KYUNGPOOK Math. J. 49(2009), 31-39

Fractional Integrals and Generalized Olsen Inequalities

Hendra Gunawan*
Department of Mathematics, Bandung Institute of Technology, Bandung 40132, Indonesia
e-mail : hgunawan@math.itb.ac.id

Eridani
Department of Mathematics, Airlangga University Surabaya 60115, Indonesia
e-mail : eridani@unair.ac.id

Abstract. Let T_ρ be the generalized fractional integral operator associated to a function $\rho : (0,\infty) \to (0,\infty)$, as defined in [16]. For a function W on $\mathbb{R}^n$, we shall be interested in the boundedness of the multiplication operator $f \mapsto W \cdot T_\rho f$ on generalized Morrey spaces. Under some assumptions on ρ, we obtain an inequality for $W \cdot T_\rho$, which can be viewed as an extension of Olsen's and Kurata-Nishigaki-Sugano's results.

1. Introduction

For $0 < \alpha < n$, let I_α denote the Riesz potential or the (classical) fractional integral operator, which is given by the formula

$$I_\alpha f(x) := \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy.$$

Formally, through its Fourier transform, the operator I_α can be recognized as a multiple of the Laplacian to the power of $-\frac{\alpha}{2}$, that is,

$$I_\alpha f = \kappa(-\Delta)^{-\alpha/2} f,$$

where $\kappa = \kappa(n,\alpha)$ (see, for instance, [2], [13], [22], [24]). A well-known result for I_α is the Hardy-Littlewood-Sobolev inequality, which was proved by Hardy and Littlewood [8], [10] and Sobolev [23] around the 1930's.

Theorem 1.1 (Hardy-Littlewood; Sobolev)**.** *For $1 < p < \frac{n}{\alpha}$, we have the inequality*

$$\|I_\alpha f\|_q \le C_p \|f\|_p, \tag{1.1}$$

* Corresponding author.
Received 23 April 2007; received in revised form 24 April 2007; accepted 1 May 2007.
2000 Mathematics Subject Classification: 26A33, 47B38, 42B20, 42B25, 26D10.
Key words and phrases: Fractional integral operators, Hardy-Littlewood maximal operators, multiplication operators, Olsen inequality, Morrey spaces.