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LAMPIRAN

Lampiran 1. Pendekatan Variabel N (Total Populasi)

Diberikan model matematika penyalahgunaan narkoba dengan memperhatikan tipe perawatan beserta tingkat resiko sebagai berikut :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L \quad (1.a)$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H \quad (1.b)$$

$$\frac{dI}{dt} = \frac{\beta_1 I(S_H + \eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 I T_j}{N} - (\mu + \sigma + \delta + \rho)I \quad (1.c)$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r \quad (1.d)$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j \quad (1.e)$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\beta_2 IR}{N} - \mu R \quad (1.f)$$

Total populasi didefinisikan sebagai berikut :

$$N = S_H + S_L + I + T_j + T_r + R$$

Dari sini dapat dibentuk menjadi,

$$\frac{dN}{dt} = \frac{S_H}{dt} + \frac{S_L}{dt} + \frac{dI}{dt} + \frac{T_j}{dt} + \frac{T_r}{dt} + \frac{dR}{dt} \quad (1.g)$$

Dengan mensubstitusikan persamaan (1.a) – (1.f) ke persamaan (1.g) diperoleh berikut:

$$\frac{dN}{dt} = \Lambda - \mu S_H - \mu S_L - \mu I - \mu T_j - \mu T_r - \mu R - \delta I$$

$$\frac{dN}{dt} = \Lambda - \mu N - \delta I \quad (1.h)$$

Pada analisis kestabilan laju kematian akibat narkoba diabaikan karena nilainya kecil ($\delta \approx 0$), sehingga persamaan (1.h) menjadi sebagai berikut :

$$\frac{dN}{dt} = \Lambda - \mu N$$

$$\frac{dN}{dt} + \mu N = \Lambda \qquad (1.i)$$

Penyelesaian persamaan diferensial diatas dilakukan menggunakan faktor integrasi. Misalkan faktor integrasinya adalah $e^{\int \mu\, dt} = e^{\mu t}$

Semua ruas dikalikan faktor integrasi, sehingga persamaan (1.i) dapat dituliskan sebagai berikut :

$$\frac{dN}{dt} e^{\mu t} + \mu N e^{\mu t} = \Lambda e^{\mu t}$$

$$\frac{d}{dt}(N e^{\mu t}) = \Lambda e^{\mu t}$$

$$d(N e^{\mu t}) = \Lambda e^{\mu t} dt$$

$$\int d(N e^{\mu t}) = \int \Lambda e^{\mu t} dt$$

$N e^{\mu t} = \frac{\Lambda}{\mu} e^{\mu t} + C$, dengan C konstanta

$$N e^{\mu t} e^{-\mu t} = \frac{\Lambda}{\mu} e^{\mu t} e^{-\mu t} + C e^{-\mu t}$$

$$N(t) = \frac{\Lambda}{\mu} + C e^{-\mu t} \qquad (1.j)$$

Karena pada persamaan (1.j) masih terdapat t maka didekati menggunakan limit untuk $t \to \infty$, sehingga diperoleh :

$$\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \left(\frac{\Lambda}{\mu} + C e^{-\mu t}\right)$$

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu} + \lim_{t\to\infty} C e^{-\mu t}$$

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu} + 0$$

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu}$$

Lampiran 2. Perhitungan Titik Setimbang Non Endemik ($I = 0$)

Diberikan sistem persamaan sebagai berikut :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L = 0 \quad (2.a)$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H = 0 \quad (2.b)$$

$$\frac{dI}{dt} = \frac{\beta_1 I(S_H + \eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 I T_j}{N} - (\mu + \sigma + \rho)I = 0 \quad (2.c)$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0 \quad (2.d)$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = 0 \quad (2.e)$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\beta_2 IR}{N} - \mu R = 0 \quad (2.f)$$

Titik setimbang non endemik atau bebas narkoba diperoleh dengan mensubstitusikan $I = 0$ kedalam persamaan (2.a) – (2.f), sehingga persamaan (2.d) menjadi sebagai berikut :

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0$$

$$0 - 0 - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0$$

$$-(\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0$$

$$\alpha_2 T_r = (\mu + \alpha_1 + \gamma_1)T_j$$

$$T_r = \frac{(\mu + \alpha_1 + \gamma_1)T_j}{\alpha_2} \quad (2.g)$$

Dari persamaan (2.e) saat diperoleh sebagai berikut :

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = 0$$

$$0 - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = 0 \quad (2.h)$$

dengan mensubstitusikan persamaan (2.g) ke persamaan (2.h) diperoleh berikut :

$$-(\mu+\alpha_2+\gamma_2)\frac{(\mu+\alpha_1+\gamma_1)T_j}{\alpha_2}+\alpha_1 T_j=0$$

$$\frac{-(\mu+\alpha_2+\gamma_2)(\mu+\alpha_1+\gamma_1)T_{j+}\alpha_1\alpha_2 T_j}{\alpha_2}=0$$

$$-(\mu+\alpha_2+\gamma_2)(\mu+\alpha_1+\gamma_1)T_{j+}\alpha_1\alpha_2 T_j=0$$

$$T_j(-(\mu+\alpha_2+\gamma_2)(\mu+\alpha_1+\gamma_1)+\alpha_1\alpha_2)=0$$

$$T_j(-(\mu+\gamma_2)(\mu+\alpha_1+\gamma_1)-\alpha_2(\mu+\gamma_1)-\alpha_1\alpha_2+\alpha_1\alpha_2)=0$$

$$-\big((\mu+\gamma_2)(\mu+\alpha_1+\gamma_1)+\alpha_2(\mu+\gamma_1)\big)T_j=0$$

karena $-\big((\mu+\gamma_2)(\mu+\alpha_1+\gamma_1)+\alpha_2(\mu+\gamma_1)\big)<0$ maka didapatkan

$$T_j=0 \qquad (2.i)$$

dengan mensubstitusikan persamaan (2.i) kedalam (2.g) diperoleh :

$$T_r=\frac{(\mu+\alpha_1+\gamma_1)T_j}{\alpha_2}$$

$$T_r=\frac{(\mu+\alpha_1+\gamma_1)0}{\alpha_2}$$

$$T_r=0 \qquad (2.j)$$

Dengan mensubstitusikan $=0$, persamaan (2.i) dan (2.j) kedalam persamaan (2.f) diperoleh :

$$\frac{dR}{dt}=\gamma_1 T_j+\gamma_2 T_r+\rho I-\frac{\beta_2 IR}{N}-\mu R=0$$

$$0+0+0-0-\mu R=0$$

$$-\mu R=0$$

Karena $-\mu<0$ maka

$$R=0 \qquad (2.k)$$

Dengan mensubstitusikan $I = 0$ kedalam persamaan (2.a), maka diperoleh :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L = 0$$

$$p\Lambda - 0 - (\mu + \omega_1)S_H + \omega_2 S_L = 0$$

$$p\Lambda + \omega_2 S_L = (\mu + \omega_1)S_H$$

$$S_H = \frac{p\Lambda + \omega_2 S_L}{(\mu + \omega_1)} \qquad (2.l)$$

Dengan mensubstitusikan $I = 0$ dan persamaan (2.l) kedalam persamaan (2.b) maka diperoleh :

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H = 0$$

$$(1-p)\Lambda - 0 - (\mu + \omega_2)S_L + \frac{\omega_1(p\Lambda + \omega_2 S_L)}{(\mu + \omega_1)} = 0$$

$$\frac{(\mu+\omega_1)(1-p)\Lambda - (\mu+\omega_1)(\mu+\omega_2)S_L + \omega_1(p\Lambda + \omega_2 S_L)}{(\mu+\omega_1)} = 0$$

$$\frac{(\mu+\omega_1)(1-p)\Lambda - (\mu^2 + \mu\omega_1 + \mu\omega_2 + \omega_1\omega_2)S_L + \omega_1 p\Lambda + \omega_1\omega_2 S_L}{(\mu+\omega_1)} = 0$$

$$\frac{\mu(1-p)\Lambda + \omega_1(1-p)\Lambda - \mu(\mu+\omega_1+\omega_2)S_L - \omega_1\omega_2 S_L + \omega_1 p\Lambda + \omega_1\omega_2 S_L}{(\mu+\omega_1)} = 0$$

$$\frac{\mu(1-p)\Lambda + \omega_1\Lambda - \mu(\mu+\omega_1+\omega_2)S_L}{(\mu+\omega_1)} = 0$$

$$\mu(1-p)\Lambda + \omega_1\Lambda - \mu(\mu + \omega_1 + \omega_2)S_L = 0$$

$$\mu(1-p)\Lambda + \omega_1\Lambda = \mu(\mu + \omega_1 + \omega_2)S_L$$

$$S_L = \frac{\mu(1-p)\Lambda + \omega_1\Lambda}{\mu(\mu+\omega_1+\omega_2)} = \frac{\Lambda(\mu(1-p)+\omega_1)}{\mu(\mu+\omega_1+\omega_2)} \qquad (2.m)$$

Dengan mensubstitusikan persamaan (2.m) kedalam (2.l) sehingga didapatkan :

$$S_H = \frac{p\Lambda + \omega_2 S_L}{(\mu+\omega_1)}$$

$$S_H = \frac{p\Lambda + \frac{\omega_2\Lambda(\mu(1-p)+\omega_1)}{\mu(\mu+\omega_1+\omega_2)}}{(\mu+\omega_1)}$$

$$S_H = \frac{p\Lambda\mu(\mu+\omega_1+\omega_2) + \omega_2\Lambda(\mu(1-p)+\omega_1)}{\mu(\mu+\omega_1+\omega_2)(\mu+\omega_1)}$$

$$S_H = \frac{p\Lambda\mu\omega_2+\mu(1-p)\omega_2\Lambda+p\Lambda\mu(\mu+\omega_1)+\omega_1\omega_2\Lambda}{\mu(\mu+\omega_1+\omega_2)(\mu+\omega_1)}$$

$$S_H = \frac{\Lambda\mu\omega_2+\omega_1\omega_2\Lambda+p\Lambda\mu(\mu+\omega_1)}{\mu(\mu+\omega_1+\omega_2)(\mu+\omega_1)}$$

$$S_H = \frac{(\Lambda\omega_2+p\Lambda\mu)(\mu+\omega_1)}{\mu(\mu+\omega_1+\omega_2)(\mu+\omega_1)}$$

$$S_H = \frac{\Lambda(\mu p+\omega_2)}{\mu(\mu+\omega_1+\omega_2)} \qquad (2.n)$$

Berdasarkan persamaan (2.i), (2.j), (2.k), (2.m) dan (2.n) diperoleh titik setimbang non endemik (E_0) yaitu

$$E_0 = \left({S_H}^0, {S_L}^0, I^0, {T_j}^0, {T_r}^0, R^0\right) = \left(\frac{\Lambda(\mu p+\omega_2)}{\mu(\mu+\omega_1+\omega_2)}, \frac{\Lambda(\mu(1-p)+\omega_1)}{\mu(\mu+\omega_1+\omega_2)}, 0,0,0,0\right)$$

Lampiran 3. Perhitungan *Basic Reproduction Number* (R_0)

Berdasarkan sistem persamaan berikut :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L \quad (3.a)$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H \quad (3.b)$$

$$\frac{dI}{dt} = \frac{\beta_1 I(S_H + \eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 I T_j}{N} - (\mu + \sigma + \rho)I \quad (3.c)$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r \quad (3.d)$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j \quad (3.e)$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\beta_2 IR}{N} - \mu R \quad (3.f)$$

diperoleh titik setimbang non endemik yaitu

$$E_0 = \left({S_H}^0, {S_L}^0, I^0, {T_j}^0, {T_r}^0, R^0\right) = \left(\frac{\Lambda(\mu p + \omega_2)}{\mu(\mu + \omega_1 + \omega_2)}, \frac{\Lambda(\mu(1-p) + \omega_1)}{\mu(\mu + \omega_1 + \omega_2)}, 0,0,0,0\right)$$

Perhitungan R_0 dilakukan dengan menggunakan metode *Next Generation Matrix* (NGM). Populasi pelopor penyebaran narkoba adalah $I, T_j, dan\, T_r$ sehingga didefinisikan

$$x = \begin{pmatrix} I \\ T_j \\ T_r \end{pmatrix}.$$

Dari sini diperoleh

$$\frac{dx}{dt} = \begin{pmatrix} dI/dt \\ dT_j/dt \\ dT_r/dt \end{pmatrix} = \mathcal{F}(x) - \mathcal{Z}(x) \quad (3.g)$$

dengan $\mathcal{F}(x)$ adalah matriks transmisi, yaitu matriks yang elemennya berisi subpopulasi tahapan awal penyalahgunaan narkoba akibat kontak dengan individu I.

Sedangkan, $\mathcal{Z}(x)$ adalah matriks transisi, yaitu matriks yang berisi subpopulasi manusia yang menyalahgunakan narkoba. Dengan mensubstitusikan persamaan (3.c) – (3.e) kedalam persamaan (3.g) diperoleh :

$$\frac{dx}{dt} = \begin{pmatrix} \frac{\beta_1 I(S_H+\eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 IT_j}{N} - (\mu+\sigma+\rho)I \\ (1-q)\sigma I - \frac{\beta_3 IT_j}{N} - (\mu+\alpha_1+\gamma_1)T_j + \alpha_2 T_r \\ q\sigma I - (\mu+\alpha_2+\gamma_2)T_r + \alpha_1 T_j \end{pmatrix}$$

$$\frac{dx}{dt} = \begin{pmatrix} \frac{\mu\beta_1 I(S_H+\eta S_L)}{\Lambda} + \frac{\mu\beta_2 IR}{\Lambda} + \frac{\mu\beta_3 IT_j}{\Lambda} - (\mu+\sigma+\rho)I \\ (1-q)\sigma I - \frac{\mu\beta_3 IT_j}{\Lambda} - (\mu+\alpha_1+\gamma_1)T_j + \alpha_2 T_r \\ q\sigma I - (\mu+\alpha_2+\gamma_2)T_r + \alpha_1 T_j \end{pmatrix}$$

sehingga didapatkan matriks $\mathcal{F}$ dan $\mathcal{Z}$ masing-masing sebagai berikut :

$$\mathcal{F} = \begin{pmatrix} \frac{\mu\beta_1 I(S_H+\eta S_L)}{\Lambda} + \frac{\mu\beta_2 IR}{\Lambda} + \frac{\mu\beta_3 IT_j}{\Lambda} \\ -\frac{\mu\beta_3 IT_j}{\Lambda} \\ 0 \end{pmatrix} \text{ dan } \mathcal{Z} = \begin{pmatrix} (\mu+\sigma+\rho)I \\ -(1-q)\sigma I + (\mu+\alpha_1+\gamma_1)T_j - \alpha_2 T_r \\ -q\sigma I + (\mu+\alpha_2+\gamma_2)T_r - \alpha_1 T_j \end{pmatrix}$$

Berikut matriks jacobian untuk $\mathcal{F}$ dan $\mathcal{Z}$ masing-masing sebagai berikut :

$$\mathbb{F} = \begin{pmatrix} \frac{\mu\beta_1(S_H+\eta S_L)}{\Lambda} + \frac{\mu\beta_2 R}{\Lambda} + \frac{\mu\beta_3 T_j}{\Lambda} & \frac{\mu\beta_3 I}{\Lambda} & 0 \\ -\frac{\mu\beta_3 T_j}{\Lambda} & -\frac{\mu\beta_3 I}{\Lambda} & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ dan } \mathbb{Z} = \begin{pmatrix} (\mu+\sigma+\rho) & 0 & 0 \\ -(1-q)\sigma & \mu+\alpha_1+\gamma_1 & -\alpha_2 \\ -q\sigma & -\alpha_1 & \mu+\alpha_2+\gamma_2 \end{pmatrix}$$

Berikut ini matriks jacobian untuk $\mathcal{F}$ dan $\mathcal{Z}$ di sekitar titik setimbang E_0

$$\mathbb{F} = \begin{pmatrix} \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{\mu+\omega_1+\omega_2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ dan } \mathbb{Z} = \begin{pmatrix} (\mu+\sigma+\rho) & 0 & 0 \\ -(1-q)\sigma & \mu+\alpha_1+\gamma_1 & -\alpha_2 \\ -q\sigma & -\alpha_1 & \mu+\alpha_2+\gamma_2 \end{pmatrix}$$

Pada metode NGM R_0 diperoleh dengan menentukan nilai eigen terbesar dari $R = \mathbb{F}\mathbb{Z}^{-1}$

Dari sini diperoleh

$$\mathbb{Z}^{-1} = \frac{1}{\det \mathbb{Z}}(Adj\ \mathbb{Z})$$

$$\mathbb{Z}^{-1} = \begin{pmatrix} \frac{1}{\mu+\sigma+\rho} & 0 & 0 \\ \frac{\sigma(\alpha_2+(1-q)(\mu+\gamma_2))}{h(\mu+\sigma+\rho)} & \frac{\mu+\alpha_2+\gamma_2}{h} & \frac{\alpha_2}{h} \\ \frac{\sigma(\alpha_1+q(\mu+\gamma_1))}{h(\mu+\sigma+\rho)} & \frac{\alpha_1}{h} & \frac{\mu+\alpha_1+\gamma_1}{h} \end{pmatrix}$$

dengan $h = (\mu+\gamma_1)(\mu+\alpha_2+\gamma_2) + \alpha_1(\mu+\gamma_2)$

Sehingga diperoleh $R = \mathbb{F}\mathbb{Z}^{-1}$ yang diuraikan sebagai berikut :

$$R = \mathbb{F}\mathbb{Z}^{-1}$$

$$R = \begin{pmatrix} \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{\mu+\omega_1+\omega_2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\mu+\sigma+\rho} & 0 & 0 \\ \frac{\sigma(\alpha_2+(1-q)(\mu+\gamma_2))}{h(\mu+\sigma+\rho)} & \frac{\mu+\alpha_2+\gamma_2}{h} & \frac{\alpha_2}{h} \\ \frac{\sigma(\alpha_1+q(\mu+\gamma_1))}{h(\mu+\sigma+\rho)} & \frac{\alpha_1}{h} & \frac{\mu+\alpha_1+\gamma_1}{h} \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Selanjutnya akan dicari nilai eigen dari R dengan uraian perhitungan sebagai berikut :

$$\det(R - \lambda I) = 0$$

$$\det\begin{pmatrix} \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} - \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix} = 0$$

$$\left(\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} - \lambda\right)(\lambda^2) = 0 \qquad (3.h)$$

Berdasarkan persamaan (3.h) didapatkan nilai eigen $\lambda_1 = \frac{\beta_1\left(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\right)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}$, $\lambda_2 = 0$, $\lambda_3 = 0$, sehingga diperoleh *basic reproduction number* berikut :

$R_0 = maks\{\lambda_i\}$, dengan $i = 1,2,3$

$$R_0 = \frac{\beta_1\left(\mu p + \omega_2 + \eta(\mu(1-p) + \omega_1)\right)}{(\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2)}$$

Lampiran 4. Perhitungan Titik Setimbang Endemik ($I \neq 0$)

Diberikan sistem persamaan diferensial sebagai berikut :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L = 0 \quad (4.a)$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H = 0 \quad (4.b)$$

$$\frac{dI}{dt} = \frac{\beta_1 I(S_H + \eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 I T_j}{N} - (\mu + \sigma + \rho)I = 0 \quad (4.c)$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0 \quad (4.d)$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = 0 \quad (4.e)$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\beta_2 IR}{N} - \mu R = 0 \quad (4.f)$$

Keadaan endemik terjadi ketika terdapat penyalahguna narkoba yang tidak mendepatkan perawatan atau $I > 0$. Persamaan (4.e) dapat diuraikan sebagai berikut :

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = 0$$

$$\Leftrightarrow q\sigma I + \alpha_1 T_j = (\mu + \alpha_2 + \gamma_2)T_r$$

$$\Leftrightarrow T_r = \frac{q\sigma I + \alpha_1 T_j}{\mu + \alpha_2 + \gamma_2} \quad (4.g)$$

Dengan mensubstitusikan persamaan (4.g) ke (4.d) maka diperoleh

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = 0$$

$$\Leftrightarrow (1-q)\sigma I - \frac{\mu\beta_3 I T_j}{\Lambda} - (\mu + \alpha_1 + \gamma_1)T_j + \frac{\alpha_2(q\sigma I + \alpha_1 T_j)}{\mu + \alpha_2 + \gamma_2} = 0$$

$$\Leftrightarrow (1-q)\sigma I - \frac{\mu\beta_3 I T_j}{\Lambda} - (\mu + \alpha_1 + \gamma_1)T_j + \frac{\alpha_2 q\sigma I}{\mu + \alpha_2 + \gamma_2} + \frac{\alpha_1\alpha_2 T_j}{\mu + \alpha_2 + \gamma_2} = 0$$

$$\Leftrightarrow (1-q)\sigma I + \frac{\alpha_2 q\sigma I}{\mu + \alpha_2 + \gamma_2} = \frac{\mu\beta_3 I T_j}{\Lambda} + (\mu + \alpha_1 + \gamma_1)T_j - \frac{\alpha_1\alpha_2 T_j}{\mu + \alpha_2 + \gamma_2}$$

$$\Leftrightarrow \frac{(\mu + \alpha_2 + \gamma_2)(1-q)\sigma I + \alpha_2 q\sigma I}{\mu + \alpha_2 + \gamma_2} = T_j\left(\frac{\mu\beta_3 I(\mu + \alpha_2 + \gamma_2) + \Lambda(\mu + \alpha_1 + \gamma_1)(\mu + \alpha_2 + \gamma_2) - \Lambda\alpha_1\alpha_2}{\Lambda(\mu + \alpha_2 + \gamma_2)}\right)$$

$$\Leftrightarrow \frac{(\mu+\gamma_2)(1-q)\sigma I+\alpha_2(1-q)\sigma I+\alpha_2 q\sigma I}{\mu+\alpha_2+\gamma_2} = T_j\left(\frac{\mu\beta_3 I(\mu+\alpha_2+\gamma_2)+\Lambda\big((\mu+\gamma_1)(\mu+\alpha_2+\gamma_2)+\alpha_1(\mu+\gamma_2)\big)+\Lambda\alpha_1\alpha_2-\Lambda\alpha_1\alpha_2}{\Lambda(\mu+\alpha_2+\gamma_2)}\right)$$

$$\Leftrightarrow (\mu+\gamma_2)(1-q)\sigma I+\alpha_2\sigma I = T_j\left(\frac{\mu\beta_3 I(\mu+\alpha_2+\gamma_2)+\Lambda\big((\mu+\gamma_1)(\mu+\alpha_2+\gamma_2)+\alpha_1(\mu+\gamma_2)\big)}{\Lambda(\mu+\alpha_2+\gamma_2)}\right)(\mu+\alpha_2+\gamma_2)$$

$$\Leftrightarrow \sigma I\big((\mu+\gamma_2)(1-q)+\alpha_2\big) = T_j\left(\frac{(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)}{\Lambda}\right)$$

$$\Leftrightarrow T_j = \frac{\Lambda\sigma I\big((\mu+\gamma_2)(1-q)+\alpha_2\big)}{(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)} \qquad (4.h)$$

Persamaan (4.h) disubstitusikan ke persamaan (4.g) sehingga didapatkan berikut :

$$T_r = \frac{q\sigma I+\alpha_1 T_j}{\mu+\alpha_2+\gamma_2}$$

$$\Leftrightarrow T_r = \frac{q\sigma I+\frac{\alpha_1\Lambda\sigma I\big((\mu+\gamma_2)(1-q)+\alpha_2\big)}{(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)}}{\mu+\alpha_2+\gamma_2}$$

$$\Leftrightarrow T_r = \frac{q\sigma I[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]+\alpha_1\Lambda\sigma I\big((\mu+\gamma_2)(1-q)+\alpha_2\big)}{(\mu+\alpha_2+\gamma_2)[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]}$$

$$\Leftrightarrow T_r = \frac{q\sigma I\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)(\mu+\alpha_2+\gamma_2)+q\sigma I\Lambda\alpha_1(\mu+\gamma_2)+\alpha_1\Lambda\sigma I(\mu+\gamma_2)(1-q)+\alpha_1\alpha_2\Lambda\sigma I}{(\mu+\alpha_2+\gamma_2)[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]}$$

$$\Leftrightarrow T_r = \frac{q\sigma I\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)(\mu+\alpha_2+\gamma_2)+\sigma I\Lambda\alpha_1(\mu+\gamma_2)+\alpha_1\alpha_2\Lambda\sigma I}{(\mu+\alpha_2+\gamma_2)[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]}$$

$$\Leftrightarrow T_r = \frac{q\sigma I\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)(\mu+\alpha_2+\gamma_2)+\sigma I\Lambda\alpha_1(\mu+\alpha_2+\gamma_2)}{(\mu+\alpha_2+\gamma_2)[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]}$$

$$\Leftrightarrow T_r = \frac{[q\sigma I\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)+\sigma I\Lambda\alpha_1](\mu+\alpha_2+\gamma_2)}{(\mu+\alpha_2+\gamma_2)[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)]}$$

$$\Leftrightarrow T_r = \frac{\sigma I[q\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)+\Lambda\alpha_1]}{(\mu\beta_3 I+\Lambda(\mu+\gamma_1))(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)} \qquad (4.i)$$

Persamaan (4.h) dan (4.i) disubstitusikan kedalam persamaan (4.f) sehingga diperoleh berikut :

$$\frac{dR}{dt} = \gamma_1 T_j+\gamma_2 T_r+\rho I-\frac{\beta_2 IR}{N}-\mu R = 0$$

$$\Leftrightarrow \gamma_1 T_j+\gamma_2 T_r+\rho I-\frac{\mu\beta_2 IR}{\Lambda}-\mu R = 0$$

$$\Leftrightarrow \gamma_1 T_j+\gamma_2 T_r+\rho I = \frac{\mu\beta_2 IR}{\Lambda}+\mu R$$

$$\Leftrightarrow \gamma_1 T_j+\frac{\gamma_2(q\sigma I+\alpha_1 T_j)}{\mu+\alpha_2+\gamma_2}+\rho I = R\left(\frac{\mu\beta_2 I+\Lambda\mu}{\Lambda}\right)$$

$$\Leftrightarrow R = \frac{\Lambda\left(\gamma_1 T_j+\frac{\gamma_2(q\sigma I+\alpha_1 T_j)}{\mu+\alpha_2+\gamma_2}+\rho I\right)}{\mu\beta_2 I+\Lambda\mu}$$

$$\Leftrightarrow R = \frac{\Lambda\left(\gamma_1 T_j(\mu+\alpha_2+\gamma_2)+\gamma_2(q\sigma I+\alpha_1 T_j)+\rho I(\mu+\alpha_2+\gamma_2)\right)}{(\mu+\alpha_2+\gamma_2)(\mu\beta_2 I+\Lambda\mu)}$$

$$\Leftrightarrow R = \frac{\Lambda\left(\gamma_1 T_j(\mu+\alpha_2+\gamma_2)+\gamma_2(q\sigma I+\alpha_1 T_j)+\rho I(\mu+\alpha_2+\gamma_2)\right)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)} \qquad (4.j)$$

Persamaan (4.a) dapat diuraikan sebagai berikut sehingga didapatkan S_H:

$$\frac{dS_H}{dt} = p\Lambda - \frac{\mu\beta_1 I S_H}{\Lambda} - (\mu+\omega_1)S_H + \omega_2 S_L = 0$$

$$\Leftrightarrow p\Lambda + \omega_2 S_L = \frac{\mu\beta_1 I S_H}{\Lambda} + (\mu+\omega_1)S_H$$

$$\Leftrightarrow p\Lambda + \omega_2 S_L = \frac{\mu\beta_1 I+\Lambda(\mu+\omega_1)}{\Lambda} S_H$$

$$\Leftrightarrow S_H = \frac{\Lambda(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)} \qquad (4.k)$$

Dengan mensubstitusikan persamaan (4.k) ke (4.b) diperoleh sebagai berikut :

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\mu\eta\beta_1 I S_L}{\Lambda} - (\mu+\omega_2)S_L + \omega_1 S_H = 0$$

$$\Leftrightarrow (1-p)\Lambda + \omega_1 S_H = \frac{\mu\eta\beta_1 I S_L}{\Lambda} + (\mu+\omega_2)S_L$$

$$\Leftrightarrow (1-p)\Lambda + \frac{\omega_1\Lambda(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)} = \frac{\mu\eta\beta_1 I+\Lambda(\mu+\omega_2)}{\Lambda} S_L$$

$$\Leftrightarrow \Lambda\left((1-p) + \frac{\omega_1(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)}\right) = \frac{\mu\eta\beta_1 I+\Lambda(\mu+\omega_2)}{\Lambda} S_L$$

$$\Leftrightarrow \Lambda^2\left(\frac{(1-p)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\omega_1(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)}\right) = \left(\mu\eta\beta_1 I + \Lambda(\mu+\omega_2)\right)S_L$$

$$\Leftrightarrow \Lambda^2\left(\frac{(1-p)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\omega_1(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)}\right) = \left(\mu\eta\beta_1 I + \Lambda(\mu+\omega_2)\right)S_L$$

$$\Leftrightarrow \Lambda^2\left((1-p)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \omega_1(p\Lambda+\omega_2 S_L)\right) = \left(\mu\eta\beta_1 I + \Lambda(\mu+\omega_2)\right)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right)S_L$$

$$\Leftrightarrow \Lambda^2\omega_1\omega_2 S_L + \Lambda^2\left((1-p)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \omega_1 p\Lambda\right) = S_L\left((\mu\eta\beta_1 I + \Lambda\mu)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \Lambda\omega_2(\mu\beta_1 I + \Lambda\mu)\right) + \Lambda^2\omega_1\omega_2 S_L$$

$$\Leftrightarrow \Lambda^2\left((1-p)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \omega_1 p\Lambda\right) = S_L\left((\mu\eta\beta_1 I + \Lambda\mu)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \Lambda\omega_2(\mu\beta_1 I + \Lambda\mu)\right) + \Lambda^2\omega_1\omega_2 S_L - \Lambda^2\omega_1\omega_2 S_L$$

$$\Leftrightarrow \Lambda^2\left((1-p)(\mu\beta_1 I + \Lambda\mu) + \omega_1(1-p)\Lambda + \omega_1 p\Lambda\right) = S_L\left((\mu\eta\beta_1 I + \Lambda\mu)\left(\mu\beta_1 I + \Lambda(\mu+\omega_1)\right) + \Lambda\omega_2(\mu\beta_1 I + \Lambda\mu)\right)$$

$$\Leftrightarrow S_L = \frac{\Lambda^2\left(\mu(1-p)(\beta_1 I+\Lambda)+\omega_1\Lambda\right)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)} \qquad (4.l)$$

Persamaan (4.l) disubstitusikan kedalam persamaan (4.k) sehingga diperoleh :

$$S_H = \frac{\Lambda(p\Lambda+\omega_2 S_L)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)}$$

$$\Leftrightarrow S_H = \frac{\Lambda\left(p\Lambda+\frac{\omega_2\Lambda^2(\mu(1-p)(\beta_1 I+\Lambda)+\omega_1\Lambda)}{b}\right)}{\mu\beta_1 I+\Lambda(\mu+\omega_1)}$$

dengan $b = (\mu\eta\beta_1 I + \Lambda\mu)\big(\mu\beta_1 I + \Lambda(\mu + \omega_1)\big) + \Lambda\omega_2(\mu\beta_1 I + \Lambda\mu)$, maka

$$S_H = \frac{bp\Lambda^2+\omega_2\Lambda^3(\mu(1-p)(\beta_1 I+\Lambda)+\omega_1\Lambda)}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{bp\Lambda^2+\omega_2\Lambda^3\big(\mu(1-p)(\beta_1 I+\Lambda)\big)+\omega_1\omega_2\Lambda^4}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2\Big((\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\Big)+\omega_2\Lambda^3\big(\mu(1-p)(\beta_1 I+\Lambda)\big)+\omega_1\omega_2\Lambda^4}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+p\Lambda^3\omega_2(\mu\beta_1 I+\Lambda\mu)+\omega_2\Lambda^3\big(\mu(1-p)(\beta_1 I+\Lambda)\big)+\omega_1\omega_2\Lambda^4}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\mu p\omega_2\Lambda^3(\beta_1 I+\Lambda)+\mu(1-p)\omega_2\Lambda^3(\beta_1 I+\Lambda)+\omega_1\omega_2\Lambda^4}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\omega_2\Lambda^3(\mu\beta_1 I+\Lambda\mu)+\omega_1\omega_2\Lambda^4}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\omega_2\Lambda^3(\mu\beta_1 I+\Lambda\mu+\Lambda\omega_1)}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\omega_2\Lambda^3(\mu\beta_1 I+\Lambda(\mu+\omega_1))}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{\big(p\Lambda^2(\mu\eta\beta_1 I+\Lambda\mu)+\omega_2\Lambda^3\big)(\mu\beta_1 I+\Lambda(\mu+\omega_1))}{b\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)}$$

$$\Leftrightarrow S_H = \frac{\Lambda^2(\mu p(\eta\beta_1 I+\Lambda)+\Lambda\omega_2)}{b}$$

$$\Leftrightarrow S_H = \frac{\Lambda^2(\mu p(\eta\beta_1 I+\Lambda)+\Lambda\omega_2)}{(\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)} \quad (4.m)$$

Dengan mensubstitusikan persamaan (4.h), (4.j), (4.l), dan (4.m) ke persamaan (4.c) diperoleh uraian sebagai berikut :

$$\frac{dI}{dt} = 0$$

$$\frac{\mu\beta_1 I(S_H+\eta S_L)}{\Lambda} + \frac{\mu\beta_2 IR}{\Lambda} + \frac{\mu\beta_3 IT_j}{\Lambda} - (\mu + \sigma + \rho)I = 0$$

$$\frac{\mu I}{\Lambda}\big(\beta_1(S_H + \eta S_L) + \beta_2 R + \beta_3 T_j\big) = (\mu + \sigma + \rho)I$$

$$\beta_1(S_H + \eta S_L) + \beta_2 R + \beta_3 T_j = \frac{\Lambda(\mu+\sigma+\rho)}{\mu} \quad (4.n)$$

Misalkan,

$m_1 = \beta_1(S_H + \eta S_L)$

$m_2 = \beta_2 R + \beta_3 T_j$

m_1 dan m_2 diatas dapat diuraikan sebagai berikut :

i. $m_1 = \beta_1(S_H + \eta S_L)$

$$\Leftrightarrow m_1 = \beta_1\left(\frac{\Lambda^2(\mu p(\eta\beta_1 I+\Lambda)+\Lambda\omega_2)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)} + \frac{\eta\Lambda^2(\mu(1-p)(\beta_1 I+\Lambda)+\omega_1\Lambda)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}\right)$$

$$\Leftrightarrow m_1 = \beta_1\left(\frac{\Lambda^2(\mu p(\eta\beta_1 I+\Lambda)+\Lambda\omega_2)+\eta\Lambda^2(\mu(1-p)(\beta_1 I+\Lambda)+\omega_1\Lambda)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}\right)$$

$$\Leftrightarrow m_1 = \beta_1\Lambda^2\left(\frac{\mu p\eta\beta_1 I+\mu p\Lambda+\Lambda\omega_2+\eta\mu(1-p)\beta_1 I+\eta\mu(1-p)\Lambda+\eta\omega_1\Lambda}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}\right)$$

$$\Leftrightarrow m_1 = \beta_1\Lambda^2\left(\frac{\eta\mu\beta_1 I+\Lambda\mu(p+\eta(1-p))+\Lambda(\eta\omega_1+\omega_2)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}\right)$$

$$\Leftrightarrow m_1 = \frac{\beta_1\Lambda^2\eta\mu\beta_1 I+\beta_1\Lambda^2\left(\Lambda\mu\left(p+\eta(1-p)\right)+\Lambda(\eta\omega_1+\omega_2)\right)}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}$$

Misalkan $= \beta_1\Lambda^2\left(\Lambda\mu\left(p+\eta(1-p)\right)+\Lambda(\eta\omega_1+\omega_2)\right)$, maka diperoleh m_1 sebagai berikut :

$$m_1 = \frac{\beta_1\Lambda^2\eta\mu\beta_1 I+v}{(\mu\eta\beta_1 I+\Lambda\mu)\left(\mu\beta_1 I+\Lambda(\mu+\omega_1)\right)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)}$$

ii. $m_2 = \beta_2 R + \beta_3 T_j$

$$\Leftrightarrow m_2 = \beta_2\frac{\Lambda\left(\gamma_1 T_j(\mu+\alpha_2+\gamma_2)+\gamma_2(q\sigma I+\alpha_1 T_j)+\rho I(\mu+\alpha_2+\gamma_2)\right)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)} + \beta_3 T_j$$

$$\Leftrightarrow m_2 = \frac{\beta_2\Lambda\left(T_j(\gamma_1\mu+\gamma_1\alpha_2+\gamma_1\gamma_2+\alpha_1\gamma_2)+\gamma_2 q\sigma I+\rho I(\mu+\alpha_2+\gamma_2)\right)+\beta_3 T_j\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_2 = \frac{T_j\left(\beta_2\Lambda(\gamma_1\mu+\gamma_1\alpha_2+\gamma_1\gamma_2+\alpha_1\gamma_2)+\beta_3\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)\right)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)} + \frac{\beta_2\Lambda\left(\gamma_2 q\sigma I+\rho I(\mu+\alpha_2+\gamma_2)\right)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_2 = \frac{\Lambda\sigma I\left((\mu+\gamma_2)(1-q)+\alpha_2\right)\left(\beta_2\Lambda(\gamma_1\mu+\gamma_1\alpha_2+\gamma_1\gamma_2+\alpha_1\gamma_2)+\beta_3\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)\right)}{\mu\left[\left(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\right)(\mu+\alpha_2+\gamma_2)+\Lambda\alpha_1(\mu+\gamma_2)\right](\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)} + \frac{\beta_2\Lambda\left(\gamma_2 q\sigma I+\rho I(\mu+\alpha_2+\gamma_2)\right)}{\mu(\mu+\alpha_2+\gamma_2)(\beta_2 I+\Lambda)}$$

Misalkan :

$n_1 = (\mu+\gamma_2)(1-q)+\alpha_2$

$n_2 = \gamma_1\mu+\gamma_1\alpha_2+\gamma_1\gamma_2+\alpha_1\gamma_2$

$n_3 = \mu + \alpha_2 + \gamma_2$

Sehingga m_2 dapat dituliskan sebagai berikut :

$$m_2 = \frac{\Lambda\sigma I n_1\left(\beta_2\Lambda n_2+\beta_3\mu n_3(\beta_2 I+\Lambda)\right)}{\mu[(\mu\beta_3 I+\Lambda(\mu+\gamma_1))n_3+\Lambda\alpha_1(\mu+\gamma_2)]n_3(\beta_2 I+\Lambda)} + \frac{\beta_2\Lambda(\gamma_2 q\sigma I+\rho I n_3)}{\mu n_3(\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_2 = \frac{\Lambda\sigma I n_1\left(\beta_2\Lambda n_2+\beta_3\mu n_3(\beta_2 I+\Lambda)\right)+\beta_2\Lambda(\gamma_2 q\sigma I+\rho I n_3)[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)]}{\mu[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)]n_3(\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_2 = \frac{\Lambda^2\sigma n_1\beta_2 n_2 I+\Lambda\sigma n_1\beta_3\mu n_3 I(\beta_2 I+\Lambda)+\beta_2\Lambda(\gamma_2 q\sigma I+\rho I n_3)[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)]}{\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}$$

Persamaan (4.n) dapat dituliskan menjadi berikut :

$$\beta_1(S_H+\eta S_L)+\beta_2 R+\beta_3 T_j = \frac{\Lambda(\mu+\sigma+\rho)}{\mu}$$

$$\Leftrightarrow m_1+m_2 = \frac{\Lambda(\mu+\sigma+\rho)}{\mu} \qquad (4.o)$$

Misalkan $m_1 + m_2$ diuraikan sebagai berikut :

$$m_1+m_2 = \frac{\beta_1\Lambda^2\eta\mu\beta_1 I+v}{(\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)} + \frac{\Lambda^2\sigma n_1\beta_2 n_2 I+\Lambda\sigma n_1\beta_3\mu n_3 I(\beta_2 I+\Lambda)+\beta_2\Lambda(\gamma_2 q\sigma I+\rho I n_3)[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)]}{\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_1+m_2 =$$

$$\frac{(\beta_1\Lambda^2\eta\mu\beta_1 I+v)\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}{((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu))\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} +$$

$$\frac{\Lambda^2\sigma n_1\beta_2 n_2 I[(\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)]}{\left((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\right)\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} +$$

$$\frac{\Lambda\sigma n_1\beta_3\mu n_3 I(\beta_2 I+\Lambda)[(\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)]}{\left((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\right)\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} +$$

$$\frac{[(\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)](\beta_2\Lambda(\gamma_2 q\sigma I+\rho I n_3)[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)])}{((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu))\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}$$

$$\Leftrightarrow m_1+m_2 =$$

$$\frac{r_1+r_2+r_3+r_4}{((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu))\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}$$

dengan

$$r_1 = \Lambda^2\sigma n_1\beta_2 n_2{\beta_1}^2\mu^2\eta I^3 + (\Lambda^3\sigma n_1\beta_2 n_2\beta_1\mu^2\eta + \Lambda^3\sigma n_1\beta_2 n_2\beta_1\mu\omega_1\eta + \Lambda^3\sigma n_1\beta_2 n_2\beta_1\mu^2 + \Lambda^3\sigma n_1\beta_2 n_2\beta_1\mu\omega_2)I^2 + (\Lambda^4\sigma n_1\beta_2 n_2\mu^2 + \Lambda^4\sigma n_1\beta_2 n_2\mu\omega_1 + \Lambda^4\sigma n_1\beta_2 n_2\mu\omega_2)I$$

$$r_2 = \Lambda\sigma n_1\beta_3 n_3\beta_2{\beta_1}^2\mu^3\eta I^4 + (\Lambda^2\sigma n_1\beta_3 n_3\beta_2\beta_1\mu^3\eta + \Lambda^2\sigma n_1\beta_3 n_3\beta_2\beta_1\mu^2\omega_1\eta + \Lambda^2\sigma n_1\beta_3 n_3\beta_2\beta_1\mu^3 + \Lambda^2\sigma n_1\beta_3 n_3\beta_2\beta_1\mu^2\omega_2 + \Lambda^2\sigma n_1\beta_3 n_3{\beta_1}^2\mu^3\eta)I^3 + (\Lambda^3\mu^3\sigma n_1\beta_3 n_3\beta_2 + \Lambda^3\mu^2\omega_1\sigma n_1\beta_3 n_3\beta_2 + \Lambda^3\mu^2\omega_2\sigma n_1\beta_3 n_3\beta_2 + \Lambda^3\sigma n_1\beta_3 n_3\beta_1\mu^3\eta + \Lambda^3\sigma n_1\beta_3 n_3\beta_1\mu^2\omega_1\eta + \Lambda^3\sigma n_1\beta_3 n_3\beta_1\mu^3 + \Lambda^3\sigma n_1\beta_3 n_3\beta_1\mu^2\omega_2)I^2 + (\Lambda^4\mu^3\sigma n_1\beta_3 n_3 + \Lambda^4\mu^2\omega_1\sigma n_1\beta_3 n_3 + \Lambda^4\mu^2\omega_2\sigma n_1\beta_3 n_3)I$$

$$r_3 = \Lambda^2\beta_3 n_3\beta_2{\beta_1}^2\mu^3\eta I^3 + \left(\Lambda^3\alpha_1\gamma_2 n_3\beta_2{\beta_1}^2\mu^2\eta + \Lambda^3\alpha_1 n_3\beta_2{\beta_1}^2\mu^3\eta + \Lambda^3\gamma_1{n_3}^2\beta_2{\beta_1}^2\mu^2\eta + \Lambda^3{n_3}^2\beta_3{\beta_1}^2\mu^3\eta + \Lambda^3{n_3}^2\beta_2{\beta_1}^2\mu^3\eta + {n_3}^2\beta_2\beta_3\mu v\right)I^2 + \left(\Lambda^4\alpha_1\gamma_2 n_3{\beta_1}^2\mu^2\eta + \Lambda^4\alpha_1 n_3{\beta_1}^2\mu^3\eta + \Lambda^4\gamma_1{n_3}^2{\beta_1}^2\mu^2\eta + \Lambda^4{n_3}^2{\beta_1}^2\mu^3\eta + \Lambda\alpha_1\gamma_2 n_3\beta_2\mu v + \Lambda\alpha_1 n_3\beta_2\mu^2 v + \Lambda\gamma_1{n_3}^2\beta_2\mu v + \Lambda{n_3}^2\beta_2\mu^2 v + \Lambda{n_3}^2\beta_3\mu^2 v\right)I + (\Lambda^2\alpha_1\gamma_2 n_3\mu v + \Lambda^2\alpha_1 n_3\mu^2 v + \Lambda^2\gamma_1{n_3}^2\mu v + \Lambda^2{n_3}^2\mu^2 v)$$

$$r_4 = \left(\Lambda\beta_2{\beta_1}^2\beta_3\eta\gamma_2 n_3\mu^3 q\sigma + \Lambda\beta_2{\beta_1}^2\beta_3\eta{n_3}^2\mu^3\rho\right)I^4 + \left(\Lambda{\beta_1}^2\alpha_1\eta{\gamma_2}^2\mu^2 q\sigma + \Lambda{\beta_1}^2\alpha_1\eta\gamma_2\mu^3 q\sigma + \Lambda{\beta_1}^2\gamma_1\eta\gamma_2 n_3\mu^2 q\sigma + \Lambda{\beta_1}^2\eta\gamma_2 n_3\mu^3 q\sigma + \Lambda\beta_1\beta_3\eta\gamma_2 n_3\mu^3 q\sigma + \Lambda\beta_1\beta_3\eta\gamma_2 n_3\mu^2\omega_1 q\sigma + \Lambda{\beta_1}^2\alpha_1\eta\gamma_2 n_3\mu^2\rho + \Lambda{\beta_1}^2\alpha_1\eta n_3\mu^3\rho + \Lambda{\beta_1}^2\gamma_1\eta{n_3}^2\mu^2\rho + \Lambda{\beta_1}^2\eta{n_3}^2\mu^3\rho + \Lambda\beta_1\beta_3\eta{n_3}^2\mu^3\rho + \Lambda\beta_1\beta_3\eta{n_3}^2\mu^2\omega_1\rho + \Lambda\beta_1\beta_3\gamma_2 n_3\mu^3 q\sigma + \Lambda\beta_1\beta_3\gamma_2 n_3\mu^2\omega_2 q\sigma + \Lambda\beta_1\beta_3{n_3}^2\mu^3\rho + \Lambda\beta_1\beta_3{n_3}^2\mu^2\omega_2\rho\right)\Lambda\beta_2 I^3 + (\Lambda^2\alpha_1\beta_1\eta{\gamma_2}^2\mu^2 q\sigma + \Lambda^2\alpha_1\beta_1\eta{\gamma_2}^2\mu\omega_1 q\sigma + \Lambda^2\alpha_1\beta_1\eta\gamma_2\mu^3 q\sigma + \Lambda^2\alpha_1\beta_1\eta\gamma_2\mu^2\omega_1 q\sigma + \Lambda^2\gamma_1\beta_1\eta\gamma_2 n_3\mu^2 q\sigma + \Lambda^2\gamma_1\beta_1\eta\gamma_2 n_3\mu\omega_1 q\sigma + \Lambda^2\beta_1\eta\gamma_2 n_3\mu^3 q\sigma + \Lambda^2\beta_1\eta\gamma_2 n_3\mu^2\omega_1 q\sigma + \Lambda^2\alpha_1\beta_1\eta\gamma_2 n_3\mu^2\rho + \Lambda^2\alpha_1\beta_1\eta\gamma_2 n_3\mu\omega_1\rho + \Lambda^2\alpha_1\beta_1\eta n_3\mu^3\rho + \Lambda^2\alpha_1\beta_1\eta n_3\mu^2\omega_1\rho + \Lambda^2\alpha_1\beta_1{\gamma_2}^2\mu^2 q\sigma + \Lambda^2\alpha_1\beta_1{\gamma_2}^2\mu\omega_2 q\sigma + \Lambda^2\alpha_1\beta_1\gamma_2\mu^3 q\sigma + \Lambda^2\alpha_1\beta_1\mu^2\gamma_2\omega_2 q\sigma + \Lambda^2\beta_1\eta\gamma_1{n_3}^2\mu^2\rho + \Lambda^2\beta_1\eta\gamma_1{n_3}^2\mu\omega_1\rho + \Lambda^2\beta_1\eta{n_3}^2\mu^3\rho + \Lambda^2\beta_1\eta{n_3}^2\mu^2\omega_1\rho + \Lambda^2\gamma_1\beta_1\gamma_2 n_3\mu^2 q\sigma + \Lambda^2\gamma_1\beta_1\gamma_2 n_3\mu\omega_2 q\sigma + \Lambda^2\beta_1\gamma_2 n_3\mu^3 q\sigma + \Lambda^2\beta_1\gamma_2 n_3\mu^2\omega_2 q\sigma + \Lambda^2\beta_3\gamma_2 n_3\mu^3 q\sigma + \Lambda^2\beta_3\gamma_2 n_3\mu^2\omega_1 q\sigma + \Lambda^2\beta_3\gamma_2 n_3\mu^2\omega_2 q\sigma + \Lambda^2\alpha_1\beta_1\gamma_2 n_3\mu^2\rho + \Lambda^2\alpha_1\beta_1\gamma_2 n_3\mu\omega_2\rho + \Lambda^2\alpha_1\beta_1 n_3\mu^3\rho + \Lambda^2\alpha_1\beta_1 n_3\mu^2\omega_2\rho + \Lambda^2\beta_1\gamma_1{n_3}^2\mu^2\rho + \Lambda^2\beta_1\gamma_1{n_3}^2\mu\omega_2\rho + \Lambda^2\beta_1{n_3}^2\mu^3\rho + \Lambda^2\beta_1{n_3}^2\mu^2\omega_2\rho + \Lambda^2\beta_3{n_3}^2\mu^3\rho + \Lambda^2\beta_3{n_3}^2\mu^2\omega_1\rho + \Lambda^2\beta_3{n_3}^2\mu^2\omega_2\rho)\Lambda\beta_2 I^2 + (\alpha_1{\gamma_2}^2\mu^2 q\sigma + \alpha_1{\gamma_2}^2\mu\omega_1 q\sigma + \alpha_1{\gamma_2}^2\mu\omega_2 q\sigma + \alpha_1\gamma_2\mu^3 q\sigma + \alpha_1\gamma_2\mu^2\omega_1 q\sigma +$$

$\alpha_1\gamma_2\mu^2\omega_2 q\sigma + \gamma_1\gamma_2 n_3\mu^2 q\sigma + \gamma_1\gamma_2 n_3\mu\omega_1 q\sigma + \gamma_1\gamma_2 n_3\mu\omega_2 q\sigma +$
$\gamma_2 n_3\mu^3 q\sigma + \gamma_2 n_3\mu^2\omega_1 q\sigma + \gamma_2 n_3\mu^2\omega_2 q\sigma + \alpha_1\gamma_2 n_3\mu^2\rho + \alpha_1\gamma_2 n_3\mu\omega_1\rho +$
$\alpha_1\gamma_2 n_3\mu\omega_2\rho + \alpha_1 n_3\mu^3\rho + \alpha_1 n_3\mu^2\omega_1\rho + \alpha_1 n_3\mu^2\omega_2\rho + \gamma_1 {n_3}^2\mu^2\rho +$
$\gamma_1 {n_3}^2\mu\omega_1\rho + \gamma_1 {n_3}^2\mu\omega_2\rho + {n_3}^2\mu^3\rho + {n_3}^2\mu^2\omega_1\rho + {n_3}^2\mu^2\omega_2\rho)\beta_2\Lambda^4 I.$

Sehingga persamaan (4.o) dapat diuraikan sebagai berikut :

$$m_1 + m_2 = \frac{\Lambda(\mu+\sigma+\rho)}{\mu}$$

$$\Leftrightarrow \frac{r_1+r_2+r_3+r_4}{((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu))\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} = \frac{\Lambda(\mu+\sigma+\rho)}{\mu}$$

$$\Leftrightarrow \frac{r_1+r_2+r_3+r_4}{((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu))\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} - \frac{\Lambda(\mu+\sigma+\rho)}{\mu} = 0$$

$$\Leftrightarrow \frac{\mu(r_1+r_2+r_3+r_4)-\Lambda(\mu+\sigma+\delta+\rho)\big((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\big)\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)}{\big((\mu\eta\beta_1 I+\Lambda\mu)(\mu\beta_1 I+\Lambda(\mu+\omega_1))+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\big)\mu n_3[n_3(\mu\beta_3 I+\Lambda(\mu+\gamma_1))+\Lambda\alpha_1(\mu+\gamma_2)](\beta_2 I+\Lambda)} = 0$$

$$\Leftrightarrow \frac{\mu(r_1+r_2+r_3+r_4)-r_5}{\Big((\mu\eta\beta_1 I+\Lambda\mu)\big(\mu\beta_1 I+\Lambda(\mu+\omega_1)\big)+\Lambda\omega_2(\mu\beta_1 I+\Lambda\mu)\Big)\mu n_3\Big[n_3\big(\mu\beta_3 I+\Lambda(\mu+\gamma_1)\big)+\Lambda\alpha_1(\mu+\gamma_2)\Big](\beta_2 I+\Lambda)} = 0 \quad (4.p)$$

dengan,

$r_5 = \Lambda{\beta_1}^2\beta_2\beta_3\eta{n_3}^2\mu^4 s I^4 + \big({\beta_1}^2\beta_2\alpha_1\eta\gamma_2 n_3\mu^3 s + {\beta_1}^2\beta_2\alpha_1\eta n_3\mu^4 s + {\beta_1}^2\beta_2\eta\gamma_1{n_3}^2\mu^3 s + {\beta_1}^2\beta_2\eta{n_3}^2\mu^4 s +$
${\beta_1}^2\beta_3\eta{n_3}^2\mu^4 s + \beta_1\beta_2\beta_3\eta{n_3}^2\mu^4 s + \beta_1\beta_2\beta_3\eta{n_3}^2\mu^3\omega_1 s + \beta_1\beta_2\beta_3{n_3}^2\mu^4 s + \beta_1\beta_2\beta_3{n_3}^2\mu^3\omega_2 s\big)\Lambda^2 I^3 +$
$\big({\beta_1}^2\alpha_1\eta\gamma_2 n_3\mu^3 s + {\beta_1}^2\alpha_1\eta n_3\mu^4 s + \beta_1\beta_2\alpha_1\eta\gamma_2 n_3\mu^3 s + \beta_1\beta_2\alpha_1\eta\gamma_2 n_3\mu^2\omega_1 s + \beta_1\beta_2\alpha_1\eta n_3\mu^4 s +$
$\beta_1\beta_2\alpha_1\eta n_3\mu^3\omega_1 s + {\beta_1}^2\eta\gamma_1{n_3}^2\mu^3 s + {\beta_1}^2\eta{n_3}^2\mu^4 s + \beta_1\beta_2\eta\gamma_1{n_3}^2\mu^3 s + \beta_1\beta_2\gamma_1\eta{n_3}^2\mu^2\omega_1 s + \beta_1\beta_2\eta{n_3}^2\mu^4 s +$
$\beta_1\beta_2\eta{n_3}^2\mu^3\omega_1 s + \beta_1\beta_3\eta{n_3}^2\mu^4 s + \beta_1\beta_3\eta{n_3}^2\mu^3\omega_1 s + \beta_1\beta_2\alpha_1\gamma_2 n_3\mu^3 s + \beta_1\beta_2\alpha_1\gamma_2 n_3\mu^2\omega_2 s + \beta_1\beta_2\alpha_1 n_3\mu^4 s +$
$\beta_1\beta_2\alpha_1\gamma_2 n_3\mu^3\omega_2 s + \beta_1\beta_2{n_3}^2\mu^3\gamma_1 s + \beta_1\beta_2\gamma_1{n_3}^2\mu^3\omega_2 s + \beta_1\beta_2{n_3}^2\mu^4 s + \beta_1\beta_2{n_3}^2\mu^3\omega_2 s + \beta_1\beta_3{n_3}^2\mu^4 s +$
$\beta_1\beta_3{n_3}^2\mu^3\omega_2 s + \beta_2\beta_3{n_3}^2\mu^4 s + \beta_2\beta_3{n_3}^2\mu^3\omega_1 s + \beta_2\beta_3{n_3}^2\mu^3\omega_2 s\big)\Lambda^3 I^2 + (\alpha_1\beta_1\eta\gamma_2 n_3\mu^3 s +$
$\alpha_1\beta_1\eta\gamma_2 n_3\mu^2\omega_1 s + \alpha_1\beta_1\eta n_3\mu^4 s + \alpha_1\beta_1\eta n_3\mu^3\omega_1 s + \beta_1\eta\gamma_1{n_3}^2\mu^3 s + \beta_1\eta\gamma_1{n_3}^2\mu^2\omega_1 s + \beta_1\eta{n_3}^2\mu^4 s +$
$\beta_1\eta{n_3}^2\mu^3\omega_1 s + \alpha_1\beta_1\gamma_2 n_3\mu^3 s + \alpha_1\beta_1\gamma_2 n_3\mu^2\omega_2 s + \alpha_1\beta_1 n_3\mu^4 s + \alpha_1\beta_1 n_3\mu^3\omega_2 s + \alpha_1\beta_2\gamma_2 n_3\mu^3 s +$
$\alpha_1\beta_2\gamma_2 n_3\mu^2\omega_1 s + \alpha_1\beta_2\gamma_2 n_3\mu^2\omega_2 s + \alpha_1\beta_2 n_3\mu^4 s + \alpha_1\beta_2 n_3\mu^3\omega_1 s + \alpha_1\beta_2 n_3\mu^3\omega_2 s + \beta_1\gamma_1{n_3}^2\mu^3 s +$
$\beta_1\gamma_1{n_3}^2\mu^2\omega_2 s + \beta_1{n_3}^2\mu^4 s + \beta_1{n_3}^2\mu^3\omega_2 s + \beta_2\gamma_1{n_3}^2\mu^3 s + \beta_2\gamma_1{n_3}^2\mu^2\omega_1 s + \beta_2\gamma_1{n_3}^2\mu^2\omega_2 s + \beta_2{n_3}^2\mu^4 s +$
$\beta_2{n_3}^2\mu^3\omega_1 s + \beta_2{n_3}^2\mu^3\omega_2 s + \beta_3{n_3}^2\mu^4 s + \beta_3{n_3}^2\mu^3\omega_1 s + \beta_3{n_3}^2\mu^3\omega_2 s)\Lambda^4 I + \Lambda^5 s(\alpha_1\gamma_2 n_3\mu^3 + \alpha_1\gamma_2 n_3\mu^2\omega_1 +$
$\alpha_1\gamma_2 n_3\mu^2\omega_2 + \alpha_1 n_3\mu^4 + \alpha_1 n_3\mu^3\omega_1 + \alpha_1 n_3\mu^3\omega_2 + \gamma_1{n_3}^2\mu^3 + \gamma_1{n_3}^2\mu^2\omega_1 + \gamma_1{n_3}^2\mu^2\omega_2 + {n_3}^2\mu^4 + {n_3}^2\mu^3\omega_1 +$
${n_3}^2\mu^3\omega_2)$

$s = \mu + \sigma + \rho$

Berdasarkan persamaan (4.p) dapat diketahui bahwa $\Big((\mu\eta\beta_1 I + \Lambda\mu)\big(\mu\beta_1 I + \Lambda(\mu + \omega_1)\big) + \Lambda\omega_2(\mu\beta_1 I + \Lambda\mu)\Big)\mu n_3\big[n_3\big(\mu\beta_3 I + \Lambda(\mu + \gamma_1)\big) + \Lambda\alpha_1(\mu + \gamma_2)\big](\beta_2 I + \Lambda) > 0$ maka diperoleh persamaan sebagai berikut :

$$\mu(r_1 + r_2 + r_3 + r_4) - r_5 = 0 \qquad (4.q)$$

Persamaan (4.q) apabila diuraikan, dapat diperoleh persamaan polinomial berikut:

$$B_1 I^{*4} + B_2 I^{*3} + B_3 I^{*2} + B_4 I^* + B_5 = 0 \qquad (4.r)$$

dengan

$$B_1 = -\Lambda{\beta_1}^2\beta_2\beta_3\eta n_3\mu^5(q\sigma + \mu + \alpha_2 + \gamma_2)$$

$$B_2 = \Lambda^2\sigma n_1\mu^3\beta_1\beta_3 n_3(\mu(\beta_2\eta + \beta_1\eta + \beta_2) + \beta_2(\omega_1\eta + \omega_2)) + \Lambda^2{\beta_1}^2\beta_2\mu^3\eta(\sigma n_1 n_2 + \beta_3 n_3\mu) + \Lambda^2\beta_1\beta_2\mu^3\big(\gamma_2 q\sigma + n_3(\rho - s)\big)(\beta_1\eta(\alpha_1\gamma_2 + \alpha_1\mu + \gamma_1 n_3 + n_3\mu) + \beta_3 n_3(\eta\mu + \eta\omega_1 + \mu + \omega_2)) - \Lambda^2{\beta_1}^2\beta_3\eta{n_3}^2\mu^4 s$$

$$B_3 = \Lambda^3\beta_1\mu^2\big(\sigma\beta_2 n_1 n_2 + \beta_3 n_3\mu(\sigma n_1 - n_3 s)\big)(\eta(\mu + \omega_1) + \mu + \omega_2) + \Lambda^3\beta_2\mu^3\beta_3 n_3(q\sigma\gamma_2 + n_3(\rho - s) + \sigma n_1)(\mu + \omega_1 + \omega_2) + \Lambda^3\beta_2\mu^2\big(q\sigma\gamma_2 + n_3(\rho - s)\big)\big(\alpha_1\beta_1(\eta\gamma_2\mu + \eta\gamma_2\omega_1 + \eta\mu^2 + \eta\mu\omega_1 + \gamma_2\mu + \gamma_2\omega_2 + \mu^2 + \mu\omega_2) + \beta_1 n_3(\eta\gamma_1\mu + \eta\gamma_1\omega_1 + \eta\mu^2 + \eta\mu\omega_1 + \gamma_1\mu + \gamma_1\omega_2 + \mu^2 + \mu\omega_2)\big) + \Lambda^3{\beta_1}^2 n_3\mu^3\eta(\beta_2 - s)\big(\alpha_1(\gamma_2 + \mu) + n_3(\gamma_1 + \mu)\big) + {n_3}^2\beta_3\mu(\Lambda^3{\beta_1}^2\mu^3\eta + \beta_2 v)$$

$$B_4 = \mu^2\Big[\Lambda^4\big(\sigma n_1(\beta_2 n_2 + \beta_3 n_3\mu) + \beta_2\big(\gamma_2 q\sigma + n_3(\rho - s)\big)\big(\alpha_1(\gamma_2 + \mu) + n_3(\gamma_1 + \mu)\big) - \beta_3{n_3}^2\mu s\big)(\mu + \omega_1 + \omega_2) + \Lambda n_3\Big(\Big(\Lambda^3{\beta_1}^2\mu\eta + \beta_2 v - \Lambda^3\beta_1 s\big(\mu + \omega_2 + \eta(\mu + \omega_1)\big)\Big)\big(\alpha_1(\gamma_2 + \mu) + n_3(\gamma_1 + \mu)\big) + \Lambda{n_3}^2\beta_3\mu^2 v\Big)\Big]$$

$$B_5 = \Lambda^2 n_3\mu^2(\alpha_1\gamma_2 + \alpha_1\mu + \gamma_1 n_3 + n_3\mu)\big(v - \Lambda^3 s(\mu + \omega_1 + \omega_2)\big)$$

Berdasarkan sifat polinomial berderajat empat, persamaan (4.r) memiliki satu akar positif jika perkalian semua akarnya kurang dari nol. Misalkan akar-akar dari persamaan (4.r) adalah I_1, I_2, I_3, dan I_4, sedangkan perkalian akar-akar dari persamaan (4.r) adalah $\frac{B_5}{B_1}$. Jelas bahwa $B_1 < 0$ karena semua parameter bernilai positif, sehingga akan ditentukan syarat agar $B_5 > 0$ yaitu sebagai berikut :

$B_5 > 0$

$\Leftrightarrow \Lambda^2 n_3 \mu^2 (\alpha_1 \gamma_2 + \alpha_1 \mu + \gamma_1 n_3 + n_3 \mu)\big(v - \Lambda^3 s(\mu + \omega_1 + \omega_2)\big) > 0$.

Karena semua parameter bernilai positif, jelas bahwa $\Lambda^2 n_3 \mu^2 (\alpha_1 \gamma_2 + \alpha_1 \mu + \gamma_1 n_3 + n_3 \mu) > 0$ sehingga didapatkan $v - \Lambda^3 s(\mu + \omega_1 + \omega_2) > 0$ dan diuraikan sebagai berikut :

$v - \Lambda^3 s(\mu + \omega_1 + \omega_2) > 0$

$\Leftrightarrow v > \Lambda^3 s(\mu + \omega_1 + \omega_2)$

$\Leftrightarrow \Lambda^3 \beta_1 \big(\mu\big(p + \eta(1-p)\big) + \eta\omega_1 + \omega_2\big) > \Lambda^3 (\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2)$

$\Leftrightarrow \Lambda^3 \Big(\beta_1 \big(\mu\big(p + \eta(1-p)\big) + \eta\omega_1 + \omega_2\big) - (\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2)\Big) > 0$

$\Leftrightarrow \Lambda^3 (\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2)(R_0 - 1) > 0$.

Jelas bahwa $\Lambda^3 (\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2) > 0$ karena semua parameter positif, sehingga diperoleh syarat yaitu

$R_0 - 1 > 0$

$\Leftrightarrow R_0 > 1$.

Jadi syarat eksistensi untuk I^*adalah $R_0 > 1$, sehingga titik setimbang endemik eksis jika dan hanya jika $R_0 > 1$.

Lampiran 5. Analisa Kestabilan Titik Setimbang Non Endemik

Berikut model matematika penyalahgunaan narkoba dengan memperhatikan tipe perawatan beserta tingkat resiko :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\beta_1 I S_H}{N} - (\mu + \omega_1)S_H + \omega_2 S_L = f_1 \tag{5.a}$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\eta\beta_1 I S_L}{N} - (\mu + \omega_2)S_L + \omega_1 S_H = f_2 \tag{5.b}$$

$$\frac{dI}{dt} = \frac{\beta_1 I(S_H + \eta S_L)}{N} + \frac{\beta_2 IR}{N} + \frac{\beta_3 I T_j}{N} - (\mu + \sigma + \rho)I = f_3 \tag{5.c}$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\beta_3 I T_j}{N} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = f_4 \tag{5.d}$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = f_5 \tag{5.e}$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\beta_2 IR}{N} - \mu R = f_6 \tag{5.f}$$

Berikut diberikan matriks Jacobian dari model matematika penyalahgunaan narkoba dengan memperhatikan tipe perawatan beserta tingkat resiko adalah sebagai berikut:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial S_H} & \frac{\partial f_1}{\partial S_L} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial T_j} & \frac{\partial f_1}{\partial T_r} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S_H} & \frac{\partial f_2}{\partial S_L} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial T_j} & \frac{\partial f_2}{\partial T_r} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S_H} & \frac{\partial f_3}{\partial S_L} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial T_j} & \frac{\partial f_3}{\partial T_r} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S_H} & \frac{\partial f_4}{\partial S_L} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial T_j} & \frac{\partial f_4}{\partial T_r} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S_H} & \frac{\partial f_5}{\partial S_L} & \frac{\partial f_5}{\partial I} & \frac{\partial f_5}{\partial T_j} & \frac{\partial f_5}{\partial T_r} & \frac{\partial f_5}{\partial R} \\ \frac{\partial f_6}{\partial S_H} & \frac{\partial f_6}{\partial S_L} & \frac{\partial f_6}{\partial I} & \frac{\partial f_6}{\partial T_j} & \frac{\partial f_6}{\partial T_r} & \frac{\partial f_6}{\partial R} \end{bmatrix}$$

$$J=\begin{bmatrix} -\frac{\mu\beta_1 I}{\Lambda}-(\mu+\omega_1) & \omega_2 & -\frac{\mu\beta_1 S_H}{\Lambda} & 0 & 0 & 0 \\ \omega_1 & -\frac{\mu\eta\beta_1 I}{\Lambda}-(\mu+\omega_2) & -\frac{\mu\eta\beta_1 S_L}{\Lambda} & 0 & 0 & 0 \\ \frac{\mu\beta_1 I}{\Lambda} & \frac{\mu\eta\beta_1 I}{\Lambda} & d_9-d_1 & \frac{\mu\beta_3 I}{\Lambda} & 0 & \frac{\mu\beta_2 I}{\Lambda} \\ 0 & 0 & (1-q)\sigma-\frac{\mu\beta_3 T_j}{\Lambda} & -\frac{\mu\beta_3 I}{\Lambda}-d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho-\frac{\mu\beta_2 R}{\Lambda} & \gamma_1 & \gamma_2 & -\frac{\mu\beta_2 I}{\Lambda}-\mu \end{bmatrix}$$

dengan

$d_1=\mu+\sigma+\rho$

$d_2=\mu+\alpha_1+\gamma_1$

$d_3=\mu+\alpha_2+\gamma_2$

$d_9=\frac{\mu(\beta_1(S_H+\eta S_L)+\beta_2 R+\beta_3 T_j)}{\Lambda}$

Titik setimbang non endemik dari model matematika penyalahgunaan narkoba yaitu $E_0=\left(S_H{}^0,S_L{}^0,I^0,T_j{}^0,T_r{}^0,R^0\right)=\left(\frac{\Lambda(\mu p+\omega_2)}{\mu(\mu+\omega_1+\omega_2)},\frac{\Lambda(\mu(1-p)+\omega_1)}{\mu(\mu+\omega_1+\omega_2)},0,0,0,0\right)$. Berikut ini adalah matriks jacobian di sekitar titik setimbang non endemik (E_0) :

$$JE_0=\begin{bmatrix} -(\mu+\omega_1) & \omega_2 & -\frac{\beta_1(\mu p+\omega_2)}{\mu+\omega_1+\omega_2} & 0 & 0 & 0 \\ \omega_1 & -(\mu+\omega_2) & -\frac{\eta\beta_1(\mu(1-p)+\omega_2)}{\mu+\omega_1+\omega_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{\mu+\omega_1+\omega_2}-d_1 & 0 & 0 & 0 \\ 0 & 0 & (1-q)\sigma & -d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho & \gamma_1 & \gamma_2 & -\mu \end{bmatrix}$$

Misalkan :

$d_4=\mu+\omega_1$

$d_5=\mu+\omega_2$

$d_6=\mu+\omega_1+\omega_2$

Dari sini matriks JE_0 dapat dituliskan sebagai berikut :

$$JE_0 = \begin{bmatrix} -d_4 & \omega_2 & -\frac{\beta_1(\mu p+\omega_2)}{d_6} & 0 & 0 & 0 \\ \omega_1 & -d_5 & -\frac{\eta\beta_1(\mu(1-p)+\omega_2)}{d_6} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 & 0 & 0 & 0 \\ 0 & 0 & (1-q)\sigma & -d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho & \gamma_1 & \gamma_2 & -\mu \end{bmatrix}$$

Selanjutnya menentukan nilai eigen dari matriks JE_0 diatas :

$\det(JE_0 - \lambda I) = 0$

$$\det\left(\begin{bmatrix} -d_4 & \omega_2 & -\frac{\beta_1(\mu p+\omega_2)}{d_6} & 0 & 0 & 0 \\ \omega_1 & -d_5 & -\frac{\eta\beta_1(\mu(1-p)+\omega_2)}{d_6} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 & 0 & 0 & 0 \\ 0 & 0 & (1-q)\sigma & -d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho & \gamma_1 & \gamma_2 & -\mu \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{bmatrix} -d_4-\lambda & \omega_2 & -\frac{\beta_1(\mu p+\omega_2)}{d_6} & 0 & 0 & 0 \\ \omega_1 & -d_5-\lambda & -\frac{\eta\beta_1(\mu(1-p)+\omega_2)}{d_6} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 - \lambda & 0 & 0 & 0 \\ 0 & 0 & (1-q)\sigma & -d_2-\lambda & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3-\lambda & 0 \\ 0 & 0 & \rho & \gamma_1 & \gamma_2 & -\mu-\lambda \end{bmatrix} = 0$$

$$\left(\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 - \lambda\right)(-\mu-\lambda)\begin{vmatrix} -d_4-\lambda & \omega_2 & 0 & 0 \\ \omega_1 & -d_5-\lambda & 0 & 0 \\ 0 & 0 & -d_2-\lambda & \alpha_2 \\ 0 & 0 & \alpha_1 & -d_3-\lambda \end{vmatrix} = 0$$

Misalkan :

$$M = \begin{pmatrix} -d_4-\lambda & \omega_2 & 0 & 0 \\ \omega_1 & -d_5-\lambda & 0 & 0 \\ 0 & 0 & -d_2-\lambda & \alpha_2 \\ 0 & 0 & \alpha_1 & -d_3-\lambda \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -d_4-\lambda & \omega_2 \\ \omega_1 & -d_5-\lambda \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -d_2-\lambda & \alpha_2 \\ \alpha_1 & -d_3-\lambda \end{pmatrix}$$

Maka,

$$\det(M) = \det\begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$$

$$= \det(A_1)\cdot\det(A_2)$$

$$= \big((-d_4-\lambda)(-d_5-\lambda)-\omega_1\omega_2\big)\big((-d_2-\lambda)(-d_3-\lambda)-\alpha_1\alpha_2\big)$$

$$= \big((-\mu-\omega_1-\lambda)(-\mu-\omega_2-\lambda)-\omega_1\omega_2\big)\big((-\mu-\alpha_1-\gamma_1-\lambda)(-\mu-\alpha_2-\gamma_2-\lambda)-\alpha_1\alpha_2\big)$$

$$= [(-\mu-\lambda)(-\mu-\omega_2-\lambda)-\omega_1(-\mu-\lambda)+\omega_1\omega_2-\omega_1\omega_2][(-\mu-\gamma_1-\lambda)(-\mu-\alpha_2-\gamma_2-\lambda)-\alpha_1(-\mu-\gamma_2-\lambda)+\alpha_1\alpha_2-\alpha_1\alpha_2]$$

$$= [(-\mu-\lambda)(-\mu-\omega_1-\omega_2-\lambda)][(-\mu-\gamma_1-\lambda)(-\mu-\alpha_2-\gamma_2-\lambda)-\alpha_1(-\mu-\gamma_2-\lambda)]$$

$$= (-\mu-\lambda)(-\mu-\omega_1-\omega_2-\lambda)(\lambda^2+\lambda(2\mu+\alpha_1+\gamma_1+\alpha_2+\gamma_2)+\mu^2+\mu\alpha_2+\mu\gamma_2+\mu\gamma_1+\gamma_1\alpha_2+\gamma_1\gamma_2+\mu\alpha_1+\gamma_2\alpha_1)$$

Misalkan :

$$c_1 = 2\mu+\alpha_1+\gamma_1+\alpha_2+\gamma_2$$

$$c_2 = \mu^2+\mu\alpha_2+\mu\gamma_2+\mu\gamma_1+\gamma_1\alpha_2+\gamma_1\gamma_2+\mu\alpha_1+\gamma_2\alpha_1$$

maka,

$$\det(M) = (-\mu-\lambda)(-\mu-\omega_1-\omega_2-\lambda)(\lambda^2+c_1\lambda+c_2)$$

sehingga,

$\det(JE_0 - \lambda I) = 0$

$\left(\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 - \lambda\right)(-\mu-\lambda)\det(M) = 0$

$\left(\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 - \lambda\right)(-\mu-\lambda)(-\mu-\lambda)(-\mu-\omega_1-\omega_2-\lambda)(\lambda^2+c_1\lambda+c_2) = 0$

Dari persamaan diatas, diperoleh nilai eigen sebagai berikut :

(i) $\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1 - \lambda = 0$

$\lambda_1 = \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{d_6} - d_1$

$= \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{\mu+\omega_1+\omega_2} - (\mu+\sigma+\rho)$

(ii) $-\mu - \lambda = 0$

$\lambda_2 = -\mu$

(iii) $-\mu - \lambda = 0$

$\lambda_3 = -\mu$

(iv) $-\mu - \omega_1 - \omega_2 - \lambda = 0$

$\lambda_4 = -\mu - \omega_1 - \omega_2$

serta sisanya adalah persamaan karakteristik berikut :

$\lambda^2 + c_1\lambda + c_2 = 0$ (5.g)

Titik setimbang non endemik stabil asimtotis jika dan hanya jika $Re(\lambda_i) < 0$.

Pada persamaan (ii),(iii) dan (iv) jelas bahwa $\lambda_2, \lambda_3, \lambda_4 < 0$ karena semua parameter bernilai positif. Berdasarkan kriteria Routh-Hurwitz, persamaan (5.g) mempunyai akar-akar dengan bagian real negatif jika dan hanya jika $c_1 > 0$ dan $c_2 > 0$.

i. $c_1 = 2\mu + \alpha_1 + \gamma_1 + \alpha_2 + \gamma_2$

Jelas bahwa $c_1 > 0$ karena semua parameter bernilai positif

ii. $c_2 = \mu^2 + \mu\alpha_2 + \mu\gamma_2 + \mu\gamma_1 + \gamma_1\alpha_2 + \gamma_1\gamma_2 + \mu\alpha_1 + \gamma_2\alpha_1$

Jelas bahwa $c_2 > 0$ karena semua parameter bernilai positif.

Berdasarkan i dan ii diatas maka dapat disimpulkan bahwa persamaan karakteristik (5.g) memiliki akar-akar dengan bagian real negatif.

Sebelumnya diperoleh $\lambda_1 = \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{\mu+\omega_1+\omega_2} - (\mu+\sigma+\delta+\rho)$. Agar titik setimbang non endemik stabil asimtotis maka :

$\lambda_1 < 0$

$\Leftrightarrow \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{\mu+\omega_1+\omega_2} - (\mu+\sigma+\rho) < 0$

$\Leftrightarrow (\mu+\sigma+\rho)\left(\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_2))}{(\mu+\omega_1+\omega_2)(\mu+\sigma+\rho)} - 1\right) < 0$

$\Leftrightarrow (\mu+\sigma+\rho)(R_0 - 1) < 0$

Jelas bahwa $\mu+\sigma+\rho > 0$ sehingga diperoleh syarat sebagai berikut :

$R_0 - 1 < 0$

$\Leftrightarrow R_0 < 1$ (5.h)

Berdasarkan (5.h) diperoleh syarat kestabilan yaitu $R_0 < 1$. Jadi, titik setimbang non endemik stabil asimtotis jika dan hanya jika $R_0 < 1$.

Lampiran 6. Perhitungan kestabilan titik setimbang endemik

Berikut model matematika penyalahgunaan narkoba dengan memperhatikan tipe perawatan beserta tingkat resiko :

$$\frac{dS_H}{dt} = p\Lambda - \frac{\mu\beta_1 IS_H}{\Lambda} - (\mu + \omega_1)S_H + \omega_2 S_L = f_1 \quad (6.a)$$

$$\frac{dS_L}{dt} = (1-p)\Lambda - \frac{\mu\eta\beta_1 IS_L}{\Lambda} - (\mu + \omega_2)S_L + \omega_1 S_H = f_2 \quad (6.b)$$

$$\frac{dI}{dt} = \frac{\mu\beta_1 I(S_H + \eta S_L)}{\Lambda} + \frac{\mu\beta_2 IR}{\Lambda} + \frac{\mu\beta_3 IT_j}{\Lambda} - (\mu + \sigma + \rho)I = f_3 \quad (6.c)$$

$$\frac{dT_j}{dt} = (1-q)\sigma I - \frac{\mu\beta_3 IT_j}{\Lambda} - (\mu + \alpha_1 + \gamma_1)T_j + \alpha_2 T_r = f_4 \quad (6.d)$$

$$\frac{dT_r}{dt} = q\sigma I - (\mu + \alpha_2 + \gamma_2)T_r + \alpha_1 T_j = f_5 \quad (6.e)$$

$$\frac{dR}{dt} = \gamma_1 T_j + \gamma_2 T_r + \rho I - \frac{\mu\beta_2 IR}{\Lambda} - \mu R = f_6 \quad (6.f)$$

Berikutdiberikanmatriks Jacobian daripersamaan (6.a) – (6.f) sebagaiberikut :

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial S_H} & \frac{\partial f_1}{\partial S_L} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial T_j} & \frac{\partial f_1}{\partial T_r} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S_H} & \frac{\partial f_2}{\partial S_L} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial T_j} & \frac{\partial f_2}{\partial T_r} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S_H} & \frac{\partial f_3}{\partial S_L} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial T_j} & \frac{\partial f_3}{\partial T_r} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S_H} & \frac{\partial f_4}{\partial S_L} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial T_j} & \frac{\partial f_4}{\partial T_r} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S_H} & \frac{\partial f_5}{\partial S_L} & \frac{\partial f_5}{\partial I} & \frac{\partial f_5}{\partial T_j} & \frac{\partial f_5}{\partial T_r} & \frac{\partial f_5}{\partial R} \\ \frac{\partial f_6}{\partial S_H} & \frac{\partial f_6}{\partial S_L} & \frac{\partial f_6}{\partial I} & \frac{\partial f_6}{\partial T_j} & \frac{\partial f_6}{\partial T_r} & \frac{\partial f_6}{\partial R} \end{bmatrix}$$

$$J = \begin{bmatrix} -\frac{\mu\beta_1 I}{\Lambda} - d_4 & \omega_2 & -\frac{\mu\beta_1 S_H}{\Lambda} & 0 & 0 & 0 \\ \omega_1 & -\frac{\mu\eta\beta_1 I}{\Lambda} - d_5 & -\frac{\mu\eta\beta_1 S_L}{\Lambda} & 0 & 0 & 0 \\ \frac{\mu\beta_1 I}{\Lambda} & \frac{\mu\eta\beta_1 I}{\Lambda} & d_9 - d_1 & \frac{\mu\beta_3 I}{\Lambda} & 0 & \frac{\mu\beta_2 I}{\Lambda} \\ 0 & 0 & (1-q)\sigma - \frac{\mu\beta_3 T_j}{\Lambda} & -\frac{\mu\beta_3 I}{\Lambda} - d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho - \frac{\mu\beta_2 R}{\Lambda} & \gamma_1 & \gamma_2 & -\frac{\mu\beta_2 I}{\Lambda} - \mu \end{bmatrix}$$

dengan

$$d_1 = \mu + \sigma + \rho$$

$$d_2 = \mu + \alpha_1 + \gamma_1$$

$$d_3 = \mu + \alpha_2 + \gamma_2$$

$$d_4 = \mu + \omega_1$$

$$d_5 = \mu + \omega_2$$

$$d_9 = \frac{\mu(\beta_1 I(S_H + \eta S_L) + \beta_2 IR + \beta_3 IT_j)}{\Lambda}$$

Diberikan titik setimbang endemik (E_1) sebagai berikut :

$$S_H{}^* = \frac{\Lambda^2(\mu p(\eta\beta_1 I^* + \Lambda) + \Lambda\omega_2)}{\mu\big((\eta\beta_1 I^* + \Lambda)(\mu\beta_1 I^* + \Lambda(\mu + \omega_1)) + \Lambda\omega_2(\beta_1 I^* + \Lambda)\big)}$$

$$S_L{}^* = \frac{\Lambda^2(\mu(1-p)(\beta_1 I^* + \Lambda) + \omega_1\Lambda)}{\mu\big((\eta\beta_1 I^* + \Lambda)(\mu\beta_1 I^* + \Lambda(\mu + \omega_1)) + \Lambda\omega_2(\beta_1 I^* + \Lambda)\big)}$$

$$T_j{}^* = \frac{\Lambda\sigma I^*\big((\mu + \gamma_2)(1-q) + \alpha_2\big)}{(\mu\beta_3 I^* + \Lambda(\mu + \gamma_1))(\mu + \alpha_2 + \gamma_2) + \Lambda\alpha_1(\mu + \gamma_2)}$$

$$T_r{}^* = \frac{\sigma I^*[q(\mu\beta_3 I^* + \Lambda(\mu + \gamma_1)) + \Lambda\alpha_1]}{(\mu\beta_3 I^* + \Lambda(\mu + \gamma_1))(\mu + \alpha_2 + \gamma_2) + \Lambda\alpha_1(\mu + \gamma_2)}$$

$$R^* = \frac{\Lambda\big(\gamma_1 T_j{}^*(\mu + \alpha_2 + \gamma_2) + \gamma_2(q\sigma I^* + \alpha_1 T_j{}^*) + \rho I^*(\mu + \alpha_2 + \gamma_2)\big)}{\mu(\mu + \alpha_2 + \gamma_2)(\beta_2 I^* + \Lambda)}.$$

Berikut ini adalah matriks jacobian di sekitar titik setimbang endemik (E_1) :

$$J_{E_1} = \begin{bmatrix} -\frac{\mu\beta_1 I^*}{\Lambda} - d_4 & \omega_2 & -\frac{\beta_1\Lambda(\mu p(\eta\beta_1 I^* + \Lambda) + \Lambda\omega_2)}{d_7} & 0 & 0 & 0 \\ \omega_1 & -\frac{\mu\eta\beta_1 I^*}{\Lambda} - d_5 & -\frac{\eta\beta_1\Lambda(\mu(1-p)(\beta_1 I^* + \Lambda) + \omega_1\Lambda)}{d_7} & 0 & 0 & 0 \\ \frac{\mu\beta_1 I^*}{\Lambda} & \frac{\mu\eta\beta_1 I^*}{\Lambda} & \frac{\mu(r_1 + r_2 + r_3 + r_4)}{\Lambda r_6} - d_1 & \frac{\mu\beta_3 I^*}{\Lambda} & 0 & \frac{\mu\beta_2 I^*}{\Lambda} \\ 0 & 0 & (1-q)\sigma - \frac{\mu\beta_3\sigma I^*((\mu + \gamma_2)(1-q) + \alpha_2)}{d_8} & -\frac{\mu\beta_3 I^*}{\Lambda} - d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho - \frac{\beta_2(\gamma_1 T_j{}^* d_3 + \gamma_2(q\sigma I^* + \alpha_1 T_j{}^*) + \rho I^* d_3)}{d_3(\beta_2 I^* + \Lambda)} & \gamma_1 & \gamma_2 & -\frac{\mu\beta_2 I^*}{\Lambda} - \mu \end{bmatrix}$$

dengan :

$$d_7 = (\eta\beta_1 I^* + \Lambda)\big(\mu\beta_1 I^* + \Lambda(\mu + \omega_1)\big) + \Lambda\omega_2(\beta_1 I^* + \Lambda)$$

$$d_8 = (\mu\beta_3 I^* + \Lambda(\mu + \gamma_1))(\mu + \alpha_2 + \gamma_2) + \Lambda\alpha_1(\mu + \gamma_2).$$

Selanjutnya menentukan nilai eigen dari matriks J_{E_1} diatas :

$$\det\big(J_{E_1} - \lambda I\big) = 0$$

$$\Leftrightarrow \det\left(\begin{bmatrix} -\frac{\mu\beta_1 I^*}{\Lambda} - d_4 & \omega_2 & -\frac{\beta_1\Lambda(\mu p(\eta\beta_1 I^* + \Lambda) + \Lambda\omega_2)}{d_7} & 0 & 0 & 0 \\ \omega_1 & -\frac{\mu\eta\beta_1 I^*}{\Lambda} - d_5 & -\frac{\eta\beta_1\Lambda(\mu(1-p)(\beta_1 I^* + \Lambda) + \omega_1\Lambda)}{d_7} & 0 & 0 & 0 \\ \frac{\mu\beta_1 I^*}{\Lambda} & \frac{\mu\eta\beta_1 I^*}{\Lambda} & \frac{\mu(r_1 + r_2 + r_3 + r_4)}{\Lambda r_6} - d_1 & \frac{\mu\beta_3 I^*}{\Lambda} & 0 & \frac{\mu\beta_2 I^*}{\Lambda} \\ 0 & 0 & (1-q)\sigma - \frac{\mu\beta_3\sigma I^*\left((\mu+\gamma_2)(1-q) + \alpha_2\right)}{d_8} & -\frac{\mu\beta_3 I^*}{\Lambda} - d_2 & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 & 0 \\ 0 & 0 & \rho - \frac{\beta_2\left(\gamma_1 T_j^* d_3 + \gamma_2\left(q\sigma I^* + \alpha_1 T_j^*\right) + \rho I^* d_3\right)}{d_3(\beta_2 I^* + \Lambda)} & \gamma_1 & \gamma_2 & -\frac{\mu\beta_2 I^*}{\Lambda} - \mu \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow \begin{vmatrix} -\frac{\mu\beta_1 I^*}{\Lambda} - d_4 - \lambda & \omega_2 & -\frac{\beta_1\Lambda(\mu p(\eta\beta_1 I^* + \Lambda) + \Lambda\omega_2)}{d_7} & 0 & 0 & 0 \\ \omega_1 & -\frac{\mu\eta\beta_1 I^*}{\Lambda} - d_5 - \lambda & -\frac{\eta\beta_1\Lambda(\mu(1-p)(\beta_1 I^* + \Lambda) + \omega_1\Lambda)}{d_7} & 0 & 0 & 0 \\ \frac{\mu\beta_1 I^*}{\Lambda} & \frac{\mu\eta\beta_1 I^*}{\Lambda} & \frac{\mu(r_1 + r_2 + r_3 + r_4)}{\Lambda r_6} - d_1 - \lambda & \frac{\mu\beta_3 I^*}{\Lambda} & 0 & \frac{\mu\beta_2 I^*}{\Lambda} \\ 0 & 0 & (1-q)\sigma - \frac{\mu\beta_3\sigma I^*\left((\mu+\gamma_2)(1-q) + \alpha_2\right)}{d_8} & -\frac{\mu\beta_3 I^*}{\Lambda} - d_2 - \lambda & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 - \lambda & 0 \\ 0 & 0 & \rho - \frac{\beta_2\left(\gamma_1 T_j^* d_3 + \gamma_2\left(q\sigma I^* + \alpha_1 T_j^*\right) + \rho I^* d_3\right)}{d_3(\beta_2 I^* + \Lambda)} & \gamma_1 & \gamma_2 & -\frac{\mu\beta_2 I^*}{\Lambda} - \mu - \lambda \end{vmatrix}$$

$$= 0.$$

Misalkan

$$y_1 = \frac{\mu\beta_1 I^*}{\Lambda} + d_4$$

$$y_2 = \frac{\mu\beta_1 I^*}{\Lambda}$$

$y_3 = \frac{\mu\eta\beta_1 I^*}{\Lambda} + d_5$

$y_4 = \frac{\mu\eta\beta_1 I^*}{\Lambda}$

$y_5 = \frac{\beta_1\Lambda(\mu p(\eta\beta_1 I^* + \Lambda) + \Lambda\omega_2)}{d_7}$

$y_6 = \frac{\eta\beta_1\Lambda(\mu(1-p)(\beta_1 I^* + \Lambda) + \omega_1\Lambda)}{d_7}$

$y_7 = \frac{\mu(r_1 + r_2 + r_3 + r_4)}{\Lambda r_6}$

$y_8 = \frac{\mu\beta_3\sigma I^*\left((\mu + \gamma_2)(1-q) + \alpha_2\right)}{d_8}$

$y_9 = \frac{\beta_2\left(\gamma_1 {T_j}^* d_3 + \gamma_2\left(q\sigma I^* + \alpha_1 {T_j}^*\right) + \rho I^* d_3\right)}{d_3(\beta_2 I^* + \Lambda)}$

$y_{10} = \frac{\mu\beta_3 I^*}{\Lambda}$

$y_{11} = \frac{\mu\beta_3 I^*}{\Lambda} + d_2$

$y_{12} = \frac{\mu\beta_2 I^*}{\Lambda}$

$y_{13} = \frac{\mu\beta_2 I^*}{\Lambda} + \mu$

Dengan demikian, $\det\left(J_{E_1} - \lambda I\right) = 0$ dapat dituliskan sebagai berikut :

$\det\left(J_{E_1} - \lambda I\right) = 0$

$$\Leftrightarrow \begin{vmatrix} -y_1 - \lambda & \omega_2 & -y_5 & 0 & 0 & 0 \\ \omega_1 & -y_3 - \lambda & -y_6 & 0 & 0 & 0 \\ y_2 & y_4 & y_7 - d_1 - \lambda & y_{10} & 0 & y_{12} \\ 0 & 0 & (1-q)\sigma - y_8 & -y_{11} - \lambda & \alpha_2 & 0 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3 - \lambda & 0 \\ 0 & 0 & \rho - y_9 & \gamma_1 & \gamma_2 & -y_{13} - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (-y_{13}-\lambda)\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 & 0 & 0 \\ \omega_1 & -y_3-\lambda & -y_6 & 0 & 0 \\ y_2 & y_4 & y_7-d_1-\lambda & y_{10} & 0 \\ 0 & 0 & (1-q)\sigma-y_8 & -y_{11}-\lambda & \alpha_2 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3-\lambda \end{vmatrix} -$$

$$y_{12}\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 & 0 & 0 \\ \omega_1 & -y_3-\lambda & -y_6 & 0 & 0 \\ 0 & 0 & (1-q)\sigma-y_8 & -y_{11}-\lambda & \alpha_2 \\ 0 & 0 & q\sigma & \alpha_1 & -d_3-\lambda \\ 0 & 0 & \rho-y_9 & \gamma_1 & \gamma_2 \end{vmatrix} = 0$$

$$\Leftrightarrow (-y_{13}-\lambda)\left((-d_3-\lambda)\left((-y_{11}-\lambda)\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 \\ \omega_1 & -y_3-\lambda & -y_6 \\ y_2 & y_4 & y_7-d_1-\lambda \end{vmatrix} -\right.\right.$$

$$\left. y_{10}\big((1-q)\sigma-y_8\big)\big((-y_1-\lambda)(-y_3-\lambda)-\omega_1\omega_2\big)\right) -$$

$$\alpha_2\left(\alpha_1\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 \\ \omega_1 & -y_3-\lambda & -y_6 \\ y_2 & y_4 & y_7-d_1-\lambda \end{vmatrix} - y_{10}q\sigma\big((-y_1-\lambda)(-y_3-\lambda)-\right.$$

$$\left.\left.\omega_1\omega_2\big)\right)\right) - y_{12}\big((-y_1-\lambda)(-y_3-\lambda)-$$

$$\omega_1\omega_2\big)\begin{vmatrix} (1-q)\sigma-y_8 & -y_{11}-\lambda & \alpha_2 \\ q\sigma & \alpha_1 & -d_3-\lambda \\ \rho-y_9 & \gamma_1 & \gamma_2 \end{vmatrix} = 0$$

Untuk $\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 \\ \omega_1 & -y_3-\lambda & -y_6 \\ y_2 & y_4 & y_7-d_1-\lambda \end{vmatrix}$ diperoleh hasil sebagai berikut :

$$\begin{vmatrix} -y_1-\lambda & \omega_2 & -y_5 \\ \omega_1 & -y_3-\lambda & -y_6 \\ y_2 & y_4 & y_7-d_1-\lambda \end{vmatrix} = -\lambda^3-\lambda^2(d_1+y_1+y_3-y_7)-\lambda(d_1y_1+$$

$$d_1y_3+y_1y_3+y_4y_6+\omega_1\omega_2+y_2y_5-y_1y_7-$$

$$y_3y_7)-(d_1y_1y_3-y_1y_3y_7+y_1y_4y_6+$$

$$\omega_1\omega_2y_7-\omega_1\omega_2d_1+\omega_1y_4y_5+\omega_2y_2y_6+$$

$$y_2y_3y_5)$$

Sedangkan untuk $\begin{vmatrix} (1-q)\sigma - y_8 & -y_{11}-\lambda & \alpha_2 \\ q\sigma & \alpha_1 & -d_3-\lambda \\ \rho - y_9 & \gamma_1 & \gamma_2 \end{vmatrix}$ diperoleh hasil sebagai berikut :

$$\begin{vmatrix} (1-q)\sigma - y_8 & -y_{11}-\lambda & \alpha_2 \\ q\sigma & \alpha_1 & -d_3-\lambda \\ \rho - y_9 & \gamma_1 & \gamma_2 \end{vmatrix} = \lambda^2(\rho - y_9) + \lambda(\rho y_{11} + \rho d_3 + (1-q)\sigma\gamma_1 + q\sigma\gamma_2 - y_9 y_{11} - y_9 d_3 - y_8\gamma_1) + (1-q)\sigma\alpha_1\gamma_2 + (1-q)\sigma\gamma_1 d_3 - y_8\alpha_1\gamma_2 - y_8\gamma_1 d_3 + q\sigma\gamma_2 y_{11} + q\sigma\alpha_2\gamma_1 + \rho y_{11} d_3 - \rho\alpha_1\alpha_2 - y_9 y_{11} d_3 + y_9\alpha_1\alpha_2$$

Sehingga $\det\left(J_{E_1} - \lambda I\right) = 0$ dapat dituliskan sebagai berikut :

$$\det\left(J_{E_1} - \lambda I\right) = 0$$

$$\Leftrightarrow (-y_{13} - \lambda)\Big((-d_3 - \lambda)\Big((-y_{11} - \lambda)[-\lambda^3 - \lambda^2(d_1 + y_1 + y_3 - y_7) - \lambda(d_1y_1 + d_1y_3 + y_1y_3 + y_4y_6 + \omega_1\omega_2 + y_2y_5 - y_1y_7 - y_3y_7) - (d_1y_1y_3 - y_1y_3y_7 + y_1y_4y_6 + \omega_1\omega_2y_7 - \omega_1\omega_2d_1 + \omega_1y_4y_5 + \omega_2y_2y_6 + y_2y_3y_5)] - y_{10}\big((1-q)\sigma - y_8\big)\big((-y_1 - \lambda)(-y_3 - \lambda) - \omega_1\omega_2\big)\Big) - \alpha_2\Big(\alpha_1[-\lambda^3 - \lambda^2(d_1 + y_1 + y_3 - y_7) - \lambda(d_1y_1 + d_1y_3 + y_1y_3 + y_4y_6 + \omega_1\omega_2 + y_2y_5 - y_1y_7 - y_3y_7) - (d_1y_1y_3 - y_1y_3y_7 + y_1y_4y_6 + \omega_1\omega_2y_7 - \omega_1\omega_2d_1 + \omega_1y_4y_5 + \omega_2y_2y_6 + y_2y_3y_5)] - y_{10}q\sigma\big((-y_1 - \lambda)(-y_3 - \lambda) - \omega_1\omega_2\big)\Big)\Big) - y_{12}\big((-y_1 - \lambda)(-y_3 - \lambda) - \omega_1\omega_2\big)(\lambda^2(\rho - y_9) + \lambda(\rho y_{11} + \rho d_3 + (1-q)\sigma\gamma_1 + q\sigma\gamma_2 - y_9y_{11} - y_9d_3 - y_8\gamma_1) + (1-q)\sigma\alpha_1\gamma_2 + (1-q)\sigma\gamma_1d_3 - y_8\alpha_1\gamma_2 - y_8\gamma_1d_3 + q\sigma\gamma_2y_{11} + q\sigma\alpha_2\gamma_1 + \rho y_{11}d_3 - \rho\alpha_1\alpha_2 - y_9y_{11}d_3 + y_9\alpha_1\alpha_2) = 0$$

$$\Leftrightarrow \lambda^6 + G_1\lambda^5 + G_2\lambda^4 + G_3\lambda^3 + G_4\lambda^2 + G_5\lambda + G_6 = 0$$

Dengan uraian sebagai berikut :

$$G_1 = d_1 + d_3 + y_{13} + y_1 + y_3 + y_{11} - y_7$$

$G_2 = d_1y_{13} + d_3y_{13} + y_{13}y_1 + y_{13}y_3 + y_{13}y_{11} - y_{13}y_7d_1y_1 + d_1y_3 + y_1y_3 + y_4y_6 + \omega_1\omega_2 + y_2y_5 + d_1y_{11} + y_1y_{11} + y_3y_{11} - y_7y_{11} - y_1y_7 - y_3y_7 - (1-q)\sigma y_{10} + y_8y_{10} + d_1d_3 + d_3y_1 + d_3y_3 + d_3y_{11} - d_3y_7 - \alpha_1\alpha_2 - y_{12}\rho + y_9y_{12}$

$G_3 = d_1y_{13}y_1 + d_1y_{13}y_3 + y_{13}y_1y_3 + y_{13}y_4y_6 + y_{13}\omega_1\omega_2 + y_{13}y_2y_5 + d_1y_{13}y_{11} + y_{13}y_1y_{11} + y_{13}y_3y_{11} - y_{13}y_7y_{11} - y_{13}y_1y_7 - y_{13}y_3y_7 - (1-q)\sigma y_{13}y_{10} + y_{13}y_8y_{10} + d_1d_3y_{13} + d_3y_{13}y_1 + d_3y_{13}y_3 + d_3y_{13}y_{11} - d_3y_{13}y_7 + d_1y_1y_3 - y_1y_3y_7 + y_1y_4y_6 + \omega_1\omega_2y_7 - \omega_1\omega_2d_1 + \omega_1y_4y_5 + \omega_2y_2y_6 + y_2y_3y_5 + d_1y_1y_{11} + d_1y_3y_{11} + y_1y_3y_{11} + y_4y_6y_{11} + \omega_1\omega_2y_{11} + y_2y_5y_{11} - y_1y_7y_{11} - y_3y_7y_{11} - (1-q)\sigma y_1y_{10} - (1-q)\sigma y_3y_{10} + y_1y_8y_{10} + y_3y_8y_{10} + d_1d_3y_1 + d_1d_3y_3 + d_3y_1y_3 + d_3y_4y_6 + d_3\omega_1\omega_2 + d_3y_2y_5 + d_3d_1y_{11} + d_3y_1y_{11} + d_3y_3y_{11} - d_3y_7y_{11} - d_3y_1y_7 - d_3y_3y_7 - (1-q)\sigma d_3y_{10} + d_3y_8y_{10} - [\alpha_1\alpha_2d_1 + \alpha_1\alpha_2y_1 + \alpha_1\alpha_2y_3 + \alpha_2y_{10}q\sigma + \alpha_1\alpha_2y_{13} - \alpha_1\alpha_2y_7 + \rho y_{11}y_{12} + \rho d_3y_{12} + (1-q)\sigma\gamma_1y_{12} + q\sigma\gamma_2y_{12} - y_9y_{11}y_{12} - y_9d_3y_{12} - y_8\gamma_1y_{12} + y_1y_{12}\rho + y_3y_{12}\rho - y_1y_9y_{12} - y_3y_9y_{12}]$

$G_4 = d_1y_{13}y_1y_3 - y_{13}y_1y_3y_7 + y_{13}y_1y_4y_6 + y_{13}\omega_1\omega_2y_7 - \omega_1\omega_2d_1y_{13} + y_{13}\omega_1y_4y_5 + y_{13}\omega_2y_2y_6 + y_{13}y_2y_3y_5 + d_1y_{13}y_1y_{11} + d_1y_{13}y_3y_{11} + y_{13}y_1y_3y_{11} + y_{13}y_4y_6y_{11} + y_{13}\omega_1\omega_2y_{11} + y_{13}y_2y_5y_{11} - y_{13}y_1y_7y_{11} - y_{13}y_3y_7y_{11} - (1-q)\sigma y_{13}y_1y_{10} - (1-q)\sigma y_{13}y_3y_{10} + y_{13}y_1y_8y_{10} + y_{13}y_3y_8y_{10} + d_1d_3y_{13}y_1 + d_1d_3y_{13}y_3 + d_3y_{13}y_1y_3 + d_3y_{13}y_4y_6 + d_3y_{13}\omega_1\omega_2 + d_3y_{13}y_2y_5 + d_3d_1y_{13}y_{11} + d_3y_{13}y_1y_{11} + d_3y_{13}y_3y_{11} - d_3y_{13}y_7y_{11} - d_3y_{13}y_1y_7 - d_3y_{13}y_3y_7 - (1-q)\sigma d_3y_{13}y_{10} + d_3y_{13}y_8y_{10} + d_1y_1y_3y_{11} - y_1y_3y_7y_{11} + y_1y_4y_6y_{11} + \omega_1\omega_2y_7y_{11} - \omega_1\omega_2d_1y_{11} + \omega_1y_4y_5y_{11} + \omega_2y_2y_6y_{11} + y_2y_3y_5y_{11} - (1-q)\sigma y_1y_3y_{10} - \omega_1\omega_2y_8y_{10} + (1-q)\sigma\omega_1\omega_2y_{10} + y_1y_3y_8y_{10} + d_1d_3y_1y_3 - d_3y_1y_3y_7 + d_3y_1y_4y_6 + d_3\omega_1\omega_2y_7 - \omega_1\omega_2d_1d_3 + d_3\omega_1y_4y_5 + d_3\omega_2y_2y_6 + d_3y_2y_3y_5 + d_1d_3y_1y_{11} + d_1d_3y_3y_{11} + d_3y_1y_3y_{11} + d_3y_4y_6y_{11} + d_3\omega_1\omega_2y_{11} + d_3y_2y_5y_{11} - d_3y_1y_7y_{11} - d_3y_3y_7y_{11} - (1-q)\sigma d_3y_1y_{10} - (1-q)\sigma d_3y_3y_{10} + d_3y_1y_8y_{10} + d_3y_3y_8y_{10} - [\alpha_1\alpha_2d_1y_1 + \alpha_1\alpha_2d_1y_3 + \alpha_1\alpha_2y_1y_3 + \alpha_1\alpha_2y_4y_6 + \alpha_1\alpha_2\omega_1\omega_2 + \alpha_1\alpha_2y_2y_5 + \alpha_2y_1y_{10}q\sigma + \alpha_2y_3y_{10}q\sigma + \alpha_1\alpha_2y_{13}d_1 + \alpha_1\alpha_2y_{13}y_1 + \alpha_1\alpha_2y_{13}y_3 + \alpha_2y_{13}y_{10}q\sigma - \alpha_1\alpha_2y_{13}y_7 - \alpha_1\alpha_2y_1y_7 - \alpha_1\alpha_2y_3y_7 + (1-q)\sigma\alpha_1\gamma_2y_{12} + (1-q)\sigma\gamma_1d_3y_{12} - y_8y_{12}\alpha_1\gamma_2 - y_8y_{12}\gamma_1d_3 + q\sigma\gamma_2y_{11}y_{12} + q\sigma\alpha_2\gamma_1y_{12} + \rho y_{11}y_{12}d_3 - \rho\alpha_1\alpha_2y_{12} - y_9y_{11}y_{12}d_3 + y_9y_{12}\alpha_1\alpha_2 + (y_1y_{12} + y_3y_{12})(\rho y_{11} +$

$\rho d_3 + (1-q)\sigma\gamma_1 + q\sigma\gamma_2 - y_9 y_{11} - y_9 d_3 - y_8\gamma_1) + y_1 y_3 y_{12}\rho - \omega_1\omega_2 y_{12}\rho - y_1 y_3 y_{12} y_9 + \omega_1\omega_2 y_9 y_{12}]$

$$G_5 = d_1 y_{13} y_1 y_3 y_{11} - y_{13} y_1 y_3 y_7 y_{11} + y_{13} y_1 y_4 y_6 y_{11} + y_{13}\omega_1\omega_2 y_7 y_{11} - \omega_1\omega_2 d_1 y_{13} y_{11} + y_{13}\omega_1 y_4 y_5 y_{11} + y_{13}\omega_2 y_2 y_6 y_{11} + y_{13} y_2 y_3 y_5 y_{11} - (1-q)\sigma y_{13} y_1 y_3 y_{10} - y_{13}\omega_1\omega_2 y_8 y_{10} + (1-q)\sigma y_{13}\omega_1\omega_2 y_{10} + y_{13} y_1 y_3 y_8 y_{10} + d_1 d_3 y_{13} y_1 y_3 - d_3 y_{13} y_1 y_3 y_7 + d_3 y_{13} y_1 y_4 y_6 + d_3 y_{13}\omega_1\omega_2 y_7 - \omega_1\omega_2 d_1 d_3 y_{13} + d_3 y_{13}\omega_1 y_4 y_5 + d_3 y_{13}\omega_2 y_2 y_6 + d_3 y_{13} y_2 y_3 y_5 + d_1 d_3 y_{13} y_1 y_{11} + d_1 d_3 y_{13} y_3 y_{11} + d_3 y_{13} y_1 y_3 y_{11} + d_3 y_{13} y_4 y_6 y_{11} + d_3 y_{13}\omega_1\omega_2 y_{11} + d_3 y_{13} y_2 y_5 y_{11} - d_3 y_{13} y_1 y_7 y_{11} - d_3 y_{13} y_3 y_7 y_{11} - (1-q)\sigma d_3 y_{13} y_1 y_{10} - (1-q)\sigma d_3 y_{13} y_3 y_{10} + d_3 y_{13} y_1 y_8 y_{10} + d_3 y_{13} y_3 y_8 y_{10} + d_1 d_3 y_1 y_3 y_{11} - d_3 y_1 y_3 y_7 y_{11} + d_3 y_1 y_4 y_6 y_{11} + d_3\omega_1\omega_2 y_7 y_{11} - \omega_1\omega_2 d_1 d_3 y_{11} + d_3\omega_1 y_4 y_5 y_{11} + d_3\omega_2 y_2 y_6 y_{11} + d_3 y_2 y_3 y_5 y_{11} - (1-q)\sigma d_3 y_1 y_3 y_{10} - d_3\omega_1\omega_2 y_8 y_{10} + (1-q)\sigma d_3\omega_1\omega_2 y_{10} + d_3 y_1 y_3 y_8 y_{10} - \big[\alpha_1\alpha_2 d_1 y_1 y_3 - \alpha_1\alpha_2 y_1 y_3 y_7 + \alpha_1\alpha_2 y_1 y_4 y_6 + \alpha_1\alpha_2\omega_1\omega_2 y_7 - \alpha_1\alpha_2\omega_1\omega_2 d_1 + \alpha_1\alpha_2\omega_1 y_4 y_5 + \alpha_1\alpha_2\omega_2 y_2 y_6 + \alpha_1\alpha_2 y_2 y_3 y_5 + y_{10}\alpha_2 q\sigma y_1 y_3 + \alpha_1\alpha_2 y_{13} d_1 y_1 + \alpha_1\alpha_2 y_{13} d_1 y_3 + \alpha_1\alpha_2 y_{13} y_1 y_3 + \alpha_1\alpha_2 y_{13} y_4 y_6 + \alpha_1\alpha_2 y_{13}\omega_1\omega_2 + \alpha_1\alpha_2 y_{13} y_2 y_5 + \alpha_2 y_{13} y_1 y_{10} q\sigma + \alpha_2 y_{13} y_3 y_{10} q\sigma - \alpha_2 y_{13}\alpha_1 y_1 y_7 - \alpha_2 y_{13}\alpha_1 y_3 y_7 - \alpha_2 y_{10} q\sigma\omega_1\omega_2 + \big((1-q)\sigma\alpha_1\gamma_2 + (1-q)\sigma\gamma_1 d_3 - y_8\alpha_1\gamma_2 - y_8\gamma_1 d_3 + q\sigma\gamma_2 y_{11} + q\sigma\alpha_2\gamma_1 + \rho y_{11} d_3 - \rho\alpha_1\alpha_2 - y_9 y_{11} d_3 + y_9\alpha_1\alpha_2\big)(y_1 y_{12} + y_3 y_{12}) + (y_1 y_3 y_{12} - \omega_1\omega_2 y_{12})(\rho y_{11} + \rho d_3 + (1-q)\sigma\gamma_1 + q\sigma\gamma_2 - y_9 y_{11} - y_9 d_3 - y_8\gamma_1)\big]$$

$$G_6 = d_1 d_3 y_{13} y_1 y_3 y_{11} - d_3 y_{13} y_1 y_3 y_7 y_{11} + d_3 y_{13} y_1 y_4 y_6 y_{11} + d_3 y_{13}\omega_1\omega_2 y_7 y_{11} - \omega_1\omega_2 d_1 d_3 y_{13} y_{11} + d_3 y_{13}\omega_1 y_4 y_5 y_{11} + d_3 y_{13}\omega_2 y_2 y_6 y_{11} + d_3 y_{13} y_2 y_3 y_5 y_{11} - (1-q)\sigma d_3 y_{13} y_1 y_3 y_{10} - d_3 y_{13}\omega_1\omega_2 y_8 y_{10} + (1-q)\sigma d_3 y_{13}\omega_1\omega_2 y_{10} + d_3 y_{13} y_1 y_3 y_8 y_{10} - \big[\alpha_1\alpha_2 y_{13} d_1 y_1 y_3 - \alpha_1\alpha_2 y_{13} y_1 y_3 y_7 + \alpha_1\alpha_2 y_{13} y_1 y_4 y_6 + \alpha_1\alpha_2 y_{13}\omega_1\omega_2 y_7 - \alpha_1\alpha_2 y_{13}\omega_1\omega_2 d_1 + \alpha_1\alpha_2 y_{13}\omega_1 y_4 y_5 + \alpha_1\alpha_2 y_{13}\omega_2 y_2 y_6 + \alpha_1\alpha_2 y_{13} y_2 y_3 y_5 + \alpha_2 y_{13} y_{10} q\sigma y_1 y_3 - \alpha_2 y_{13} y_{10} q\sigma\omega_1\omega_2 + \big((1-q)\sigma\alpha_1\gamma_2 + (1-q)\sigma\gamma_1 d_3 - y_8\alpha_1\gamma_2 - y_8\gamma_1 d_3 + q\sigma\gamma_2 y_{11} + q\sigma\alpha_2\gamma_1 + \rho y_{11} d_3 - \rho\alpha_1\alpha_2 - y_9 y_{11} d_3 + y_9\alpha_1\alpha_2\big)(y_1 y_3 y_{12} - \omega_1\omega_2 y_{12})\big]$$

Lampiran 7. Perhitungan Indeks Sensitivitas Parameter

Diberikan *basic reproduction number* (R_0) model matematika penyalahgunaan narkoba dengan memperhatikan tipe perawatan beserta tingkat resiko sebagai berikut :

$$R_0 = \frac{\beta_1\big(\mu p + \omega_2 + \eta(\mu(1-p) + \omega_1)\big)}{(\mu + \sigma + \rho)(\mu + \omega_1 + \omega_2)}.$$

Terdapat 9 parameter yang mempengaruhi R_0 diatas yaitu $\beta_1, \mu, p, \omega_1, \omega_2, \eta, \sigma, \delta,$ dan ρ. Indeks sensitivitas menurut **Chitnis, dkk (2008)** dirumuskan sebagai berikut:

$$e_m = \left(\frac{\partial R_0}{\partial m}\right)\frac{m}{R_0}$$

dengan,

m : parameter yang akan dianalisis

e_m : indeks sensitivitas parameter m.

Setelah mendapatkan indeks sensitivitas yang dihitung dengan rumus diatas, kemudian mensubstitusikan nilai parameter pada Tabel 4.3 kedalam indeks sensitivitas yang diperoleh. Berikut uraian perhitungan masing-masing indeks sensitivitas parameter yang telah disebutkan sebelumnya :

1. Untuk parameter β_1

$$e_{\beta_1} = \left(\frac{\partial R_0}{\partial \beta_1}\right)\frac{\beta_1}{R_0}$$

$$e_{\beta_1} = \left(\frac{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{\beta_1}{\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_{\beta_1} = \left(\frac{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{\beta_1(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}$$

$$e_{\beta_1} = \left(\frac{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}$$

$$e_{\beta_1} = 1$$

2. Untuk parameter σ

$$e_\sigma = \left(\frac{\partial R_0}{\partial \sigma}\right)\frac{\sigma}{R_0}$$

$$e_\sigma = \left(-\frac{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)^2(\mu+\omega_1+\omega_2)}\right)\frac{\sigma}{\frac{\beta_1\left(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\right)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_\sigma = \left(-\frac{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)^2(\mu+\omega_1+\omega_2)}\right)\frac{\sigma(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}$$

$$e_\sigma = -\frac{\sigma}{\mu+\sigma+\rho}$$

$$e_\sigma = -\frac{0.02827}{0.02+0.02827+0.0082}$$

$$e_\sigma = -\frac{0.02827}{0.05647}$$

$$e_\sigma = -0.50$$

3. Untuk parameter ω_2

$$e_{\omega_2} = \left(\frac{\partial R_0}{\partial \omega_2}\right)\frac{\omega_2}{R_0}$$

$$e_{\omega_2} = \left(\frac{\beta_1}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} - \frac{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2}\right)\frac{\omega_2}{\frac{\beta_1\left(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\right)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_{\omega_2} = \left(\frac{\beta_1\big(\mu+\omega_1+\omega_2-\mu p-\omega_2-\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2}\right)\frac{\omega_2(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}$$

$$e_{\omega_2} = \left(\frac{\omega_2\big(\mu+\omega_1+\omega_2-\mu p-\omega_2-\eta(\mu(1-p)+\omega_1)\big)(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}\right)$$

$$e_{\omega_2} = \frac{0.65(0.87-0.6762064)(0.05647)(0.87)}{(0.05647)(0.87)^2(0.6762064)}$$

$$e_{\omega_2} = 0.21$$

4. Untuk parameter ω_1

$$e_{\omega_1} = \left(\frac{\partial R_0}{\partial \omega_1}\right)\frac{\omega_1}{R_0}$$

$$e_{\omega_1} = \left(\frac{\eta\beta_1}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} - \frac{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2}\right)\frac{\omega_1}{\frac{\beta_1\left(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\right)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_{\omega_1} = \left(\frac{\beta_1\big(\eta(\mu+\omega_1+\omega_2)-\mu p-\omega_2-\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2}\right)\frac{\omega_1(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}$$

$$e_{\omega_1} = \frac{\omega_1(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)\big(\eta(\mu+\omega_1+\omega_2)-\mu p-\omega_2-\eta(\mu(1-p)+\omega_1)\big)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2\big(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)\big)}$$

$$e_{\omega_1} = \frac{0.2(0.05647)(0.87)(0.09(0.87)-0.6762064)}{(005647)(0.87)^2(0.6762064)}$$

$$e_{\omega_1} = -0.20$$

5. Untuk parameter μ

$$e_\mu = \left(\frac{\partial R_0}{\partial \mu}\right)\frac{\mu}{R_0}$$

$$e_\mu = \left(\frac{\beta_1(\eta(1-p)+p)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)} - \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)^2} - \frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)^2(\mu+\omega_1+\omega_2)}\right)\frac{\mu}{\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_\mu = \frac{\mu[(\eta(1-p)+p)(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)-(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))((\mu+\sigma+\rho)+(\mu+\omega_1+\omega_2))]}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}$$

$$e_\mu = \frac{0.02[0.41032(0.05647)(0.87)-0.6762064(0.05647+0.87)]}{(0.05647)(0.87)(0.6762064)}$$

$$e_\mu = -0.37$$

6. Untuk parameter ρ

$$e_\rho = \left(\frac{\partial R_0}{\partial \rho}\right)\frac{\rho}{R_0}$$

$$e_\rho = \left(-\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)^2(\mu+\omega_1+\omega_2)}\right)\frac{\rho}{\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_\rho = \left(-\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)^2(\mu+\omega_1+\omega_2)}\right)\frac{\rho(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}$$

$$e_\rho = -\frac{\rho}{(\mu+\sigma+\rho)}$$

$$e_\rho = -\frac{0.0082}{0.05647}$$

$$e_\rho = -0.15$$

7. Untuk parameter η

$$e_\eta = \left(\frac{\partial R_0}{\partial \eta}\right)\frac{\eta}{R_0}$$

$$e_\eta = \left(\frac{\beta_1(\mu(1-p)+\omega_1)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{\eta}{\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_\eta = \left(\frac{\beta_1(\mu(1-p)+\omega_1)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{\eta(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}$$

$$e_\eta = \frac{\eta(\mu(1-p)+\omega_1)}{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}$$

$$e_\eta = \frac{0.09(0.21296)}{0.6762064}$$

$$e_\eta = 0.03$$

8. Untuk parameter p

$$e_p = \left(\frac{\partial R_0}{\partial p}\right)\frac{p}{R_0}$$

$$e_p = \left(\frac{\beta_1\mu(1-\eta)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{p}{\frac{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}}$$

$$e_p = \left(\frac{\beta_1\mu(1-\eta)}{(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}\right)\frac{p(\mu+\sigma+\rho)(\mu+\omega_1+\omega_2)}{\beta_1(\mu p+\omega_2+\eta(\mu(1-p)+\omega_1))}$$

$$e_p = \frac{\mu p(1-\eta)}{\mu p+\omega_2+\eta(\mu(1-p)+\omega_1)}$$

$$e_p = \frac{0.02(0.352)(1-0.09)}{0.6762064}$$

$$e_p = 0.01$$

Lampiran 8. Kode Program Bidang Fase $I - T_j$

1. Kode Program Pendefinisian Model

```
function xdot=barusimulasinarkoba(t,x)
global p q lambda beta1 beta2 beta3 mu omega1 omega2 eta sigma
rho gamma1 gamma2 alpha1 alpha2;

xdot=zeros(6,1);
xdot(1)=p*lambda-beta1*x(3)*x(1)*mu/lambda-
(mu+omega1)*x(1)+omega2*x(2);
xdot(2)=(1-p)*lambda-eta*beta1*x(3)*x(2)*mu/lambda-
(mu+omega2)*x(2)+omega1*x(1);
xdot(3)=beta1*x(3)*(x(1)+eta*x(2))*mu/lambda+beta2*x(3)*x(6)*mu
/lambda+beta3*x(3)*x(4)*mu/lambda-(mu+sigma+rho)*x(3);
xdot(4)=(1-q)*sigma*x(3)-beta3*x(3)*x(4)*mu/lambda-
(mu+alpha1+gamma1)*x(4)+alpha2*x(5);
xdot(5)=q*sigma*x(3)-(mu+alpha2+gamma2)*x(5)+alpha1*x(4);
xdot(6)=gamma1*x(4)+gamma2*x(5)+rho*x(3)-
beta2*x(3)*x(6)*mu/lambda-mu*x(6);

end
```

2. Kode Program Utama

```
clc;
close all;
clear all;

global p q lambda beta1 beta2 beta3 mu omega1 omega2 eta
sigma rho gamma1 gamma2 alpha1 alpha2;

p=0.352;
q=0.352;
lambda=5000;
beta1=0.3;
beta2=0.15;
beta3=0.1;
mu=0.02;
omega1=0.2;
omega2=0.65;
eta=0.09;
sigma=0.02827;
rho=0.0082;
gamma1=0.01;
gamma2=0.3142;
alpha1=0.02961;
alpha2=0.003;

Ro=beta1*(eta*(omega1+mu*(1-p))+omega2+mu*p)/
((mu+omega1+omega2)*(mu+sigma+rho))
```

Lampiran 8-2

```
[t1,x1]=ode45(@barusimulasinarkoba,[0 500],[200 250 100
85 70 60]);
[t2,x2]=ode45(@barusimulasinarkoba,[0 500],[1000 3000
800 600 700 100]);
[t3,x3]=ode45(@barusimulasinarkoba,[0 500],[9000 8000
5000 7000 6000 500]);

plot(x1(:,1),x1(:,3),x2(:,1),x2(:,3),x3(:,1),x3(:,3),'Li
neWidth',1.5);
xlabel('Populasi Sh');
ylabel('Populasi I');
legend('Nilai Awal 1','Nilai Awal 2','Nilai Awal 3');
```

Lampiran 9. Kode Program Analisis Sensitivitas

```
clc;
close all;
clear all;

global p mu omega1 omega2 delta rho sigma1 sigma2 sigma3 eta;
sigma1=0.002827;
sigma2=0.02827;
sigma3=0.2827;
beta1=[0.1 0.8];

p=0.352;
omega1=0.2;
omega2=0.65;
mu=0.02;
eta=0.09;
rho=0.0082;

Ro1=beta1*(eta*(omega1+mu*(1-
p))+omega2+mu*p)/((mu+omega1+omega2)*(mu+sigma1+rho));
Ro2=beta1*(eta*(omega1+mu*(1-
p))+omega2+mu*p)/((mu+omega1+omega2)*(mu+sigma2+rho));
Ro3=beta1*(eta*(omega1+mu*(1-
p))+omega2+mu*p)/((mu+omega1+omega2)*(mu+sigma3+rho));

plot(beta1,Ro1,beta1,Ro2,beta1,Ro3,'LineWidth',2)
xlabel('\beta_1')
ylabel('R_0')
legend('\sigma=0.002827','\sigma=0.02827','\sigma=0.2827')
```

Lampiran 10. Kode Program Simulasi Model Matematika Penyalahgunaan Narkoba dengan Memperhatikan Tipe Perawatan beserta Tingkat Resiko Tanpa Adanya Kontrol

```
% A MATLAB example described in detail in the technical report
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %
% DOTcvp - Dynamic Optimization Toolbox with CVP approach for          %
%  handling continuous and mixed-integer dynamic optimization problems %
% Copyright (C) 2007-2010                                              %
% Tomas Hirmajer et al., thirmajer@gmail.com                           %
%                                                                      %
% The DOTcvp toolbox is completely free of charge under the creative   %
% commons license. The conditions of the license can be found on the   %
% following web page:                                                  %
% http://creativecommons.org/licenses/by-nc-nd/3.0/                    %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %

clear mex; clear all; close all;
% --------------------------------------------------- %
% Initialization:
% --------------------------------------------------- %
data.name                 = 'narkobatanpakontrol';
data.compiler             = 'None'; %['None'|'FORTRAN']

% --------------------------------------------------- %
% Settings for IVP (ODEs, sensitivities):
% --------------------------------------------------- %
data.odes.Def_FORTRAN     = {}; %this option is needed only for FORTRAN
parameters definition, e.g. {'double precision k10, k20, ..'}
data.odes.parameters      = {}; %constant parameters before ODE {'T=300','..}
data.odes.Def_MATLAB      = {}; %this option is needed only for MATLAB
parameters definition
data.odes.res(1)          = {'0.352*5000-0.3*y(3)*y(1)*0.02/5000-
(0.02+0.2)*y(1)+0.65*y(2)'};
data.odes.res(2)          = {'(1-0.352)*5000-
0.09*0.3*y(3)*y(2)*0.02/5000-(0.02+0.65)*y(2)+0.2*y(1)'};
data.odes.res(3)          =
{'0.3*y(3)*(y(1)+0.09*y(2))*0.02/5000+0.15*y(3)*y(6)*0.02/5000+0.1
*y(3)*y(4)*0.02/5000-(0.02+0.02827+0.0082)*y(3)'};
data.odes.res(4)          = {'(1-0.352)*0.02827*y(3)-
0.1*y(3)*y(4)*0.02/5000-(0.02+0.02961+0.01)*y(4)+0.003*y(5)'};
data.odes.res(5)          = {'0.352*0.02827*y(3)-
(0.02+0.003+0.3142)*y(5)+0.02961*y(4)'};
data.odes.res(6)          = {'0.01*y(4)+0.3142*y(5)+0.0082*y(3)-
0.15*y(3)*y(6)*0.02/5000-0.02*y(6)'};
data.odes.res(7)          = {'y(3)'};
data.odes.black_box       = {'None','1','FunctionName'};
%['None'|'Full'],[penalty coefficient for all constraints],[a
black box model function name]
data.odes.ic              = [20000 25000 15000 5000 10000 3000 0];
data.odes.NUMs            = size(data.odes.res,2); %number of
state variables (y)
data.odes.t0              = 0.0; %initial time
data.odes.tf              = 100.0; %final time
```

```
data.odes.NonlinearSolver = 'Newton'; %['Newton'|'Functional']
/Newton for stiff problems; Functional for non-stiff problems
data.odes.LinearSolver    = 'Dense'; %direct ['Dense'|'Diag'|'Band'];
iterative ['GMRES'|'BiCGStab'|'TFQMR'] /for the Newton NLS
data.odes.LMM             = 'Adams'; %['Adams'|'BDF'] /Adams for
non-stiff problems; BDF for stiff problems
data.odes.MaxNumStep      = 500; %maximum number of steps
data.odes.RelTol          = 1*10^(-7); %IVP relative tolerance level
data.odes.AbsTol          = 1*10^(-7); %IVP absolute tolerance level
data.sens.SensAbsTol      = 1*10^(-7); %absolute tolerance for
sensitivity variables
data.sens.SensMethod      = 'Simultaneous';
%['Staggered'|'Staggered1'|'Simultaneous']
data.sens.SensErrorControl= 'on'; %['on'|'off']

% --------------------------------------------------- %
% NLP definition:
% --------------------------------------------------- %
data.nlp.RHO              = 20; %number of time intervals
data.nlp.problem          = 'min'; %['min'|'max']
data.nlp.J0               = 'y(7)'; %cost function: min-max(cost function)
data.nlp.u0               = [0]; %initial value for control values
data.nlp.lb               = [0]; %lower bounds for control values
data.nlp.ub               = [1]; %upper bounds for control values
data.nlp.p0               = []; %initial values for time-
independent parameters
data.nlp.lbp              = []; %lower bounds for time-independent parameters
data.nlp.ubp              = []; %upper bounds for time-independent parameters
data.nlp.solver           = 'FMINCON';
%['FMINCON'|'IPOPT'|'SRES'|'DE'|'ACOMI'|'MISQP'|'MITS']
data.nlp.SolverSettings   = 'None'; %insert the name of the file that
contains settings for NLP solver, if does not exist use ['None']
data.nlp.NLPtol           = 1*10^(-5); %NLP tolerance level
data.nlp.GradMethod       = 'SensitivityEq';
%['SensitivityEq'|'FiniteDifference'|'None']
data.nlp.MaxIter          = 1000; %Maximum number of iterations
data.nlp.MaxCPUTime       = 60*60*0.50; %Maximum CPU time of the
optimization (60*60*0.25) = 15 minutes
data.nlp.approximation    = 'PWC'; %['PWC'|'PWL'] PWL only for:
FMINCON & without the free time problem
data.nlp.FreeTime         = 'off'; %['on'|'off'] set 'on' if free
time is considered
data.nlp.t0Time           = [data.odes.tf/data.nlp.RHO]; %initial
size of the time intervals, e.g. [data.odes.tf/data.nlp.RHO] or
for the each time interval separately [dt1 dt2 dt3]
data.nlp.lbTime           = 0.01; %lower bound of the time intervals
data.nlp.ubTime           = data.odes.tf; %upper bound of the time intervals
data.nlp.NUMc             = size(data.nlp.u0,2); %number of
control variables (u)
data.nlp.NUMi             = 0; %number of integer variables (u)
taken from the last control variables, if not equal to 0 you need
to use some MINLP solver ['ACOMI'|'MISQP'|'MITS']
data.nlp.NUMp             = size(data.nlp.p0,2); %number of time-
independent parameters (p)

% --------------------------------------------------- %
```

```
% Equality constraints (ECs):
% -------------------------------------------------- %
data.nlp.eq.status         = 'off'; %['on'|'off'] ECs
data.nlp.eq.NEC            = 1; %number of active ECs
data.nlp.eq.eq(1)          = {''};
data.nlp.eq.time(1)        = data.nlp.RHO;
data.nlp.eq.PenaltyFun     = 'off'; %['on'|'off'] ECs penalty function
data.nlp.eq.PenaltyCoe     = [1.0];
%J0=J0+data.nlp.eq.PenaltyCoe*ViolationOfEqualityConstraint /*
only for stochastic solvers */

% -------------------------------------------------- %
% Inequality /path/ constraints (INECs):
% -------------------------------------------------- %
data.nlp.ineq.status       = 'off'; %['on'|'off'] INECs
data.nlp.ineq.NEC          = 2; %number of active INECs
data.nlp.ineq.InNUM        = 1; %how many inequality constraints
are '>' else '<'
data.nlp.ineq.eq(1)        = {''};
data.nlp.ineq.eq(2)        = {''};
data.nlp.ineq.Tol          = 0.0005; %tolerance level of violation of INECs
data.nlp.ineq.PenaltyFun   = 'off'; %['on'|'off'] INECs penalty
function
data.nlp.ineq.PenaltyCoe   = [1.0 1.0];
%J0=J0+data.nlp.ineq.PenaltyCoe*ViolationOfInequalityConstraint /*
for every inequality constraint one parameter */

% -------------------------------------------------- %
% Options for setting of the final output:
% -------------------------------------------------- %
data.options.intermediate = 'off'; %['on'|'off'|'silent'] display
of the intermediate results
data.options.display      = 'on'; %['on'|'off'] display of the figures
data.options.title        = 'on'; %['on'|'off'] display of the figure title
data.options.state        = 'on'; %['on'|'off'] display of the
state trajectory
data.options.control      = 'on'; %['on'|'off'] display of the
control trajectory
data.options.ConvergCurve = 'on'; %['on'|'off'] display of the
convergence curve
data.options.Pict_Format  = 'eps'; %['eps'|'wmf'|'both'] save figures as
data.options.report       = 'on'; %['on'|'off'] save data in the dat file
data.options.commands     = {''}; %additional commands, e.g. 'figure(1),.. '
data.options.trajectories = data.odes.NUMs-1; %how many state
trajectories will be displayed
data.options.profiler     = 'off'; %['on'|'off']
data.options.multistart   = 1; %set 1 if the multistart is off,
otherwise you have to put here some integer value

data.options.action       = 'single-optimization'; %['single-
optimization'|'re-optimization'|'hybrid-strategy'|'simulation']

% -------------------------------------------------- %
% Call of the main function (you do not change this!):
% -------------------------------------------------- %
[data]=dotcvp_main(data);
```

Lampiran 11. Kode Program Simulasi Model Matematika Penyalahgunaan Narkoba dengan Memperhatikan Tipe Perawatan beserta Tingkat Resiko dengan Adanya Kontrol

```
% A MATLAB example described in detail in the technical report
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %
% DOTcvp - Dynamic Optimization Toolbox with CVP approach for        %
%  handling continuous and mixed-integer dynamic optimization problems %
% Copyright (C) 2007-2010                                            %
% Tomas Hirmajer et al., thirmajer@gmail.com                         %
%                                                                    %
% The DOTcvp toolbox is completely free of charge under the creative  %
% commons license. The conditions of the license can be found on the  %
% following web page:                                                %
% http://creativecommons.org/licenses/by-nc-nd/3.0/                  %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %

clear mex; clear all; close all;
% --------------------------------------------------- %
% Initialization:
% --------------------------------------------------- %
data.name                  = 'narkobatanpakontrol';
data.compiler              = 'None'; %['None'|'FORTRAN']

% --------------------------------------------------- %
% Settings for IVP (ODEs, sensitivities):
% --------------------------------------------------- %
data.odes.Def_FORTRAN      = {}; %this option is needed only for FORTRAN
parameters definition, e.g. {'double precision k10, k20, ..'}
data.odes.parameters       = {}; %constant parameters before ODE {'T=300','..}
data.odes.Def_MATLAB       = {}; %this option is needed only for MATLAB
parameters definition
data.odes.res(1)           = {'0.352*5000-(1-
u(1))*0.3*y(3)*y(1)*0.02/5000-(0.02+0.2)*y(1)+0.65*y(2)'};
data.odes.res(2)           = {'(1-0.352)*5000-(1-
u(1))*0.09*0.3*y(3)*y(2)*0.02/5000-(0.02+0.65)*y(2)+0.2*y(1)'};
data.odes.res(3)           = {(1-
u(1))*'0.3*y(3)*(y(1)+0.09*y(2))*0.02/5000+0.15*y(3)*y(6)*0.02/500
0+0.1*y(3)*y(4)*0.02/5000-(0.02+0.02827+0.0082)*y(3)'};
data.odes.res(4)           = {'(1-0.352)*0.02827*y(3)-
0.1*y(3)*y(4)*0.02/5000-(0.02+0.02961+0.01)*y(4)+0.003*y(5)'};
data.odes.res(5)           = {'0.352*0.02827*y(3)-
(0.02+0.003+0.3142)*y(5)+0.02961*y(4)'};
data.odes.res(6)           = {'0.01*y(4)+0.3142*y(5)+0.0082*y(3)-
0.15*y(3)*y(6)*0.02/5000-0.02*y(6)'};
data.odes.res(7)           = {'y(3) +0.5*10*(u(1)^2)'};
data.odes.black_box        = {'None','1','FunctionName'};
%['None'|'Full'],[penalty coefficient for all constraints],[a
black box model function name]
data.odes.ic               = [20000 25000 15000 5000 10000 3000 0];
data.odes.NUMs             = size(data.odes.res,2); %number of
state variables (y)
data.odes.t0               = 0.0; %initial time
data.odes.tf               = 100.0; %final time
```

```
data.odes.NonlinearSolver = 'Newton'; %['Newton'|'Functional']
/Newton for stiff problems; Functional for non-stiff problems
data.odes.LinearSolver    = 'Dense'; %direct ['Dense'|'Diag'|'Band'];
iterative ['GMRES'|'BiCGStab'|'TFQMR'] /for the Newton NLS
data.odes.LMM             = 'Adams'; %['Adams'|'BDF'] /Adams for
non-stiff problems; BDF for stiff problems
data.odes.MaxNumStep      = 500; %maximum number of steps
data.odes.RelTol          = 1*10^(-7); %IVP relative tolerance level
data.odes.AbsTol          = 1*10^(-7); %IVP absolute tolerance level
data.sens.SensAbsTol      = 1*10^(-7); %absolute tolerance for
sensitivity variables
data.sens.SensMethod      = 'Simultaneous';
%['Staggered'|'Staggered1'|'Simultaneous']
data.sens.SensErrorControl= 'on'; %['on'|'off']

% --------------------------------------------------- %
% NLP definition:
% --------------------------------------------------- %
data.nlp.RHO              = 20; %number of time intervals
data.nlp.problem          = 'min'; %['min'|'max']
data.nlp.J0               = 'y(7)'; %cost function: min-max(cost function)
data.nlp.u0               = [0]; %initial value for control values
data.nlp.lb               = [0]; %lower bounds for control values
data.nlp.ub               = [1]; %upper bounds for control values
data.nlp.p0               = []; %initial values for time-
independent parameters
data.nlp.lbp              = []; %lower bounds for time-independent parameters
data.nlp.ubp              = []; %upper bounds for time-independent parameters
data.nlp.solver           = 'FMINCON';
%['FMINCON'|'IPOPT'|'SRES'|'DE'|'ACOMI'|'MISQP'|'MITS']
data.nlp.SolverSettings   = 'None'; %insert the name of the file that
contains settings for NLP solver, if does not exist use ['None']
data.nlp.NLPtol           = 1*10^(-5); %NLP tolerance level
data.nlp.GradMethod       = 'SensitivityEq';
%['SensitivityEq'|'FiniteDifference'|'None']
data.nlp.MaxIter          = 1000; %Maximum number of iterations
data.nlp.MaxCPUTime       = 60*60*0.50; %Maximum CPU time of the
optimization (60*60*0.25) = 15 minutes
data.nlp.approximation    = 'PWC'; %['PWC'|'PWL'] PWL only for:
FMINCON & without the free time problem
data.nlp.FreeTime         = 'off'; %['on'|'off'] set 'on' if free
time is considered
data.nlp.t0Time           = [data.odes.tf/data.nlp.RHO]; %initial
size of the time intervals, e.g. [data.odes.tf/data.nlp.RHO] or
for the each time interval separately [dt1 dt2 dt3]
data.nlp.lbTime           = 0.01; %lower bound of the time intervals
data.nlp.ubTime           = data.odes.tf; %upper bound of the time intervals
data.nlp.NUMc             = size(data.nlp.u0,2); %number of
control variables (u)
data.nlp.NUMi             = 0; %number of integer variables (u)
taken from the last control variables, if not equal to 0 you need
to use some MINLP solver ['ACOMI'|'MISQP'|'MITS']
data.nlp.NUMp             = size(data.nlp.p0,2); %number of time-
independent parameters (p)

% --------------------------------------------------- %
```

```
% Equality constraints (ECs):
% --------------------------------------------------- %
data.nlp.eq.status        = 'off'; %['on'|'off'] ECs
data.nlp.eq.NEC           = 1; %number of active ECs
data.nlp.eq.eq(1)         = {''};
data.nlp.eq.time(1)       = data.nlp.RHO;
data.nlp.eq.PenaltyFun    = 'off'; %['on'|'off'] ECs penalty function
data.nlp.eq.PenaltyCoe    = [1.0];
%J0=J0+data.nlp.eq.PenaltyCoe*ViolationOfEqualityConstraint /*
only for stochastic solvers */

% --------------------------------------------------- %
% Inequality /path/ constraints (INECs):
% --------------------------------------------------- %
data.nlp.ineq.status      = 'off'; %['on'|'off'] INECs
data.nlp.ineq.NEC         = 2; %number of active INECs
data.nlp.ineq.InNUM       = 1; %how many inequality constraints
are '>' else '<'
data.nlp.ineq.eq(1)       = {''};
data.nlp.ineq.eq(2)       = {''};
data.nlp.ineq.Tol         = 0.0005; %tolerance level of violation of INECs
data.nlp.ineq.PenaltyFun  = 'off'; %['on'|'off'] INECs penalty
function
data.nlp.ineq.PenaltyCoe  = [1.0 1.0];
%J0=J0+data.nlp.ineq.PenaltyCoe*ViolationOfInequalityConstraint /*
for every inequality constraint one parameter */

% --------------------------------------------------- %
% Options for setting of the final output:
% --------------------------------------------------- %
data.options.intermediate = 'off'; %['on'|'off'|'silent'] display
of the intermediate results
data.options.display      = 'on'; %['on'|'off'] display of the figures
data.options.title        = 'on'; %['on'|'off'] display of the figure title
data.options.state        = 'on'; %['on'|'off'] display of the
state trajectory
data.options.control      = 'on'; %['on'|'off'] display of the
control trajectory
data.options.ConvergCurve = 'on'; %['on'|'off'] display of the
convergence curve
data.options.Pict_Format  = 'eps'; %['eps'|'wmf'|'both'] save figures as
data.options.report       = 'on'; %['on'|'off'] save data in the dat file
data.options.commands     = {''}; %additional commands, e.g. 'figure(1),.. '
data.options.trajectories = data.odes.NUMs-1; %how many state
trajectories will be displayed
data.options.profiler     = 'off'; %['on'|'off']
data.options.multistart   = 1; %set 1 if the multistart is off,
otherwise you have to put here some integer value

data.options.action       = 'single-optimization'; %['single-
optimization'|'re-optimization'|'hybrid-strategy'|'simulation']

% --------------------------------------------------- %
% Call of the main function (you do not change this!):
% --------------------------------------------------- %
[data]=dotcvp_main(data);
```