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# THE FRACTIONAL INTEGRAL OPERATORS ON MORREY SPACES OVER Q-HOMOGENEOUS METRIC MEASURE SPACE

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## **ABSTRACT**

This paper establishes necessary and sufficient condition for the boundedness of the fractional integral operator  $I_{\alpha}f$  on Morrey spaces over metric measure spaces which satisfies the Q-homogeneous and its corollary.

**Key words:** Morrey Space Classic; Metric Measure Space; Q-Homogeneous.

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## 1. INTRODUCTION

We consider to a topological space  $X := (X, \delta, \mu)$ , endowed with complete measure  $\mu$  such that the space of compactly supported continuous functions is dense in  $L^1(X, \mu)$  and there exists a function (metric)  $\delta: X \times X \to [0, \infty)$  satisfying the following conditions.

- 1.  $\delta(x,y) = 0$  if and only if x = y;
- 2.  $\delta(x,y) > 0$  for all  $x \neq y, x, y \in X$ ;
- 3.  $\delta(x,y) = \delta(y,x)$ ;
- 4.  $\delta(x, y) \leq \{\delta(x, z) + \delta(z, y)\}$

for every  $x, y, z \in X$ . We have an assumptions that the balls  $B(a, r) := \{x \in X : \delta(x, a) < r\}$  are measurable, for  $a \in X, r > 0$ , and  $0 \le \mu(B(a, r)) < \infty$ . For every neighborhood V of  $x \in X$ , there exists r > 0, such that  $B(x, r) \subset V$ . We also assume that  $\mu(X) = \infty$ ,  $\mu\{a\} = 0$ 

0 and  $B(a, r_2) \setminus B(a, r_1) = \emptyset$ , for all  $a \in X$ ,  $0 < r_1 < r_2 < \infty$ . The triple  $(X, \delta, \mu)$  will be called metric measure space [7].

X is called Q-homogeneous (Q > 0) such that  $C_0 r^Q \le \mu(B) \le C_1 r^Q$  where  $C_0$  and  $C_1$  are positive constants [8].

Eridani [6,7] proved the boundedness theorem on Lebesgue spaces in  $K_{\alpha}$  and classic Morrey spaces over quasi metric space where

$$K_{\alpha} := \int_{X} \frac{f(y)}{\mu \left( B(x, \delta(x, y)) \right)^{1-\alpha}} d\mu(y)$$

with  $0 < \alpha < 1$ .

The result of [7] can be adapted to the operator  $K_{\alpha}$  with doubling condition. Let  $0 < \alpha < \beta$ , we consider the fractional integral operator  $I_{\alpha}$  given by

$$I_{\alpha}f(x) \coloneqq \int_{X} \frac{f(y)}{\delta(x,y)^{\beta-\alpha}} d\mu(y)$$

for suitable f on X

The boundedness theorem of  $I_{\alpha}$  on homogeneous classic Morrey spaces can be proved using Q-Homogeneous. In this paper, we will prove the generalization of the boundedness theorem from [6,7].

# 2. PRELIMINARIES

The following theorem is the inequality for the operator  $K_{\alpha}$  from  $\mathcal{L}^{p}(X,\mu)$  to  $\mathcal{L}^{q}(X,\nu)$  for the case of Euclidean spaces.

**Theorem 2.1** [6] Let  $(X, \delta, \mu)$  be a space of homogeneous type. Suppose that  $1 and <math>0 < \alpha < \frac{1}{p}$ . Assume that  $\nu$  is another measure on X. Then  $K_{\alpha}$  is bounded from  $\mathcal{L}^p(X, \mu)$  to  $\mathcal{L}^q(X, \nu)$  if and only if

$$\nu(B) \leq C\mu(B)^{q(\frac{1}{p}-\alpha)}$$

for all balls B in X.

Eridani and Meshki [7] proved the boundedness results of  $K_{\alpha}$  from  $\mathcal{L}^p(X,\mu)$  to the classic Morrey spaces  $\mathcal{L}^{p,\lambda}(X,\nu,\mu)$  which is defined as a set of functions  $f \in \mathcal{L}^p_{lok}(X,\nu)$  such that

$$||f: \mathcal{L}^{p,\lambda}(X,\nu,\mu)|| = \sup_{B} \left(\frac{1}{\mu(B)^{\lambda}} \int_{B} |f(y)|^{p} d\nu(y)\right)^{\frac{1}{p}} < \infty.$$

with  $\nu$  is another measure on X, where  $1 \le p < \infty$  and  $\lambda \ge 0$ . Their theorem can be stated as the following theorem.

**Theorem 2.2** [7] Let  $(X, \delta, \mu)$  be a space of homogeneous type and let  $1 . Suppose that <math>0 < \alpha < \frac{1}{p}$ ,  $0 < \lambda_1 < 1 - \alpha p$  and  $\frac{\lambda_2}{q} = \frac{\lambda_1}{p}$ . Then  $K_{\alpha}$  is bounded from  $\mathcal{L}^{p,\lambda_1}(X,\nu,\mu)$  to  $\mathcal{L}^{q,\lambda_2}(X,\nu,\mu)$  if and only if there is a positive constant C such that

$$\nu(B) \leq C\mu(B)^{q(\frac{1}{p}-\alpha)}$$

# 3. MAIN RESULT

In this section, we formulate the main results of the paper. We begin with the case of  $\beta$ -homogeneous over metric measure space.

**Theorem 3.1** Let  $(X, \delta, \mu)$  be a  $\beta$ -homogeneous metric measure space,  $\nu$  be a measure on X,  $1 , <math>1 < \alpha < \beta$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^p(X, \mu)$  to  $\mathcal{L}^q(X, \nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\nu(B) \leq C\mu(B)^{q\left(\frac{1}{p} - \frac{\alpha}{\beta}\right)}$$

**Proof:**(Necessity) If  $x, y \in B(a, r)$  then  $\delta(x, a) < r$  and  $\delta(y, a) < r$  thus  $\delta(x, y) \le \delta(x, a) + \delta(y, a) < 2r$  thus

$$\frac{1}{(2r)^{\beta-\alpha}} \le \frac{1}{\delta(x,y)^{\beta-\alpha}}$$

the above inequality implies.

$$\frac{\mu(B)}{r^{\beta-\alpha}} = \int_{B} \frac{d\mu(y)}{(2r)^{\beta-\alpha}} \le \int_{B} \frac{d\mu(y)}{\delta(x,y)^{\beta-\alpha}} = \int_{X} \frac{\chi_{B}(y)d\mu(y)}{\delta(x,y)^{\beta-\alpha}} = CI_{\alpha}\chi_{B}(x)$$

$$r^{\alpha} \le CI_{\alpha}\chi_{B}(x)$$

$$||I_{\alpha}\chi_{B}: \mathcal{L}^{q}(v)|| \leq C||\chi_{B}: \mathcal{L}^{p}(\mu)|| \leq C\left(\int_{X}\chi_{B}(t)d\mu(t)\right)^{\frac{1}{p}} \leq C\mu(B)^{\frac{1}{p}}$$

$$\left(\int_{B}|r^{\alpha}|^{q}d\nu(x)\right)^{\frac{1}{q}} \leq C\left(\int_{B}|I_{\alpha}\chi_{B}(t)|^{q}d\nu(t)\right)^{\frac{1}{q}} \leq C||I_{\alpha}\chi_{B}: \mathcal{L}^{q}(v)|| \leq C\mu(B)^{\frac{1}{p}}$$

Thus

$$r^{\alpha} \nu(B)^{\frac{1}{q}} \le C\mu(B)^{\frac{1}{p}}$$

 $C_0 r^{\beta} \le \mu(B) \le C_1 r^{\beta}$  thus

$$\mu(B)^{\frac{\alpha}{\beta}} \le Cr^{\alpha}$$

$$\mu(B)^{\frac{\alpha}{\beta}} \nu(B)^{\frac{1}{q}} \le Cr^{\alpha} \nu(B)^{\frac{1}{q}} \le C\mu(B)^{\frac{1}{p}}$$

$$\nu(B)^{\frac{1}{q}}\mu(B)^{\frac{\alpha}{\beta}-\frac{1}{p}} \le C$$

Thus

$$\nu(B)^{\frac{1}{q}} \le C\mu(B)^{\frac{1}{p} - \frac{\alpha}{\beta}}$$

or alternatively

$$\nu(B) \le C\mu(B)^{q\left(\frac{1}{p} - \frac{\alpha}{\beta}\right)}$$

**Sufficiency:** Let  $f \geq 0$ . We define

$$S(s) := \int_{\delta(a,y) < s} f(y) d\mu(y)$$

for every  $s \in [0, r]$ . Suppose that  $S(r) < \infty$ , then  $2^m < S(r) \le 2^{m+1}$ , for some  $m \in \mathbb{Z}$ . Let

$$s_i := \sup\{t : S(t) \le 2^j\}, j \le m, and s_{m+1} := r.$$

Then  $(s_j)_{j=-\infty}^{m+1}$  is non-decreasing sequence,  $S(s_j) \leq 2^j$ ,  $S(t) \geq 2^j$  for  $t > s_j$  and

$$2^{j} \le \int_{s_{j} \le \delta(a,y) \le s_{j+1}} f(y) d\mu(y)$$

If  $\rho := \lim_{i \to -\infty} s_i$ , then

$$\delta(a,x) < r \Leftrightarrow \delta(a,x) \in [0,\rho] \cup \bigcup_{j=-\infty}^{m} (s_j,s_{j+1}],$$

if  $S(r) = \infty$  then  $m = \infty$ . Thus

$$0 \le \int_{\delta(a,y) < \rho} f(y) d\mu(y) \le S(s_j) \le 2^j$$

for every *j*, thus

$$\int_{\delta(a,y)<\rho} f(y)d\mu(y) = 0$$

from these observations, we have

$$\int_{\delta(a,x) < r} \left( I_{\alpha} f(x) \right)^{q} d\nu(x) \leq \sum_{j=-\infty}^{m} \int_{s_{j} \leq \delta(a,x) \leq s_{j+1}} \left( I_{\alpha} f(x) \right)^{q} d\nu(x)$$

$$\leq \sum_{j=-\infty}^{m} \int_{s_{j} \leq \delta(a,x) \leq s_{j+1}} \left( \int_{\delta(a,y) \leq s_{j+1}} \frac{f(y) d\mu(y)}{\delta(x,y)^{\beta-\alpha}} \right)^{q} d\nu(x)$$

$$\leq \sum_{j=-\infty}^{m} \int_{s_{j} \leq \delta(a,x) \leq s_{j+1}} \left( \sum_{k=0}^{\infty} \left( \frac{1}{s_{j+1}} \right)^{\beta-\alpha} \int_{\delta(a,y) \leq s_{j+1}} f(y) d\mu(y) \right)^{q} d\nu(x)$$

$$\leq \left( \sum_{j=-\infty}^{m} \left( \frac{1}{s_{j+1}} \right)^{\beta-\alpha} \int_{\delta(a,y) \leq s_{j+1}} f(y) d\mu(y) \right)^{q} \nu(B)$$

Using the fact that

$$\int_{\delta(a,y) \leq s_{j+1}} f(y) d\mu(y) \leq S \big( s_{j+1} \big) \leq 2^{j+2} \leq C \int_{s_{j-1} \leq \delta(a,y) \leq s_j} f(y) d\mu(y)$$

then, by using Holder's inequality, we obtain

$$\leq C\nu(B) \left( \sum_{j=-\infty}^{m} \left( \int_{s_{j-1} \leq \delta(a,y) \leq s_{j}} (f(y))^{p} d(\mu) y \right)^{\frac{1}{p}} \left( \int_{s_{j-1} \leq \delta(a,y) \leq s_{j}} 1^{q} d(\mu) y \right)^{\frac{1}{q}} \frac{1}{s_{j}^{\beta-\alpha}} \right)^{q}$$

$$\leq C\nu(B) \left( \left( \int_{s_{j-1} \leq \delta(a,y) \leq s_{j}} (f(y))^{p} d(\mu) y \right)^{\frac{1}{p}} \sum_{j=-\infty}^{m} \mu(B(x,r))^{1-\frac{1}{p}} \frac{1}{s_{j}^{\beta-\alpha}} \right)^{q}$$

$$= C\nu(B) r^{q(\alpha-\frac{\beta}{p})} \left( \left( \int_{s_{j-1} \leq \delta(a,y) \leq s_{j}} (f(y))^{p} d(\mu) y \right)^{\frac{1}{p}} \right)^{q}$$

$$\leq C\mu(B)^{q\left(\frac{1}{p}-\frac{\alpha}{\beta}\right)}r^{q\left(\alpha-\frac{\beta}{p}\right)}\left(\left(\int\limits_{s_{j-1}\leq\delta(\alpha,y)\leq s_{j}}\left(f(y)\right)^{p}d(\mu)y\right)^{\frac{1}{p}}\right)^{q}$$

$$= C \left( \left( \int_{s_{j-1} \le \delta(a,y) \le s_j} (f(y))^p d(\mu) y \right)^{\frac{1}{p}} \right)^q$$

Thus

$$||I_{\alpha}f:\mathcal{L}^{q}(\nu)|| \leq C||f:\mathcal{L}^{p}(\mu)||$$

Next, using the modified condition for measure  $\nu$ , we obtain the following result.

**Theorem 3.2** Let  $(X, \delta, \mu)$  be a Q-homogeneous metric measure space,  $\nu$  be a measure on X,  $1 , <math>1 < \alpha < \beta - \frac{Q}{p'}$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^p(X, \mu)$  to  $\mathcal{L}^q(X, \nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\nu(B) \leq Cr^{\left(\beta - \alpha - \frac{Q}{p'}\right)q}$$

with 
$$p' = \frac{p}{p-1}$$
.

**Proof.** (Necessity) Suppose that  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p}(X,\mu)$  to  $\mathcal{L}^{q}(X,\nu)$  thus

$$||I_{\alpha}f: \mathcal{L}^{q}(X,\nu)|| \leq C||f: \mathcal{L}^{p}(X,\mu)||$$

Hence,

$$\left(\int_{X} |I_{\alpha}f|^{q} d\nu\right)^{1/q} \leq C \left(\int_{X} |f(x)|^{p} d\mu\right)^{1/p}$$

 $f \coloneqq \chi_B$  where  $a \in X$  , r > 0 then

$$\left(\int_{B} \left|I_{\alpha}\chi_{B}\right|^{q} d\nu\right)^{1/q} \leq C \left(\int_{B} \left|\chi_{B}\right|^{p} d\mu\right)^{1/p}$$

$$\left(\int_{B} \left(\int_{B} \frac{\chi_{B}}{\delta(x, y)^{\beta - \alpha}} d\mu(y)\right)^{q} dv\right)^{1/q} \leq C\mu(B)^{1/p}$$

$$r^{\alpha-\beta}\mu(B)\nu(B)^{1/q} \le C\mu(B)^{1/p}$$
  
 $\nu(B)^{1/q} \le C\mu(B)^{\frac{1}{p}-1}r^{\beta-\alpha}$ 

Because  $p' = \frac{p}{p-1}$  and  $C_0 r^Q \le \mu(B) \le C_1 r^Q$  then

$$\nu(B)^{1/q} \le Cr^{-\frac{Q}{p'}}r^{\beta-\alpha}$$

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$$\nu(B) \le C r^{q\left(\beta - \alpha - \frac{Q}{p'}\right)}$$

**Sufficiency.** Let  $f \ge 0$ . For x,  $a \in X$ , next we consider the notation

$$E_1(x) := \left\{ y : \delta(a, y) < \frac{\delta(a, x)}{2a_1} \right\};$$

$$E_2(x) := \left\{ y : \frac{\delta(a, x)}{2a_1} \le \delta(a, y) \le 2a_1 \delta(a, x) \right\};$$

$$E_3(x) := \left\{ y : \delta(a, y) > a_1 \delta(a, x) \right\}.$$

Thus

$$\int_{X} (I_{\alpha}f(x)) d\nu(x)$$

$$\leq C \int_{X} \left( \int_{E_{1}(x)} |f(y)| \delta(x,y)^{\alpha-\beta} d\mu(y) \right)^{q} d\nu(x)$$

$$+ C \int_{X} \left( \int_{E_{2}(x)} |f(y)| \delta(x,y)^{\alpha-\beta} d\mu(y) \right)^{q} d\nu(x)$$

$$+ C \int_{X} \left( \int_{E_{3}(x)} |f(y)| \delta(x,y)^{\alpha-\beta} d\mu(y) \right)^{q} d\nu(x) = S_{1} + S_{2} + S_{3}$$

If  $y \in E_1(x)$ , then  $\delta(a, x) < 2a_1a_0\delta(a, x)$ . Thus obviously

$$S_{1} = \int_{\delta(a,x) < r} \left( \int_{E_{1}(x)} |f(y)| \delta(x,y)^{\alpha-\beta} d\mu(y) \right)^{q} d\nu(x)$$

$$\leq C \int_{B} \left( \int_{\delta(a,y) < \delta(a,x)} |f(y)| \delta(x,y)^{\alpha-\beta} d\mu(y) \right)^{q} d\nu(x)$$

$$\leq C \int_{B} \delta(a,x)^{q(\alpha-\beta)} \left( \int_{\delta(a,y) < \delta(a,x)} |f(y)| d\mu(y) \right)^{q} d\nu(x)$$

Thus we have

$$\int_{\delta(a,x)\geq t} \delta(a,x)^{q(\alpha-\beta)} d\nu(x) = \sum_{n=0}^{\infty} \int_{B(a,2^{k+1}t)\backslash B(a2^kt)} \left(\delta(a,x)^{q(\alpha-\beta)} d\nu(x)\right)$$

$$\leq C \sum_{n=0}^{\infty} (2^k t)^{q(\alpha-\beta)} \nu(B), = C t^{q(\alpha-\beta)} \nu(B)$$

which implies

$$\int_{\delta(\alpha,x)\leq t} 1^{(1-p')} d\mu(x) \leq C\mu(B)$$

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Thus

$$\begin{aligned} \sup_{\alpha \in X, \, t > 0} \left( \int_{\delta(\alpha, x) \ge t} \delta(\alpha, x)^{q(\alpha - \beta)} dv(x) \right)^{\frac{1}{q}} \left( \int_{\delta(\alpha, x) \le t} 1^{(1 - p')} d\mu(x) \right)^{\frac{1}{p'}} \\ & \le \left( C t^{q(\alpha - \beta)} v(B) \right)^{\frac{1}{q}} C \mu(B)^{\frac{1}{p'}} \\ & \le C t^{(\alpha - \beta)} C t^{\left(\beta - \alpha - \frac{Q}{p'}\right) q \frac{1}{q}} t^{Q\left(\frac{p - 1}{p}\right)} = C < \infty \end{aligned}$$

Now, using theorem C in [9], we have

$$S_1 \le C \left( \int_B |f(y)|^p d\mu(y) \right)^{q/p} \le C \|f\|_{\mathcal{L}^p(X,\mu)}^q$$

Next, we observe that if  $\delta(a,y) > 2a_1\delta(a,x)$ , then  $\delta(a,y) \le a_1\delta(a,x) + a_1\delta(a,y) \le \delta(a,y)/2 + a_1\delta(x,y)$ . Thus  $\delta(a,y)/2a_1 \le \delta(x,y)$ . Implies, using the condition  $v(B) \le Cr^{\left(\beta-\alpha-\frac{Q}{p'}\right)q}$ , then

$$S_{3} \leq C \int_{B(a,r)} \left( \int_{\delta(a,y) > \delta(a,x)} \frac{|f(y)|}{\delta(a,y)^{\beta-\alpha}} d\mu(y) \right)^{q} dv(x)$$

$$\leq C \int_{B(a,r)} \left( \sum_{k=0}^{\infty} \int_{B\left(a,2^{k+1}\delta(a,x)\right) \setminus B\left(a,2^{k}\delta(a,x)\right)} \frac{|f(y)|}{\delta(a,y)^{\beta-\alpha}} d\mu(y) \right)^{q} dv(x)$$

$$\leq C \int_{B(a,r)} \left[ \sum_{k=0}^{\infty} \left( \int_{B\left(a,2^{k+1}\delta(a,x)\right) \setminus B\left(a,2^{k}\delta(a,x)\right)} |f(y)|^{p} d\mu(y) \right)^{\frac{1}{p}} \right]^{q} dv(x)$$

$$\times \left( \int_{B\left(a,2^{k+1}\delta(a,x)\right) \setminus B\left(a,2^{k}\delta(a,x)\right)} \delta(a,y)^{(\alpha-\beta)p'} d\mu(y) \right)^{\frac{1}{p}} dv(x)$$

$$\leq C \|f\|_{\mathcal{L}^{p}(X,\mu)}^{q} \int_{B(a,r)} \left( \sum_{k=0}^{\infty} \left( 2^{k}\delta(a,x) \right)^{\alpha-\beta} \left( \mu B\left(a,2^{k+1}\delta(a,x)\right) \right)^{\frac{1}{p}} \right)^{q} dv(x)$$

$$\leq C \|f\|_{\mathcal{L}^{p}(X,\mu)}^{q} \int_{B(a,r)} \left( \sum_{k=0}^{\infty} \left( 2^{k}\delta(a,x) \right)^{\alpha-\beta} \left( \mu B\left(a,2^{k+1}\delta(a,x)\right) \right)^{\frac{1}{p}} \right)^{q} dv(x)$$

$$= C \|f\|_{\mathcal{L}^{p}(X,\mu)}^{q} r^{(\alpha-\beta)q} r^{\frac{Qq}{p'}} v(B)$$

$$= C \|f\|_{\mathcal{L}^{p}(X,\mu)}^{q}$$

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Hence, we conclude that

$$S_3 \le C \|f\|_{\mathcal{L}^p(X,\mu)}^q$$

To estimate  $S_2$ , we consider two cases. First assumption is that  $\alpha < \beta - \frac{Q}{p}$ . The hypothesis on the theorem  $\alpha > 0$  which implies  $0 < \alpha < \beta - \frac{Q}{p}$ . Given  $p^* = \frac{pQ}{p(\beta - \alpha - Q) + Q}$  then  $q \le p^*$ . First assumption  $q < p^*$  and suppose that

$$F_k := \{x: 2^k \le \delta(a, x) < s^{k+1}\};$$

$$G_k \coloneqq \left\{ y : \frac{2^{k-2}}{a_1} \le \delta(a, y) \setminus a_1 2^{k+2} \right\}.$$

Assume that  $\frac{p^*}{q}$ , using Holder's inequality, we obtain

$$S_{2} = \int_{X} \left( \int_{E_{2}(x)} |f(y)| \, \delta(x, y)^{\alpha - \beta} d\mu(y) \right)^{q} d\nu(x)$$

$$= C \sum_{k \in \mathbb{Z}} \int_{F_{k}} \left( \int_{E_{2}(x)} |f(y)| \, \delta(x, y)^{\alpha - \beta} d\mu(y) \right)^{q} d\nu(x)$$

$$\leq \sum_{k \in \mathbb{Z}} \left( \int_{F_{k}} \left( \int_{E_{2}(x)} |f(y)| \, \delta(a, x)^{\alpha - \beta} d\mu(y) \right)^{p^{*}} d\nu(x) \right)^{\frac{q}{p^{*}}} \times \left( \int_{F_{k}} 1^{\frac{p^{*}}{p^{*} - q}} d\nu(x) \right)^{\frac{p^{*} - q}{p^{*}}}$$

$$\leq C \sum_{k \in \mathbb{Z}} \nu(B)^{\frac{p^{*} - q}{p^{*}}} \left( \int_{X} \left( I_{\alpha} \left( |f| \chi_{G_{k}} \right) \right)^{p^{*}} d\nu(y) \right)^{\frac{q}{p^{*}}}$$

$$\leq C \sum_{k \in \mathbb{Z}} \nu(B)^{\frac{p^{*} - q}{p^{*}}} \left( \int_{G_{k}} |f(y)|^{p} d\mu(y) \right)^{\frac{q}{p}}$$

Where

$$\frac{p^* - q}{p^*} = 1 - \frac{q}{p^*}$$

$$= 1 - \frac{q(p(\beta - \alpha) - pQ + Q)}{pQ}$$

$$= 1 - \frac{Q(pq + p - q)}{pQ} + q - \frac{q}{p} = 0$$

$$\leq C \left( \int_{G_k} |f(y)|^p d\mu(x) \right)^{\frac{q}{p}}$$
  
$$\leq C \|f\|_{\mathcal{L}^p(X,\mu)}^q$$

if  $q = p^*$ , thus, we have

$$S_{2} = \int_{X} \left( \int_{E_{2}(x)} |f(y)| \, \delta(x, y)^{\alpha - \beta} d\mu(y) \right)^{q} dv(x)$$

$$= C \sum_{k \in \mathbb{Z}} \int_{F_{k}} \left( \int_{E_{2}(x)} |f(y)| \, \delta(x, y)^{\alpha - \beta} d\mu(y) \right)^{p^{*}} dv(x)$$

$$\leq C \sum_{k \in \mathbb{Z}} \left( \int_{X} \left( I_{\alpha} \left( |f| \chi_{G_{k}} \right) \right) dv(y) \right)^{p^{*}}$$

$$\leq C \sum_{k \in \mathbb{Z}} \left( \int_{G_{k}} |f(y)|^{p} d\mu(x) \right)^{\frac{q}{p}}$$

$$\leq C \left( \int_{G_{k}} |f(y)|^{p} d\mu(x) \right)^{\frac{p^{*}}{p}}$$

$$\leq C \|f\|_{\mathcal{L}^{p}(X,\mu)}^{q}$$

If  $\alpha > \beta - \frac{Q}{p'}$ , using Holder's inequality, we obtain

$$S_{2} \leq \int_{X} \left( \int_{E_{2}(x)} (f(y))^{p} d\mu(y) \right)^{\frac{q}{p}} \left( \int_{E_{2}(x)} \delta(a, x)^{(\alpha - \beta)p'} d\mu(y) \right)^{\frac{q}{p'}} d\nu(y)$$

thus we have

$$\begin{split} \int_{E_{2}(x)} \delta(a,x)^{(\alpha-\beta)p'} d\mu(y) &\leq \int_{0}^{\infty} \mu\left(B\left(a,\delta(a,x)\right) \cap \left\{y | \delta(x,y) < \lambda^{\frac{1}{(\alpha-\beta)p'}}\right\}\right) d\lambda \\ &\leq \int_{0}^{\delta(a,x)^{(\alpha-\beta)p'}} \mu\left(B\left(a,\delta(a,x)\right) \cap \left\{y | \delta(x,y) < \lambda^{\frac{1}{(\alpha-\beta)p'}}\right\}\right) d\lambda \\ &+ \int_{\delta(a,x)^{(\alpha-\beta)p'}}^{\infty} \mu\left(B\left(a,\delta(a,x)\right) \cap \left\{y | \delta(x,y) < \lambda^{\frac{1}{(\alpha-\beta)p'}}\right\}\right) d\lambda \\ &\leq C\delta(a,x)^{Q+(\alpha-\beta)p'} + \int_{\delta(a,x)^{(\alpha-\beta)p'}}^{\infty} \lambda^{\frac{1}{(\alpha-\beta)p'}} d\lambda = C\delta(a,x)^{Q+(\alpha-\beta)p'} \end{split}$$

The Fractional Integral Operators on Morrey Spaces Over Q-Homogeneous Metric Measure Space

where the positive constant C is independent of a and x. Hence, using Holder's inequality, we obtain

$$S_{2} \leq \int_{X} \left( \int_{E_{2}(x)} \delta(a, x)^{(\alpha - \beta)p'} d\mu(y) \right)^{\frac{q}{p'}} \left( \int_{E_{2}(x)} |f(y)|^{p} d\mu(y) \right)^{\frac{q}{p}} d\nu(x)$$

$$\leq \sum_{k \in \mathbb{Z}} \int_{F_{k}} \delta(a, x)^{Q + (\alpha - \beta)p'} \left( \int_{E_{2}(x)} |f(y)|^{p} d\mu(y) \right)^{\frac{q}{p}} d\nu(y)$$

$$\leq C 2^{k \left( \left( \beta - \alpha - \frac{Q}{p'} \right) q + \frac{Qq}{p'} + (\alpha - \beta)q \right)} \left( \int_{G_{k}} |f(y)|^{p} d\mu(y) \right)^{\frac{q}{p}} \leq C \left( \int_{X} |f(y)|^{p} d\mu(y) \right)^{\frac{q}{p}}$$

$$\leq C \|f\|_{\mathcal{L}^{p}(X, \mu)}^{q}$$

The proof is complete.

The similar results concerning the boundedness properties of the fractional integral operator  $I_{\alpha}$  on the classic Morrey spaces using *Q-homogeneous metric measure space* is obtained by the following theorem.

**Theorem 3.3** Let  $(X, \delta, \mu)$  be a Q -homogeneous metric measure space,  $\nu$  be a measure on X,  $1 , <math>1 < \alpha < \beta - \frac{Q}{p'}$ ,  $0 < \lambda_1 < \frac{\beta p}{q}$ , and  $\frac{Q\lambda_1}{\beta p} = \frac{\lambda_2}{q}$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p,\frac{Q\lambda_1}{\beta p}}(X,\mu)$  to  $\mathcal{L}^{q,\frac{\lambda_2}{q}}(X,\nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\nu(B) \leq C r^{\left(\beta - \alpha - \frac{Q}{p'}\right)q}$$
 with  $p' = \frac{p}{p-1}$ .

**Proof:** (Necessity) Suppose that  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(X,\mu)$  to  $\mathcal{L}^{q,\lambda_2}(X,\nu)$  which implies that

$$||I_{\alpha}f:\mathcal{L}^{q,\lambda_2}(X,\nu)|| \leq C ||f:\mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(X,\mu)||$$

Thus

$$\left(\frac{1}{\mu(B)^{\lambda_2}}\int_X |I_{\alpha}f|^q d\nu(x)\right)^{\frac{1}{q}} \le C\left(\frac{1}{\mu(B)^{\frac{Q\lambda_1}{\beta}}}\int_X |f(x)|^p d\mu(x)\right)^{\frac{1}{p}}$$

 $f := \chi_B$  where  $a \in X$  and r > 0 then

$$\left(\frac{1}{\mu(B)^{\lambda_2}} \int_X |I_\alpha \chi_B(\mathbf{x})|^q \, d\nu(\mathbf{x})\right)^{\frac{1}{q}} \leq C \left(\frac{1}{\mu(B)^{\frac{Q\lambda_1}{\beta}}} \int_X |\chi_B(\mathbf{x})|^p \, d\mu(\mathbf{x})\right)^{\frac{1}{p}}$$

$$\left(\frac{1}{\mu(B)^{\lambda_2}} \int_B \left(\int_B \frac{\chi_B}{\delta(\mathbf{x}, \mathbf{y})^{\beta - \alpha}} \, d\mu(\mathbf{y})\right)^q \, d\nu(\mathbf{x})\right)^{\frac{1}{q}} \leq C \mu(B)^{\frac{-Q\lambda_1}{p\beta}} \mu(B)^{\frac{1}{p}}$$

$$\mu(B)^{\frac{-\lambda_2}{q}} r^{\alpha - \beta} \, \mu(B) \nu(B)^{\frac{1}{q}} \leq C \mu(B)^{\frac{-Q\lambda_1}{p\beta}} \mu(B)^{\frac{1}{p}}$$
Because  $p' = \frac{p}{p-1}$ ,  $\frac{Q\lambda_1}{\beta p} = \frac{\lambda_2}{q}$  and  $C_0 r^Q \leq \mu(B) \leq C_1 r^Q$  then
$$\nu(B)^{\frac{1}{q}} \leq C \mu(B)^{\frac{1}{p'}} r^{\alpha - \beta}$$

$$\nu(B)^{\frac{1}{q}} \leq C r^{\frac{Q}{p'}} r^{\beta - \alpha}$$

$$\nu(B) \leq C r^{\left(\beta - \alpha - \frac{Q}{p'}\right)q}$$

**Sufficiency.** Given arbitrary ball B on X. Suppose that B := B(a, r) and  $\tilde{B} := (a, 2r)$  and  $f \in \mathcal{L}^{p, \frac{Q\lambda_1}{\beta}}(\mu)$ , we write

$$f = f_1 + f_2 := f_{\chi_{\widetilde{B}}} + f_{\chi_{\widetilde{B}}c}$$

$$\|f_1 \colon \mathcal{L}^p(\mu)\| = \left(\int_B |f(x)|^p d\mu(x)\right)^{\frac{1}{p}}$$

$$= \mu(B)^{\frac{Q\lambda_1}{\beta p}} \left(\frac{1}{\mu(B)^{\frac{Q\lambda_1}{\beta}}} \int_B |f(x)|^p d\mu\right)^{\frac{1}{p}}$$

$$\leq \mu(B)^{\frac{Q\lambda_1}{\beta p}} \left\|f \colon \mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(X,\mu)\right\|$$

if  $f_1 \in \mathcal{L}^p(X, \mu)$ , and using Theorem 3.2, it is obvious that

$$\left(\frac{1}{\mu(B)^{\lambda_{2}}} \int_{B} |I_{\alpha}f_{1}|^{q} d\nu(x)\right)^{\frac{1}{q}} \leq \mu(B)^{-\frac{\lambda_{2}}{q}} \left(\int_{B} |I_{\alpha}f_{1}|^{q} d\nu(x)\right)^{\frac{1}{q}} 
\leq \mu(B)^{-\frac{\lambda_{2}}{q}} ||I_{\alpha}f_{1}: \mathcal{L}^{q}(\nu)|| 
\leq C\mu(B)^{-\frac{\lambda_{2}}{q}} ||f_{1}: \mathcal{L}^{p}(\mu)||$$

The Fractional Integral Operators on Morrey Spaces Over Q-Homogeneous Metric Measure Space

$$\leq C\mu(B)^{-\frac{\lambda_2}{q}}\mu(B)^{-\frac{Q\lambda_1}{\beta p}}\left\|f:\mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(X,\mu)\right\|$$
$$\leq C\left\|f:\mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(X,\mu)\right\|$$

further we will prove,

$$|I_{\alpha}f_{2}(x)| = \left| \int_{\delta(x,y)\geq r} \frac{f(y)}{\delta(x,y)^{\beta-\alpha}} d\mu(y) \right|$$

$$\leq \int_{\delta(x,y)\geq r} \frac{|f(y)|}{\delta(x,y)^{\beta-\alpha}} d\mu(y)$$

$$\leq \sum_{k=0}^{\infty} \int_{2^{k}r\leq \delta(x,y)\leq 2^{k+1}r} \frac{|f(y)|}{\delta(x,y)^{\beta-\alpha}} d\mu(y)$$

$$\leq \sum_{k=0}^{\infty} \left( \frac{1}{2^{k}r} \right)^{\beta-\alpha} \int_{\delta(x,y)\leq 2^{k+1}r} |f(y)| d\mu(y)$$

$$\leq C \sum_{k=0}^{\infty} \left( \int_{\delta(x,y)\leq 2^{k+1}r} |f(x)|^{p} d\mu(y) \right)^{\frac{1}{p}} \left( \int_{\delta(x,y)\leq 2^{k+1}r} 1^{q} d\mu(y) \right)^{\frac{1}{q}} \frac{1}{2^{k}r^{\beta-\alpha}}$$

$$\leq C\mu(B)^{\frac{Q\lambda_{1}}{\beta p}} \left\| f: \mathcal{L}^{\frac{p,\frac{Q\lambda_{1}}{\beta p}}}(X,\mu) \right\| \sum_{k=0}^{\infty} \mu(B(x,2^{k+1}r))^{\frac{1}{q}} \frac{1}{(2^{k}r)^{\beta-\alpha}}$$

$$= \mu(B)^{\frac{Q\lambda_{1}}{\beta p}} r^{\alpha-\beta} r^{\frac{Q}{p'}} \left\| f: \mathcal{L}^{\frac{p,\frac{Q\lambda_{1}}{\beta p}}}(X,\mu) \right\|$$

Then

$$\left(\frac{1}{\mu(B)^{\lambda_2}} \int_{B} |I_{\alpha} f_2(x)|^q d\nu(x)\right)^{\frac{1}{q}} = C\nu(B)^{\frac{1}{q}} \mu(B)^{\frac{-\lambda_2}{q}} \mu(B)^{\frac{Q\lambda_1}{\beta p}} r^{\alpha-\beta} r^{\frac{Q}{p'}} \left\| f : \mathcal{L}^{p, \frac{Q\lambda_1}{\beta p}}(X, \mu) \right\|$$

$$= C \left\| f : \mathcal{L}^{p, \frac{Q\lambda_1}{\beta p}}(X, \mu) \right\|$$

The proof is complete.

The condition  $\frac{Q\lambda_1}{\beta p} = \frac{\lambda_2}{q}$  is interchangeable to the condition  $\nu(B) \leq Cr^{q(\beta-\alpha-\frac{Q}{p'})}$  Yet, the following theorem is hold obviously.

**Theorem 3.4** Let  $(X, \delta, \mu)$  be a Q -homogeneous metric measure space,  $\nu$  be a measure on X,  $1 , <math>1 < \alpha < \beta - \frac{Q}{p'}$ ,  $0 < \lambda_1 < \frac{\beta p}{q}$ , and  $\nu(B) \le Cr^{\left(\beta - \alpha - \frac{Q}{p'}\right)q}$  with  $p' = \frac{p}{p-1}$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p,\frac{Q\lambda_1}{\beta p}}(X,\mu)$  to  $\mathcal{L}^{q,\frac{\lambda_2}{q}}(X,\nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\frac{Q\lambda_1}{\beta p} = \frac{\lambda_2}{q}$$

When  $Q = \beta$ , the previous theorem implies the following corollary.

Corollary 3.5 Let  $(X, \delta, \mu)$  be a  $\beta$ -homogeneous metric measure space,  $\nu$  be a measure on X,  $0 < \lambda_1 < \frac{\beta p}{q}$ ,  $1 , and <math>\frac{\lambda_1}{p} = \frac{\lambda_2}{q}$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p,\frac{\lambda_1}{p}}(X,\mu)$  to  $\mathcal{L}^{q,\frac{\lambda_2}{q}}(X,\nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\nu(B) \leq C\mu(B)^{q\left(\frac{1}{p}-\frac{\alpha}{\beta}\right)}$$

Corollary 3.6 Let  $(X, \delta, \mu)$  be a  $\beta$ -homogeneous metric measure space,  $\nu$  be a measure on X,  $0 < \lambda_1 < \frac{\beta p}{q}$ ,  $1 , and <math>\nu(B) \le C\mu(B)^{q\left(\frac{1}{p} - \frac{\alpha}{\beta}\right)}$ . Then  $I_{\alpha}$  is bounded from  $\mathcal{L}^{p,\frac{\lambda_1}{p}}(X,\mu)$  to  $\mathcal{L}^{q,\frac{\lambda_2}{q}}(X,\nu)$  if and only if there is a constant C > 0 such that for every ball B on X,

$$\frac{\lambda_1}{p} = \frac{\lambda_2}{q}$$

# 4. CONCLUSIONS

Through our work we have been able to extend the known results for the classical fractional integral operator  $I_{\alpha}$  to the boundedness of with measure  $\mu$  and  $\nu$  on Morrey spaces over Q-homogeneous metric measure space. Our results not only cover the known results for  $I_{\alpha}$ , but also enrich the class of funtions of  $\alpha$ ,  $\lambda_1$  and  $\lambda_2$  for which the operator  $I_{\alpha}$  is bounded from the classical Morrey space  $\mathcal{L}^{p,\frac{Q\lambda_1}{\beta}}(\mu)$  to  $\mathcal{L}^{q,\lambda_2}(\nu)$ , on Q-homogeneous and the corollary  $I_{\alpha}$  is bounded from the classical Morrey space  $\mathcal{L}^{p,\lambda_1}(\mu)$  to  $\mathcal{L}^{q,\lambda_2}(\nu)$ , on  $\beta$ -homogeneous.

# **COMPETING INTERESTS**

The authors declare that there is no conflict of interests regarding the publication of this paper

# **AUTHOR'S CONTRIBUTIONS**

The author read and approved the final manuscript.

## **ACKNOWLEDGMENTS**

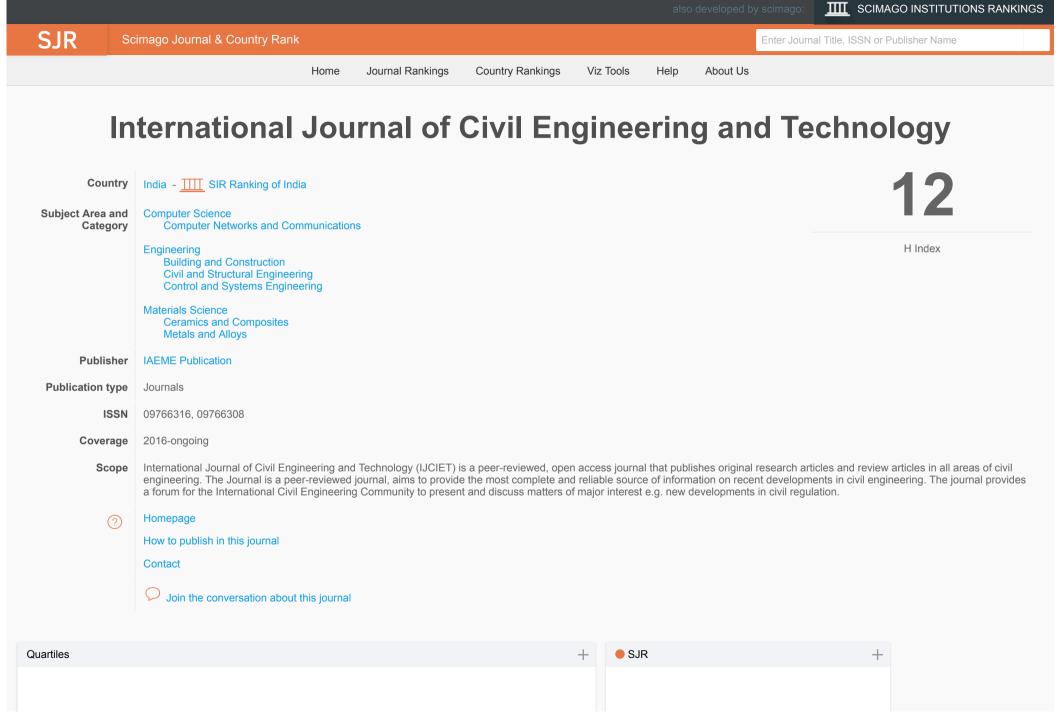
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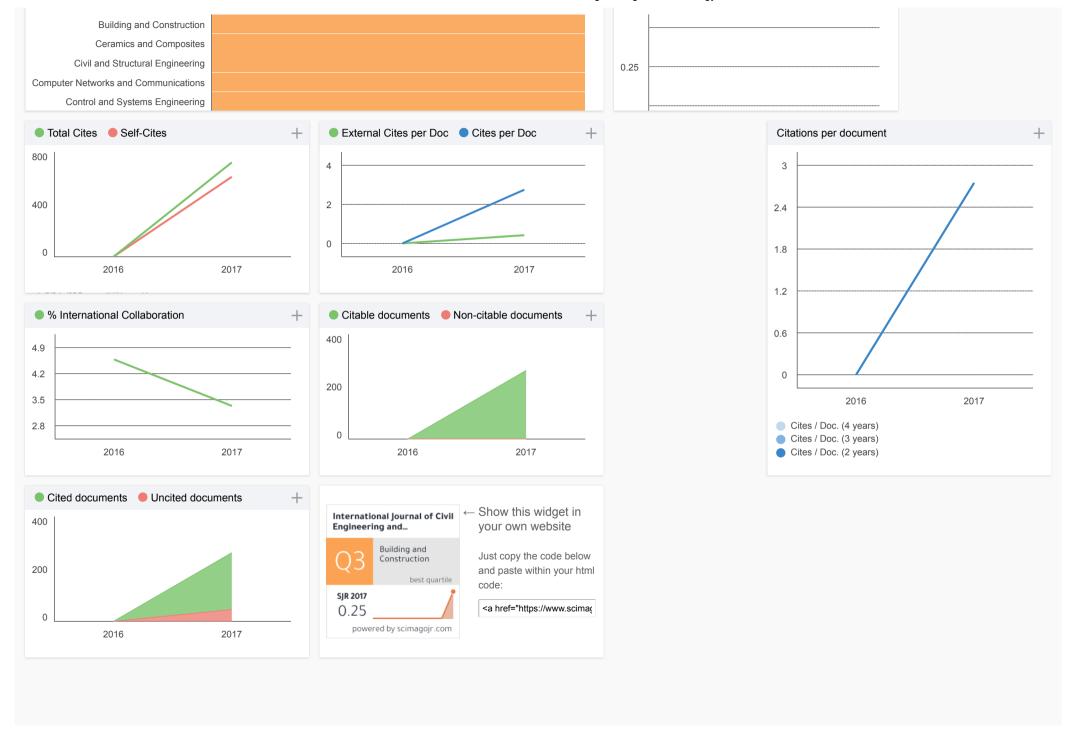
# **FUNDING**

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# **REFERENCES**

- [1] Petree, J.: On the theory of *lp* spaces. J. Funct. Ana 4, 71–87 (1969).
- [2] Adams, R.A.: Sobolev spaces. In: Pure and Applied Mathematics. Academic Press, London (1975).
- [3] Chiarenza, F., Frasca: Morrey spaces and hardy-littlewood maximal function. Rend. Mat 7(1),273–279 (1987).
- [4] Nakai, E.: Hardy-littlewood maximal operator, singular integral operators, and the riesz potential on the generalized morrey spaces. Math. Nachr 166, 95–103 (1994)
- [5] Eridani, Gunawan, H.: Fractional integrals and generalized olsen inequalities. Kyungpook Math 49(1), 31–39 (2009).
- [6] Edmunds D, V.K., Meshky, A.: Bounded and compac integral operator. vol. 543, (2002).
- [7] Eridani, V.K., Meshky, A.: Morrey spaces and fractional integral operators. Expo. Math 27(3), 227–239 (2009).
- [8] Nakai, E.: On generalized fractional integrals in the orlicz spaces homogeneous type. Scientiae Mathematicae Japonicae Online 4, 901–915 (2001).
- [9] Kokilashvili, V., Meshky, A.: On some weighted inequlities for fractional integrals on non-homogeneous spaces. Mathematics ad It's Applications 24(4), 871–885 (2005).







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Thank You Madam.

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Oludare Abiodun 7 months ago

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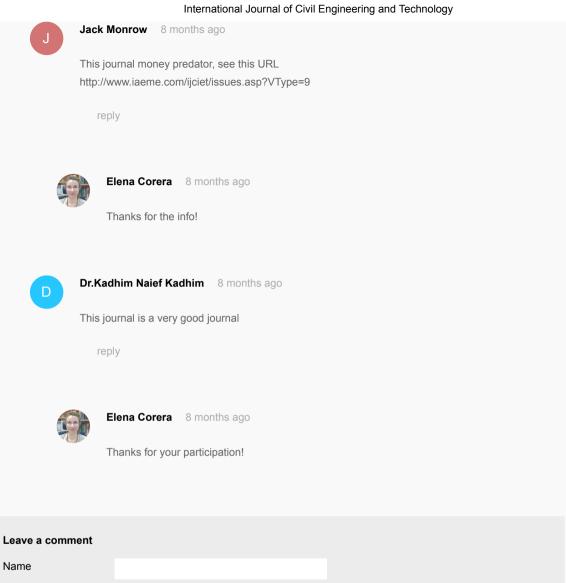
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