# THE FRACTIONAL INTEGRAL OPERATORS ON MORREY SPACES OVER $Q$-HOMOGENEOUS METRIC MEASURE SPACE 

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#### Abstract

This paper establishes necessary and sufficient condition for the boundedness of the fractional integral operator $I_{\alpha} f$ on Morrey spaces over metric measure spaces which satisfies the Q-homogeneous and its corollary.


Key words: Morrey Space Classic; Metric Measure Space; Q-Homogeneous.
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## 1. INTRODUCTION

We consider to a topological space $X:=(X, \delta, \mu)$, endowed with complete measure $\mu$ such that the space of compactly supported continuous functions is dense in $L^{1}(X, \mu)$ and there exists a function (metric) $\delta: X \times X \rightarrow[0, \infty)$ satisfying the following conditions.

1. $\delta(x, y)=0$ if and only if $x=y$;
2. $\delta(x, y)>0$ for all $x \neq y, x, y \in X$;
3. $\delta(x, y)=\delta(y, x)$;
4. $\delta(x, y) \leq\{\delta(x, z)+\delta(z, y)\}$
for every $x, y, z \in X$. We have an assumptions that the balls $B(a, r):=\{x \in X: \delta(x, a)<$ $r\}$ are measurable, for $a \in X, r>0$, and $0 \leq \mu(B(a, r))<\infty$. For every neighborhood $V$ of $x \in X$, there exists $r>0$, such that $B(x, r) \subset V$. We also assume that $\mu(X)=\infty, \mu\{a\}=$
