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CHARACTERIZATIONS FOR THE GENERALIZED FRACTIONAL INTEGRAL OPERATORS ON MORREY SPACES

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Abstract. We present some characterizations for the boundedness of the generalized fractional integral operators on Morrey spaces. The characterizations follow from two key estimates, one for the norm of some functions in Morrey spaces, and another for the values of the corresponding fractional integrals. We prove three theorems about necessary and sufficient conditions. We show that these theorems are independent by giving some examples. We also obtain counterparts for the weak generalized Morrey spaces.

1. Introduction

In this paper, for a measurable function $\rho : (0,\infty) \to (0,\infty)$, we are interested in the generalized fractional integral operator I_ρ given by the formula

$$I_\rho f(x) := \int_{\mathbb{R}^d} \frac{\rho(|x-y|)}{|x-y|^d} f(y)\,dy, \quad x \in \mathbb{R}^d,$$

for any suitable function f on $\mathbb{R}^d$. This generalized fractional integral operator was initilally investigated in [27]. Nowadays many authors have been culminating important observations about I_ρ especially in connection with Morrey spaces. These spaces cover Lebesgue spaces as special cases and seem to describe the behavior of I_ρ well. In order to highlight what we shall prove in this paper, we take up the works [3, 6, 7, 18, 22, 25, 28, 35], where we formulated sufficient conditions on ρ for I_ρ to be bounded on Morrey spaces $L_{p,\phi}$ with $1 \leqslant p < \infty$ and ϕ a function from $(0,\infty)$ to itself. We aim to show that these conditions are necessary as well. We characterize the boundedness by estimating the norm of the characteristic functions of balls and the function $\phi(|\cdot|)$, as well as the value of the corresponding fractional integrals.

Hereafter, we assume that

$$\int_0^1 \frac{\rho(s)}{s}\,ds < \infty,$$

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