(t)$ ) and susceptible to HIV, the class of the infected with HIV only and susceptible to malaria ( $I_h(t)$ ), the class of the infected with HIV and malaria both ( $I_{hm}(t)$ ), and the class of the infected with AIDS population ( $A(t)$ ). Hence, the total human population  $N(t) = S(t) + I_m(t) + I_h(t) + I_{hm}(t) + A(t)$ .

We also assume that the susceptible cannot get HIV and malaria infection simultaneously at the same time. The class of individuals with AIDS ( $A(t)$ ) and dually infected individuals ( $I_{hm}(t)$ ) are isolated, so that they cannot infect anyone.

The total vector population at time  $t$ , denoted by  $N_v(t)$ , is classified into two classes, namely, the class of susceptible vector population ( $S_v(t)$ ) and the class of infected vector ( $I_v(t)$ ). Thus,  $N_v(t) = S_v(t) + I_v(t)$ .

We suppose the malaria prevention ( $u_1(t)$ ), anti-malaria treatment ( $u_2(t)$ ) and the ARV treatment ( $u_3(t)$ ) as the control efforts to reduce the HIV and malaria infection respectively. The control functions  $u_1, u_2$  and  $u_3$  are defined on interval  $[0, t_f]$ , where  $0 \leq u_i(t) \leq 1$ ,  $t \in [0, t_f]$ ,  $i = 1, 2, 3$  and  $t_f$  denotes the final time of the controls.

the transmission diagram for deriving our model was shown in the Figure 1.



**Figure 1.** HIV and malaria co-infection transmission diagram.

The model is as follows.

$$\begin{aligned}
 \frac{dS}{dt} &= \Lambda - (1 - u_1)\beta_m S I_v - \beta_h S I_h + u_2 \alpha_1 I_m - \delta S, \\
 \frac{dI_m}{dt} &= (1 - u_1)\beta_m S I_v - u_2 \alpha_1 I_m - \phi_1 \beta_h I_m I_h - \delta I_m, \\
 \frac{dI_h}{dt} &= \beta_h S I_h + u_2 \alpha_2 I_{hm} - (1 - u_1)\phi_2 \beta_m I_h I_v - (1 - u_3)\gamma_1 I_h - \delta I_h, \\
 \frac{dI_{hm}}{dt} &= \phi_1 \beta_h I_m I_h + (1 - u_1)\phi_2 \beta_m I_h I_v - u_2 \alpha_2 I_{hm} - (\delta + \mu_d) I_{hm} - (1 - u_3)\gamma_2 I_{hm}, \\
 \frac{dA}{dt} &= (1 - u_3)\gamma_1 I_h + (1 - u_3)\gamma_2 I_{hm} - (\delta + \mu_a) A, \\
 \frac{dS_v}{dt} &= \Lambda_v - (1 - u_1)\beta_v S_v (I_m + I_{hm}) - \delta_v S_v, \\
 \frac{dI_v}{dt} &= (1 - u_1)\beta_v S_v (I_m + I_{hm}) - \delta_v I_v.
 \end{aligned}
 \tag{1}$$

**Table 1.** Parameters of model

Description	Parameter
Infection rate for HIV	$\beta_h$
Progression rate from malaria to HIV-malaria co-infection	$\phi_1$
Progression rate from HIV to HIV-malaria co-infection	$\phi_2$
Recovery rate from malaria	$\alpha_1$
Recovery rate from malaria among HIV-malaria	$\alpha_2$
Progression rate from HIV only to AIDS	$\gamma_1$
Progression rate from HIV-malaria co-infection to AIDS	$\gamma_2$
Disease AIDS induced death rate	$\mu_a$
Disease HIV-malaria induced death rate	$\mu_d$
Recruitment rate into the host population	$\Lambda$
Recruitment rate into the vector population	$\Lambda_v$
Infection rate for host	$\beta_m$
Infection rate for vector	$\beta_v$
Natural death rate for host	$\delta$
Natural death rate for vector	$\delta_v$

The region of biological interest of the model (1) is

$$\hbar = \hbar_h \times \hbar_v \subset \mathfrak{R}_+^5 \times \mathfrak{R}_+^2,$$

with

$$\hbar_h = \left\{ (S(t), I_m(t), I_h(t), I_{hm}(t), A(t)) \in \mathfrak{R}_+^5 : 0 \leq N_h(t) \leq \frac{\Lambda}{\delta} \right\},$$

and

$$\hbar_v = \left\{ (S_v(t), I_v(t)) \in \mathfrak{R}_+^2 : 0 \leq N_v(t) \leq \frac{\Lambda_v}{\delta_v} \right\}.$$

It is assumed that all parameters used in the model (1) are non-negative. The description of the parameters is given in Table 1. The region  $\hbar$  is positively invariant. Hence, model (1) is well-posed in this region since any vector fields on the boundary will not move to the exterior region. So, if it was given any non-negative initial condition in  $\mathfrak{R}_+^7$ , then the solution was defined for any time  $t \geq 0$  and the solution remain in the region. We desire to minimize the number of a HIV and malaria co-infection while keeping the costs of applying malaria preventive, anti-malaria and ARV treatment controls as low as possible. The cost function is defined as

$$J(u_1, u_2, u_3) = \int_0^{t_f} \left( I_m + I_{hm} + A + I_v + \frac{c_1}{2} u_1^2 + \frac{c_2}{2} u_2^2 + \frac{c_3}{2} u_3^2 \right) dt, \quad (2)$$

where  $c_1, c_2$  and  $c_3$  are the weighting constants for malaria preventive, anti-malaria and ARV treatment efforts, respectively. We take a quadratic form for measuring the control cost [8, 10]. The term  $c_1 u_1^2, c_2 u_2^2$  and  $c_3 u_3^2$  describe the cost associated with the malaria preventive, anti-malaria and ARV treatment controls respectively. The cost of malaria preventive is associated with the costs of vaccination, spraying of insecticide, personal protection and insecticide treated bed nets. Larger values of  $c_1, c_2$  and  $c_3$  will imply more expensive implementation cost for malaria preventive, anti-malaria and ARV treatment efforts. Our goal is to find an optimal control pair  $u_1^*, u_2^*$  and  $u_3^*$  such that

$$J(u_1^*, u_2^*, u_3^*) = \min_{\Gamma} J(u_1, u_2, u_3), \quad (3)$$

where  $\Gamma = \{(u_1, u_2, u_3) | 0 \leq u_i \leq 1, i = 1, 2, 3\}$ .

### 3. Model and sensitivity analysis

Consider the mathematical model in eq. (1) without the control functions  $u_1, u_2$  and  $u_3$ . Let

$$R_m = \frac{\sqrt{\beta_m \beta_v \Lambda \Lambda_v}}{\delta \delta_v}$$

$$R_h = \frac{\beta_h \Lambda}{\delta(\delta + \gamma_1)}.$$

Parameters  $R_m$  and  $R_h$  are basic reproduction ratio corresponding to the malaria infection, and HIV infection, respectively. These ratios define number of secondary cases of primary case throughout the infective period because of the type of infection [16, 17].

By setting  $u_1 = u_2 = u_3 = 0$ , the mathematical model in eq. (1) has four equilibria (to the coordinate  $(S, I_m, I_h, I_{hm}, A, S_v, I_v)$ ), namely,

1. The disease-free equilibrium  $E_0 = (\frac{\Lambda}{\delta}, 0, 0, 0, 0, \frac{\Lambda_v}{\delta_v}, 0)$ .
2. The malaria endemic equilibrium  $E_m = (\frac{\Lambda}{\beta_m I_v + \delta}, \frac{\beta_m \Lambda I_v}{\delta(\beta_m I_v + \delta)}, 0, 0, 0, \frac{\Lambda_v}{\beta_v I_m + \delta_v}, \frac{\beta_v \Lambda_v I_m}{\delta_v(\beta_v I_m + \delta_v)})$ .
3. The HIV endemic equilibrium  $E_h = (\frac{\gamma_1 + \delta}{\beta_h}, 0, \delta(\gamma_1 + \delta)(R_h - 1), 0, \frac{\gamma_1 I_h}{\delta + \mu_a}, \frac{\Lambda_v}{\delta_v}, 0)$ . The equilibrium  $E_h$  exists if  $R_h > 1$ .
4. The HIV-malaria endemic equilibrium  $E_{hm} = (S^*, I_m^*, I_h^*, I_{hm}^*, A^*, S_v^*, I_v^*)$ , with

$$\begin{aligned} S^* &= \frac{\phi_2 \beta_m I_v + \gamma_1 + \delta}{\beta_h}, \\ I_m^* &= \frac{\beta_m S I_v}{\phi_1 \beta_h I_h + \delta}, \\ I_h^* &= \frac{\Lambda - \beta_m S I_v - \delta S}{\beta_h S}, \\ I_{hm}^* &= \frac{\phi_1 \beta_h I_m I_h + \phi_2 \beta_m I_h I_v}{\mu_d + \delta + \gamma_2}, \\ A^* &= \frac{\gamma_1 I_h + \gamma_2 I_{hm}}{\delta + \mu_a}, \\ S_v^* &= \frac{\Lambda_v}{\beta_v (I_m + I_{hm}) + \delta_v}, \\ I_v^* &= \frac{\beta_v S_v (I_m + I_{hm})}{\delta_v}. \end{aligned}$$

The equilibrium  $E_{hm}$  exists if  $R_h > 1$  and  $\delta(\delta + \gamma_1)(R_h - 1) > \phi_2 \beta_m^2 I_v^2 + \beta_m(\delta + \gamma_1)I_v + \delta \phi_2 \beta_m I_v$ .

The following theorem give the stability criteria of the disease-free equilibrium.

**Theorem 1** *The disease-free equilibrium  $E_0$  is locally asymptotically stable whenever  $R_m, R_h < 1$ . Furthermore, the equilibrium  $E_0$  is unstable if  $R_m, R_h > 1$ .*

**Proof.** Linearizing the mathematical model in eq. (1) near the equilibrium  $E_0$  gives eigenvalues  $-\delta, -\delta_v, -(\delta + \mu_a), -(\delta + \mu_d + \gamma_2), -(\delta + \gamma_1)(R_h - 1)$  and the roots of quadratic equation  $x^2 + (\delta + \delta_v)x + \delta \delta_v(1 - T_m) = 0$ , with  $T_m = \frac{\beta_m \beta_v \Lambda \Lambda_v}{\delta^2 \delta_v^2}$ . The quadratic equation have negative roots if  $T_m < 1$  or equivalently  $R_m < 1$ . It is clear that all of the eigenvalues are negative if  $R_h < 1$  and  $R_m < 1$ . So, if  $R_m, R_h < 1$ , then the equilibrium  $E_0$  is locally asymptotic stable. Otherwise, it is unstable.

Next, we analyze the sensitivity of the basic reproduction numbers  $R_m$  and  $R_h$  to the parameters in the presented model. The purpose of this analysis was to find the parameters that have the greatest effects on the reproduction numbers. By using the method in [18], we derived the analytical expression for sensitivity index of the  $R_m$  and  $R_h$  to each parameters.

The sensitivity indices of a variable,  $R_0$ , that depends differentially on a parameter,  $l$ , is defined as

$$\Upsilon_l^{R_0} := \frac{\partial R_0}{\partial l} \times \frac{l}{R_0}. \quad (4)$$

Now, by using the parameter values in the Table 2, we have the following results in Table 3. For example, the sensitivity index of  $R_m$  with respect to  $\beta_m$  is

$$\Upsilon_{\beta_m}^{R_m} := \frac{\partial R_m}{\partial \beta_m} \times \frac{\beta_m}{R_m} = 0.5. \quad (5)$$

The sensitivity indices of  $R_m$  and  $R_h$  with respect to other parameters such as  $\Lambda$ ,  $\Lambda_v$ ,  $\beta_v$ ,  $\delta$ ,  $\delta_v$ ,  $\beta_h$ ,  $\gamma_1$ , can be derived in the same way as (5). The interpretation of the sensitivity index is as follow. Since  $\Upsilon_{\beta_m}^{R_m} = 0.5$ , that is to say, increasing (or decreasing) infection rate for malaria,  $\beta_m$ , by 10%, increases (or decreases) the reproduction number  $R_m$  by 5%. Thus, increasing (or decreasing) natural death rate  $\delta$  by 10% decreases (increases)  $R_m$  by 10%.

**Table 2.** Parameter values.

Parameter	Value	Ref.
$\Lambda$	500/year	Assumed
$\delta$	0.02/year	[13]
$\beta_h$	0.00031/year	Assumed
$\beta_m$	0.00045/year	Assumed
$\beta_v$	0.00035/year	Assumed
$\gamma_1$	0.01/year	Assumed
$\gamma_2$	0.05/year	Assumed
$\Lambda_v$	5000/year	Assumed
$\delta_v$	0.1429/year	[18]
$\phi_1$	1.1/year	Assumed
$\phi_2$	1.15/year	Assumed
$\mu_a$	0.02/year	Assumed
$\mu_d$	0.03/year	Assumed
$\alpha_1$	0.2/year	Assumed
$\alpha_2$	0.2/year	Assumed

**Table 3.** Sensitivity indices to parameter for model (1).

Parameter	Sensitivity index ( $R_m$ )	Parameter	Sensitivity index ( $R_h$ )
$\Lambda$	0.5	$\Lambda$	1
$\Lambda_v$	0.5	$\beta_h$	1
$\beta_m$	0.5	$\delta$	-1.67
$\beta_v$	0.5	$\gamma_1$	-0.33
$\delta$	-1		
$\delta_v$	-1		

#### 4. Mathematical analysis of optimal control

In this section, we consider the mathematical model in eq. (1) with its control functions  $u_1, u_2$  and  $u_3$ . We describe the cost function in eq. (2) for the model in eq. (1). The necessary conditions to find the optimal controls  $u_1^*, u_2^*$  and  $u_3^*$  such that condition (3) with constraint model (1) must satisfy come from the Pontryagin Maximum Principle [19]. The principle converts the equations (1) - (3) into minimizing Hamiltonian function  $H$  problem with respect  $(u_1, u_2, u_3)$ , that is

$$H(S, I_m, I_h, I_{hm}, A, S_v, I_v, u_1, u_2, u_3, \lambda_1, \lambda_2, \dots, \lambda_7) = I_m + I_{hm} + A + I_v + \frac{c_1}{2} u_1^2 + \frac{c_2}{2} u_2^2 + \frac{c_3}{2} u_3^2 + \sum_{i=1}^7 \lambda_i g_i,$$

where  $g_i$  is the right hand side of the mathematical model in eq. (1) which is the  $i$ -th state variable equation. The variables  $\lambda_i, i = 1, 2, \dots, 7$ , are called adjoint variables satisfying the following co-state equations

$$\begin{aligned} \frac{d\lambda_1}{dt} &= (\lambda_1 - \lambda_2)(1 - u_1)\beta_m I_v + (\lambda_1 - \lambda_3)\beta_h I_h + \lambda_1 \delta, \\ \frac{d\lambda_2}{dt} &= -1 + (\lambda_2 - \lambda_1)\alpha_1 u_2 + (\lambda_2 - \lambda_4)\phi_1 \beta_h I_h + (\lambda_6 - \lambda_7)(1 - u_1)\beta_v S_v + \lambda_2 \delta, \\ \frac{d\lambda_3}{dt} &= (\lambda_1 - \lambda_3)\beta_h S + (\lambda_2 - \lambda_4)\phi_1 \beta_h I_h + (\lambda_3 - \lambda_4)(1 - u_1)\phi_2 \beta_m I_v \\ &\quad + (\lambda_3 - \lambda_5)(1 - u_3)\gamma_1 + \lambda_3 \delta, \\ \frac{d\lambda_4}{dt} &= -1 + (\lambda_4 - \lambda_3)u_2 \alpha_2 + (\lambda_4 - \lambda_5)(1 - u_3)\gamma_2 + (\lambda_6 - \lambda_7)(1 - u_1)\beta_v S_v + \\ &\quad \lambda_4(\delta + \mu_d), \\ \frac{d\lambda_5}{dt} &= -1 + \lambda_5(\delta + \mu_d), \\ \frac{d\lambda_6}{dt} &= (\lambda_6 - \lambda_7)(1 - u_1)\beta_v(I_h + I_{hm}) + \lambda_6 \delta_v, \\ \frac{d\lambda_7}{dt} &= -1 + (\lambda_1 - \lambda_2)(1 - u_1)\beta_m S + (\lambda_3 - \lambda_4)(1 - u_1)\phi_2 \beta_m I_h + \lambda_7 \delta_v, \end{aligned} \tag{6}$$

where the transversality conditions  $\lambda_i(t_f) = 0, i = 1, \dots, 7$ .

By using Pontryagin's Maximum Principle and the existence result for the optimal control pairs, the steps to obtain the optimal controls  $u = (u_1^*, u_2^*, u_3^*)$  are as following [20, 21].

- (i) Minimize the Hamiltonian function  $H$  with respect to  $u$ , that is  $\frac{\partial H}{\partial u} = 0$  which is the stationary condition. Hence, we find

$$\begin{aligned} u_1^* &= \begin{cases} 0 & \text{for } u_1 \leq 0 \\ \frac{(\lambda_4 - \lambda_3)\phi_2 \beta_m I_v I_h + (\lambda_2 - \lambda_1)\beta_m S I_v + (\lambda_7 - \lambda_6)\beta_v S_v (I_h + I_{hm})}{c_1} & \text{for } 0 < u_1 < 1 \\ 1 & \text{for } u_1 \geq 1 \end{cases} \\ u_2^* &= \begin{cases} 0 & \text{for } u_2 \leq 0 \\ \frac{(\lambda_4 - \lambda_3)I_{hm} \alpha_2 + (\lambda_2 - \lambda_1)\alpha_1 I_h}{c_2} & \text{for } 0 < u_2 < 1 \\ 1 & \text{for } u_2 \geq 1 \end{cases} \\ u_3^* &= \begin{cases} 0 & \text{for } u_3 \leq 0 \\ \frac{(\lambda_5 - \lambda_3)\gamma_1 I_h + (\lambda_5 - \lambda_4)\gamma_2 I_{hm}}{c_3} & \text{for } 0 < u_3 < 1 \\ 1 & \text{for } u_3 \geq 1 \end{cases} \end{aligned}$$



- (ii) Solve the state system  $\dot{x}(t) = \frac{\partial H}{\partial \lambda}$  which is the mathematical model in eq. (1), where  $x = (S, I_m, I_h, I_{hm}, A, S_v, I_v)$ ,  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_7)$  and the initial condition  $x(0)$ .
- (iii) Solve the co-state system  $\dot{\lambda}(t) = -\frac{\partial H}{\partial x}$  which is the system in eq. (6) with the final condition  $\lambda_i(t_f) = 0, i = 1, \dots, 7$ .

Henceforth, we find the following theorem.

**Theorem 2** *The optimal controls  $(u_1^*, u_2^*, u_3^*)$  that minimizes the objective function  $J(u_1, u_2, u_3)$  on  $\Gamma$  is given by*

$$\begin{aligned} u_1^* &= \max \left\{ 0, \min \left( 1, \frac{(\lambda_4 - \lambda_3)\phi_2\beta_m I_v I_h + (\lambda_2 - \lambda_1)\beta_m S I_v + (\lambda_7 - \lambda_6)\beta_v S_v (I_h + I_{hm})}{c_1} \right) \right\} \\ u_2^* &= \max \left\{ 0, \min \left( 1, \frac{(\lambda_4 - \lambda_3)I_{hm}\alpha_2 + (\lambda_2 - \lambda_1)\alpha_1 I_h}{c_2} \right) \right\}, \\ u_3^* &= \max \left\{ 0, \min \left( 1, \frac{(\lambda_5 - \lambda_3)\gamma_1 I_h + (\lambda_5 - \lambda_4)\gamma_2 I_{hm}}{c_3} \right) \right\} \end{aligned}$$

where  $\lambda_i, i = 1, \dots, 7$  is the solution of the co-state equations (6) with the transversality conditions  $\lambda_i(t_f) = 0, i = 1, \dots, 7$ .

Substituting the optimal control  $(u_1^*, u_2^*, u_3^*)$  which is achieved from the state system in eq. (1) and the co-state system in eq. (6), we find an optimal system. The solutions of the optimality system will be solved numerically for some choices of the parameter. Because of lack of data, most of parameter values are assumed within realistic ranges for a typical scenario.

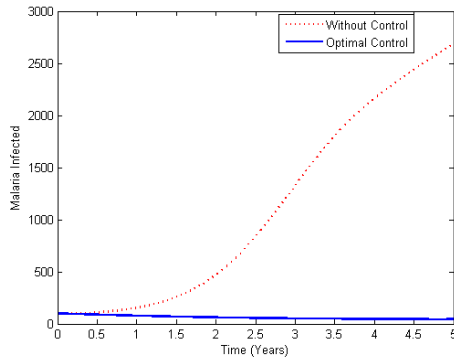
## 5. Numerical simulation

In this section, we perform some numerical simulations of the presented model in eq. (1) with and without optimal control. The optimal control strategy is found by the iterative method of the fourth order Runge-Kutta method. The state equations are initially solved by the forward Runge-Kutta method of the fourth order. Then, by using the backward Runge-Kutta method of the fourth order, we solved the co-state equations with the transversality conditions. The controls are updated by using a convex combination of the previous controls and the value from the characterizations of  $u_1^*, u_2^*$  and  $u_3^*$ . This process is reiterated and the iteration is ended if the current state, the adjoint, and the control values converge sufficiently [22].

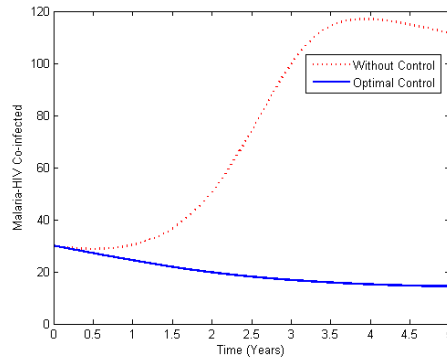
We present four scenarios. In the first situations, we study combination of the malaria preventive and the anti-malaria treatment as control strategy. In the second scenario, we consider combination of the malaria preventive and ARV treatment controls. In the third scenario, we use the combination of the anti-malaria and ARV treatment controls. In the last one, we use the combination of the malaria preventive, anti-malaria and ARV treatment as control strategy. Parameters used in these simulations given in Table 2. In these simulations, we use initial condition  $(S(0), I_m(0), I_h(0), I_{hm}(0), A(0), S_v(0), I(0)) = (500, 100, 50, 30, 50, 5000, 100)$ , weighting constants  $c_1 = 20, c_2 = 50, c_3 = 50$ .

### 5.1. First scenario

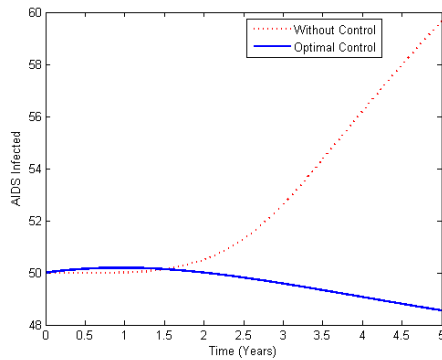
In this scenario, we apply the malaria prevention control  $u_1$  and the anti-malaria treatment control  $u_2$  to optimize the objective function  $J$ , while setting the ARV control  $u_3$  to zero. For this scenario, we found in the Figure 2-3 that the population of malaria infected ( $I_m$ ) and HIV-malaria co-infection ( $I_{hm}$ ) decreases, while the population increases when there is no any controls. Similarly, the result in the Figure 4-5 show that the number of AIDS infected ( $A$ ) and mosquito infected ( $I_v$ ) a reduction in the number infected compared with the case without control. Hence, the malaria prevention and the anti-malaria treatment control gives a significant influence in controlling infected malaria, HIV-malaria co-infection and AIDS infected. The profile of the optimal treatment controls  $u_1^*$  and  $u_2^*$  for this scenario is presented in the Figure 6.



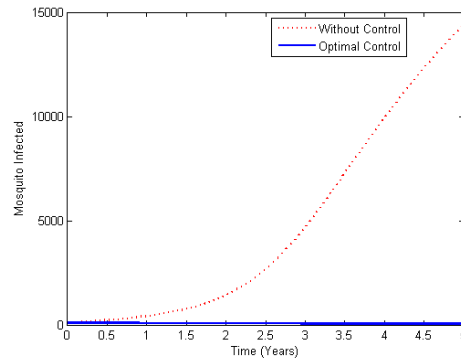
**Figure 2.** The dynamics of  $I_m$  using the controls  $u_1^*$  and  $u_2^*$ .



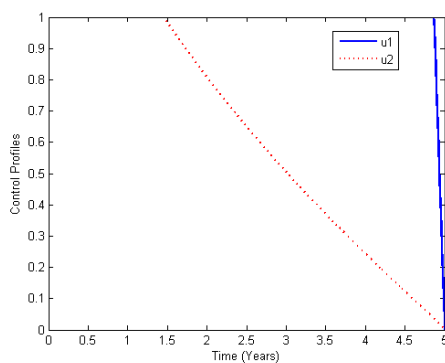
**Figure 3.** The dynamics of  $I_{hm}$  using the controls  $u_1^*$  and  $u_2^*$ .



**Figure 4.** The dynamics of  $A$  using the controls  $u_1^*$  and  $u_2^*$ .



**Figure 5.** The dynamics of  $I_v$  using the controls  $u_1^*$  and  $u_2^*$ .

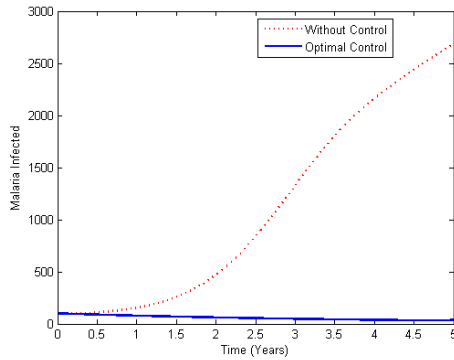


**Figure 6.** The profile of the optimal controls  $u_1^*$  and  $u_2^*$ .

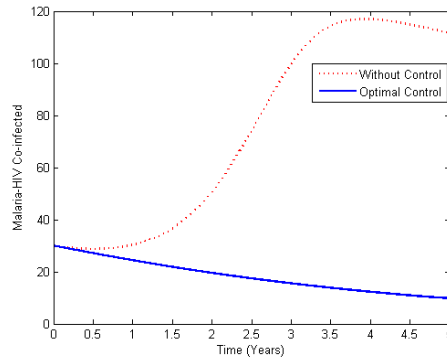
*5.2. Second scenario*

In the second scenario, we utilize the malaria prevention control  $u_1$  and the ARV treatment control  $u_3$  to optimize the objective function  $J$ , while set the anti-malaria treatment control  $u_2$  to zero. The infected populations dynamics of this scenario are presented in Figure 7-10. We observe in Figure 7-8 that this control strategy results in a significant decrease in the number of malaria infected ( $I_m$ ) and HIV-malaria co-infection ( $I_{hm}$ ) compared with the case without control. Also in Figure 9-10, this control strategy results in a significant decrease in the number

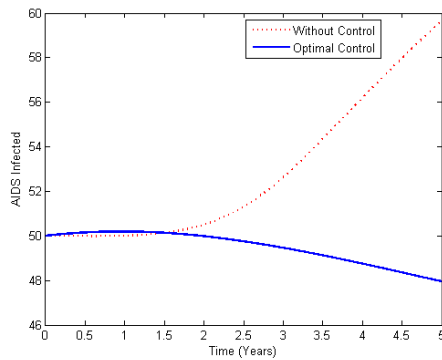
of AIDS infected ( $A$ ) and mosquito infected ( $I_v$ ) as against an increase in the uncontrolled case. The result shown in the Figure 7-10 clearly suggests that this scenario is very effective in the control of the number of the infected. The control profiles of the malaria prevention and ARV treatment is shown in the Figure 11.



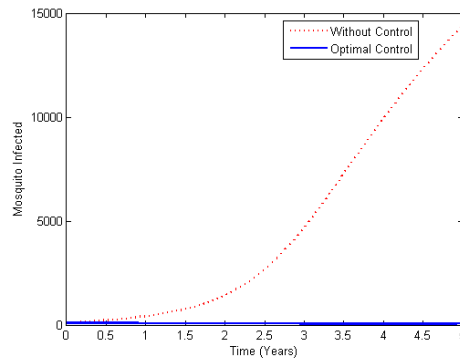
**Figure 7.** The dynamics of  $I_m$  using the controls  $u_1^*$  and  $u_3^*$ .



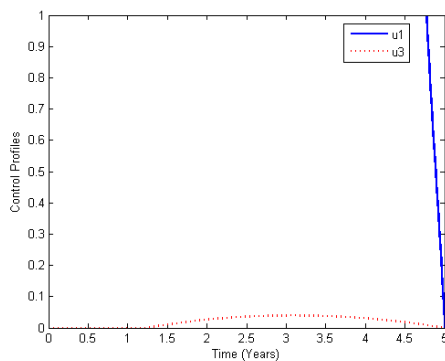
**Figure 8.** The dynamics of  $I_{hm}$  using the controls  $u_1^*$  and  $u_3^*$ .



**Figure 9.** The dynamics of  $A$  using the controls  $u_1^*$  and  $u_3^*$ .



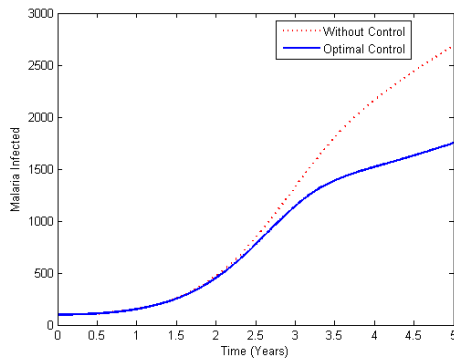
**Figure 10.** The dynamics of  $I_v$  using the controls  $u_1^*$  and  $u_3^*$ .



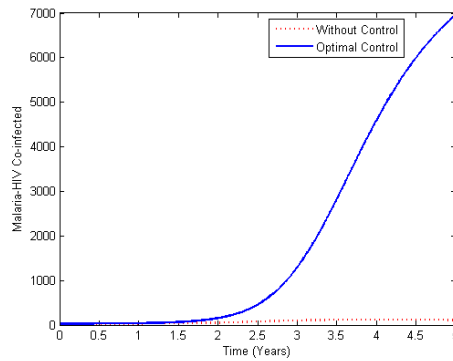
**Figure 11.** The profile of the optimal controls  $u_1^*$  and  $u_3^*$ .

5.3. Third scenario

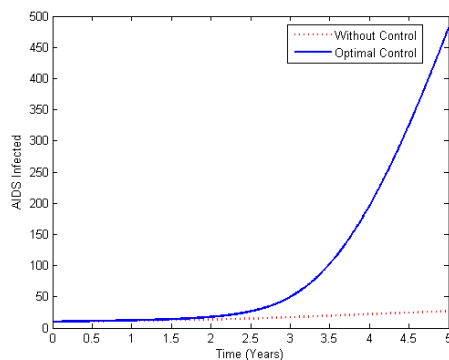
In the third scenario, we active controls  $u_2$  and  $u_3$  on the anti-malaria and ARV treatment to optimize the objective function  $J$  whereas the malaria prevention control  $u_1$  is set to zero. The dynamics of the infected populations of this scenario are given in Figure 12-15. We observe in Figure 12 that the malaria infected populations are lesser when the control strategy is used than when the control strategy is not implemented. It was also shown in Figure 13 that the number of HIV-malaria co-infection populations increases with this control strategy compared to the number without control. Figure 14-15 depict the number of AIDS infected and mosquito infected that the populations is higher with this control scenario than the cases without control. Figure 16 shows the control profiles of the anti-malaria ( $u_2$ ) and ARV treatment ( $u_3$ ) in which the control  $u_2$  is given maximum in almost 5 years, while the the optimal control  $u_3$  starts and remain at the lower bound in 5 years. The results show that the anti-malaria treatment give the positive impact to reduce the malaria infected, while the mosquito infected tend to increases using this strategy. Then, the HIV-malaria co-infection and AIDS infection populations also increase with this strategy due to there is no significant ARV treatment intervention.



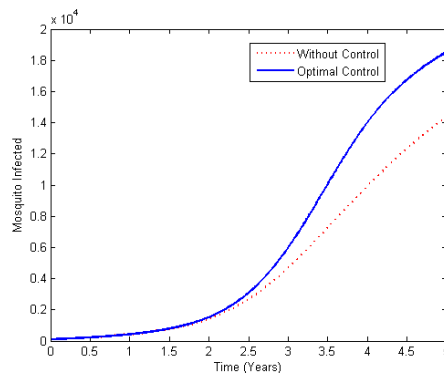
**Figure 12.** The dynamics of  $I_m$  using the controls  $u_2^*$  and  $u_3^*$ .



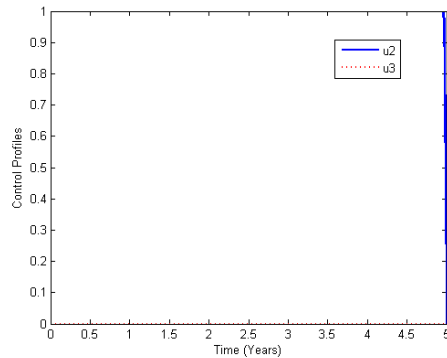
**Figure 13.** The dynamics of  $I_{hm}$  using the controls  $u_2^*$  and  $u_3^*$ .



**Figure 14.** The dynamics of  $A$  using the controls  $u_2^*$  and  $u_3^*$ .



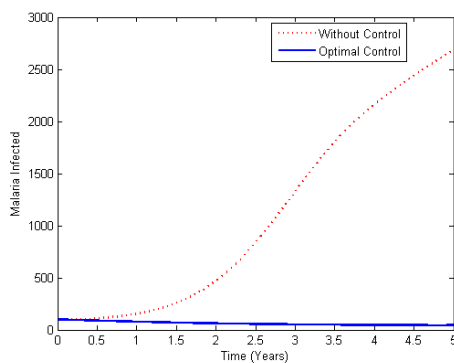
**Figure 15.** The dynamics of  $I_v$  using the controls  $u_2^*$  and  $u_3^*$ .



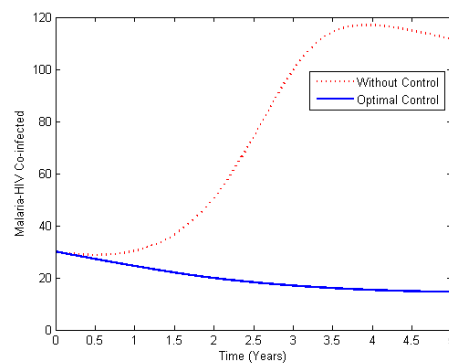
**Figure 16.** The profile of the optimal controls  $u_2^*$  and  $u_3^*$ .

5.4. *Fourth scenario*

In this scenario, all the controls ( $u_1, u_2$  and  $u_3$ ) are used to optimize the objective function  $J$ . The dynamics of the malaria infected, malaria-HIV co-infection, AIDS infection and mosquito infected are given in Figure 17-20. For this strategy, we observed in Figure 17-18 that the control strategies resulted in a decrease in the number of the malaria infected and HIV-malaria co-infection compared to the number without control. A similar reduction is observed in Figure 19-20 for AIDS infected and mosquito infected in the control strategy, while an increased number for the uncontrolled case resulted.



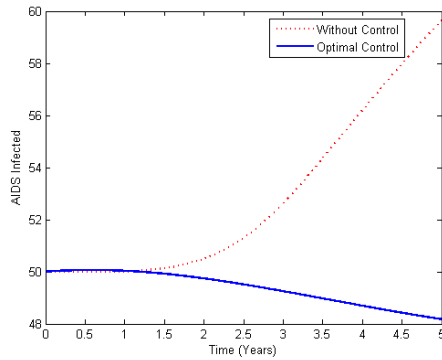
**Figure 17.** The dynamics of  $I_m$  using the controls  $u_1^*, u_2^*$  and  $u_3^*$ .



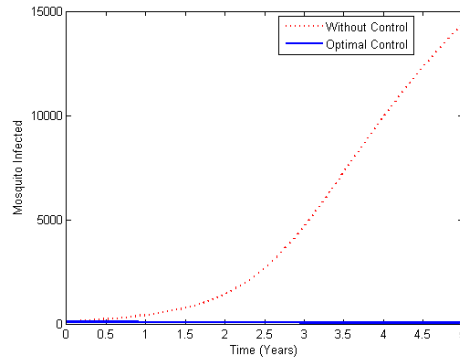
**Figure 18.** The dynamics of  $I_{hm}$  using the controls  $u_1^*, u_2^*$  and  $u_3^*$ .

The profiles of the optimal malaria prevention  $u_1^*$ , anti-malaria treatment control  $u_2^*$  and ARV control  $u_3^*$  of this scenario is given in Figure 21. To reduce HIV and malaria co-infection in 5 years, the malaria prevention should be given intensively almost 5 years before dropping gradually until reaching the lower bound in the end 5th year. Meanwhile, anti-malaria treatment should be needed intensive effort during 1.5 years before finally dropping to its lower bound. While ARV treatment control decreases from 0.1 (10%) at the beginning time before finally dropping to its lower bound at the end of the intervention.

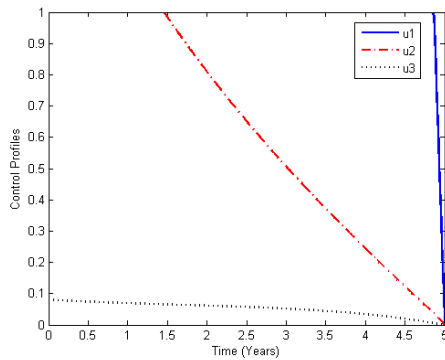
Based on the numerical results, the optimal control scenarios are arranged from the least to the costly based on the total cost function  $J$ . Table 4 shows the optimal values of the objective function  $J$  in the four scenarios. From Table 4, we conclude that the combination of the malaria



**Figure 19.** The dynamics of  $A$  using the controls  $u_1^*$ ,  $u_2^*$  and  $u_3^*$



**Figure 20.** The dynamics of  $I_v$  using the controls  $u_1^*$ ,  $u_2^*$  and  $u_3^*$



**Figure 21.** The profile of the optimal controls  $u_1^*$ ,  $u_2^*$  and  $u_3^*$

**Table 4.** Optimal values of the cost functional.

Scenario of the optimal control	Total cost functional $J$
$u_1^*$ , and $u_3^*$	$1.0359 \times 10^3$
$u_1^*$ , $u_2^*$ , and $u_3^*$	$1.1234 \times 10^3$
$u_1^*$ , and $u_2^*$	$1.1237 \times 10^3$
$u_2^*$ , and $u_3^*$	$4.5127 \times 10^4$

preventive ( $u_1^*$ ) and ARV treatment ( $u_3^*$ ) have the least cost function followed by combination all controls ( $u_1^*$ ,  $u_2^*$ , and  $u_3^*$ ) to reduce the number of malaria infected, HIV-malaria co-infection, AIDS infected and mosquito infected.

### 6. Conclusion

In this paper, we have developed a deterministic mathematical model for the spread of malaria and HIV co-infection that incorporates malaria prevention, anti-malaria and ARV treatment as optimal control strategies. For the model without controls, we obtain two thresholds  $R_m$  and  $R_h$  which are basic reproduction ratios for the malaria and HIV diseases respectively. These ratios determine the existence and stability of the equilibria of the model. If the thresholds are less than unity then the diseases free equilibrium is locally asymptotically stable. Finally, the conditions for existence of optimal control is studied analytically using the Pontryagin Maximum

Principle. From our numerical analysis of the optimal control indicates that the best strategy is to combine the malaria prevention and ARV treatments followed by combination all controls in order to reduce malaria and HIV co-infection.

### Acknowledgements

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