

A CHARACTERIZATION FOR FRACTIONAL INTEGRALS ON GENERALIZED MORREY SPACES

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Abstract. This paper concerns with the fractional integrals, which are also known as the Riesz potentials. A characterization for the boundedness of the fractional integral operators on generalized Morrey spaces will be presented. Our results can be viewed as a refinement of Nakai's^[7].

Key words: *fractional integrals, morrey spaces*

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1 Introduction

For $0 < \alpha < d$, we define the fractional integral (also known as the Riesz potential) $I_\alpha f$ by

$$I_\alpha f(x) := \int_{\mathbf{R}^d} \frac{f(y)}{|x-y|^{d-\alpha}} dy, \quad x \in \mathbf{R}^d,$$

for any suitable function f on \mathbf{R}^d . Clearly $I_\alpha f$ is well-defined for any locally bounded, compactly supported function f on \mathbf{R}^d . It is well-known that I_α is bounded from $L^p(\mathbf{R}^d)$ to $L^q(\mathbf{R}^d)$, that is,

$$\|I_\alpha f : L^q\| \leq C \|f : L^p\|,$$

if and only if

$$\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{d},$$