Expansion of Ceva Theorem in the Normed Space with the Angle of Wilson

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Expansion of Ceva Theorem in the Normed Space with the Angle of Wilson

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Abstract: *This paper discusses the expansion of Ceva theorem in the normed space. The Ceva theorem is expanded using the Wilson angle. Before entering the core issue first discusses about Wilson's angle and its properties. Furthermore, it is proved by the Ceva's Theorem by first modifying it.*

Keywords: Ceva's Theorem, normed space, Wilson angle

1. Introduction

The angle between two vectors in the Euclidean space $\mathbb{R}^2$ has been well known. In the Euclidean space the angle between two vectors is defined using the dot product [8]. An angle between two vectors has also been expended in the inner product space [7]. Furthermore, in the normed space it has also been known the angle between the two vectors among other angles P, I, g ([1], [2], [3], [4]), Thy angle [2] and Wilson angle [6].

The angle in the normed space discussed in this paper is the angle of Wilson. The Wilson angle is introduced by Valentine and Wayment (1971). A review of the Wilson angle is discussed as follows :

Let $(V, \|\cdot\|)$ be a is a normed space over the field $\mathbb{R}$, for any $a, b \in V$ Defines a nonlinear function :

$$2\langle a,b\rangle := \|a\|^2 + \|b\|^2 - \|a-b\|^2 \qquad (1)$$

From the nature of the normed space belongs :

$$\big|\|a\| - \|b\|\big|^2 \le \|a-b\|^2$$
$$\Leftrightarrow \|a\|^2 - 2\|a\|\cdot\|b\| + \|b\|^2 \le \|a-b\|^2$$
$$\Leftrightarrow \langle a,b\rangle \le \|a\|\cdot\|b\| \qquad (2)$$

meanwhile :

$$\|a-b\|^2 \le (\|a\| + \|b\|)^2$$
$$\Leftrightarrow \|a-b\|^2 - \|a\|^2 - \|b\|^2 \le 2\|a\|\cdot\|b\|$$
$$\Leftrightarrow -\langle a,b\rangle \le \|a\|\cdot\|b\| \qquad (3)$$

Of the equation (2) and (3) obtained :

$$|\langle a, b\rangle| \le \|a\|.\|b\|, \quad \forall\, a,b \in V \qquad (4)$$

fulfill the cauchy - Schwarz inequality [8]. The Wilson angle is defined as the angle between two vectors a and b satisfy

$$\angle(a,b) := \arccos\left(\frac{\|a\|^2 + \|b\|^2 - \|a-b\|^2}{2\|a\|\cdot\|b\|}\right) \qquad (5)$$

From the angle of Wilson obtained the rules of cosine :

$$\|c\|^2 = \|a\|^2 + \|b\|^2 - 2\|a\|\cdot\|b\|\cos\angle(a,b) \qquad (6)$$

Next from the equation Next from the equation (6) sine rules are obtained :

$$\|a\|\cdot\|b\|\sin\angle(a,b) = K \qquad (7)$$

With $K = 2\sqrt{s(s-\|a\|)(s-\|b\|)(s-\|c\|)}$ and $2s = \|a\| + \|b\| + \|c\|$

2. Main Result

Definition. 2.1. Let $(V,\|\cdot\|)$ be a normed space for $a,b,c \in V\setminus\{0\}$, defined $\Delta[a,b,c]$ as $\{a,b,c\}$ satisfy $a + c = b$, which completed with a Wilson angle $\angle(a,b)$, $\angle(-a,c)$, dan $\angle(b,c)$.

Definition. 2.2. Let $(V,\|\cdot\|)$ be a normed space for $d \in V\setminus\{0\}$, called the Ceva vector of $\Delta[a_1,a_2,a_3]$ if any $\alpha \in (0,1)$ so that it satisfies $\alpha a_i + d = a_j$ with $i \ne j$.

Definition. 2.3. Let $(V,\|\cdot\|)$ be a normed space for $d,e,f \in V\setminus\{0\}$, called vector ceva ally of $\Delta[a,b,c]$ if any $\alpha_i \in (0,1)$, $i = 1,2,3,4,5,6$ so that it satisfies $(1-\alpha_6)f + a = (1-\alpha_5)e$, $(1-\alpha_5)e + c = (1-\alpha_4)d$, $(1-\alpha_6)f + b = (1-\alpha_4)d$

Theorem. 2.1. Let $(V,\|.\|)$ be a normed space for $\Delta[a,b,c]$, then the following statement is equivalent.

1. Let $d,e,f \in V\setminus\{0\}$ be a so that it satisfies
$(1-\alpha_6)f+a=(1-\alpha_5)e$,
$(1-\alpha_5)e+c=(1-\alpha_4)d$,
$(1-\alpha_6)f+b=(1-\alpha_4)d$, with $\alpha_i\in(0,1)$, and i = 1,2,3,4,5,6

2. $$\frac{\sin\angle(-b,(1-\alpha_6)f)}{\sin\angle(b,(1-\alpha_4)d)}\cdot\frac{\sin\angle(c,(1-\alpha_4)d)}{\sin\angle(-c,(1-\alpha_5)e)}\cdot\frac{\sin\angle(a,(1-\alpha_5)e)}{\sin\angle(-a,(1-\alpha_6)f)}=1$$

3. $$\frac{\|-\alpha_1 a\|}{\|(1-\alpha_1)a\|}\cdot\frac{\|-\alpha_3 c\|}{\|(1-\alpha_3)c\|}\cdot\frac{\|(1-\alpha_2)b\|}{\|-\alpha_2 b\|}=1$$

Proof.

$(1\Rightarrow 2)$

pay attention $\Delta[(1-\alpha_6)f,(1-\alpha_4)d,b]$ with angle $\angle(-b,(1-\alpha_6)f)$, $\angle((1-\alpha_6)f,(1-\alpha_4)d)$ and $\angle(b,(1-\alpha_4)d)$ obtained :

$$K_1=\|-b\|\cdot\|(1-\alpha_6)f\|\sin\angle(-b,(1-\alpha_6)f) \quad (8)$$

$$K_1=\|b\|\cdot\|(1-\alpha_4)d\|\sin\angle(b,(1-\alpha_4)d) \quad (9)$$

From equation (8) dan (9) obtained :

$$\frac{\sin\angle(-b,(1-\alpha_6)f)}{\sin\angle(b,(1-\alpha_4)d)}=\frac{\|b\|\cdot\|(1-\alpha_4)d)\|}{\|-b\|\cdot\|(1-\alpha_6)f\|}=\frac{\|(1-\alpha_4)d\|}{\|(1-\alpha_6)f\|} \quad (10)$$

pay attention $\Delta[(1-\alpha_5)e,(1-\alpha_4)d,c]$ with angle $\angle(-c,(1-\alpha_5)e)$, $\angle((1-\alpha_5)e,(1-\alpha_4)d)$ and $\angle(c,(1-\alpha_4)d)$. Obtained :

$$K_2=\|c\|\cdot\|(1-\alpha_4)d\|\sin\angle(c,(1-\alpha_4)d) \quad (11)$$

$$K_2=\|-c\|\cdot\|(1-\alpha_5)e\|\sin\angle(-c,(1-\alpha_5)e) \quad (12)$$

From equation (11) and (12) obtained :

$$\frac{\sin\angle(c,(1-\alpha_4)d)}{\sin\angle(-c,(1-\alpha_5)e)}=\frac{\|-c\|\cdot\|(1-\alpha_5)e\|}{\|c\|\cdot\|(1-\alpha_4)d\|}=\frac{\|(1-\alpha_5)e\|}{\|(1-\alpha_4)d\|} \quad (13)$$

pay attention $\Delta[(1-\alpha_6)f,(1-\alpha_5)e,a]$ with angle $\angle(-a,(1-\alpha_6)f)$, $\angle((1-\alpha_5)e,(1-\alpha_6)f)$ and $\angle(a,(1-\alpha_5)e)$ obtained :

$$K_3=\|-a\|\cdot\|(1-\alpha_6)f\|\sin\angle(-a,(1-\alpha_6)f) \quad (14)$$

$$K_3=\|a\|\cdot\|(1-\alpha_5)e\|\sin\angle(a,(1-\alpha_5)e) \quad (15)$$

From equation (14) and (15) obtained :

$$\frac{\sin\angle(a,(1-\alpha_5)e)}{\sin\angle(-a,(1-\alpha_6)f)}=\frac{\|-a\|\cdot\|(1-\alpha_6)f\|}{\|a\|\cdot\|(1-\alpha_5)e\|}=\frac{\|(1-\alpha_6)f\|}{\|(1-\alpha_5)e\|} \quad (16)$$

Multiply equations (10), (13) and (16) then obtained :

$$\frac{\sin\angle(-b,(1-\alpha_6)f)}{\sin\angle(b,(1-\alpha_4)d)}\cdot\frac{\sin\angle(c,(1-\alpha_4)d)}{\sin\angle(-c,(1-\alpha_5)e)}\cdot\frac{\sin\angle(a,(1-\alpha_5)e)}{\sin\angle(-a,(1-\alpha_6)f)}=\frac{\|(1-\alpha_4)d\|}{\|(1-\alpha_6)f\|}\cdot\frac{\|(1-\alpha_5)e\|}{\|(1-\alpha_4)d\|}\cdot\frac{\|(1-\alpha_6)f\|}{\|(1-\alpha_5)e\|}=1$$

$(2\Rightarrow 3)$

$$\frac{\sin\angle(a,(1-\alpha_5)e)}{\sin\angle(-a,(1-\alpha_6)f)}\cdot\frac{\sin\angle(-b,(1-\alpha_6)f)}{\sin\angle(b,(1-\alpha_4)d)}\cdot\frac{\sin\angle(c,(1-\alpha_4)d)}{\sin\angle(-c,(1-\alpha_5)e)}=1$$

$$\Leftrightarrow \frac{\sin\angle(-\alpha_2 b,(1-\alpha_6)f)}{\sin\angle(-\alpha_1 a,(1-\alpha_6)f)}\cdot\frac{\sin\angle((1-\alpha_1)a,(1-\alpha_5)e)}{\sin\angle(-\alpha_3 c,(1-\alpha_5)e)}\cdot\frac{\sin\angle((1-\alpha_3)c,(1-\alpha_4)d)}{\sin\angle((1-\alpha_4)d,(1-\alpha_2)b)}=1$$

$$\Leftrightarrow \frac{\dfrac{K_2}{\|-\alpha_2 b\|\cdot\|(1-\alpha_6)f\|}}{\dfrac{K_1}{\|-\alpha_1 a\|\cdot\|(1-\alpha_6)f\|}}\cdot\frac{\dfrac{K_1}{\|(1-\alpha_1)a\|\cdot\|(1-\alpha_5)e\|}}{\dfrac{K_3}{\|-\alpha_3 c\|\cdot\|(1-\alpha_5)e\|}}$$

$$\frac{\dfrac{K_3}{\|(1-\alpha_3)c\|\cdot\|(1-\alpha_4)d\|}}{\dfrac{K_2}{\|(1-\alpha_4)d\|\cdot\|(1-\alpha_2)b\|}}=1$$

$$\Leftrightarrow \quad \frac{\|-\alpha_1 a\|}{\|(1-\alpha_1)a\|}\cdot\frac{\|-\alpha_3 c\|}{\|(1-\alpha_3)c\|}\cdot\frac{\|(1-\alpha_2)b\|}{\|-\alpha_2 b\|}=1$$

$(3 \Rightarrow 1)$

Suppose that $(1-\alpha_6)f + a \neq (1-\alpha_4)e$

For example $(1-\alpha_6)f + a = \beta g$ then

$$\frac{\|-\alpha_1 a\|}{\|(1-\alpha_1)a\|}\cdot\frac{\|\beta c\|}{\|(1-\beta)c\|}\cdot\frac{\|(1-\alpha_2)b\|}{\|-\alpha_2 b\|}=1 \qquad (17)$$

While it is known :

$$\frac{\|-\alpha_1 a\|}{\|(1-\alpha_1)a\|}\cdot\frac{\|-\alpha_3 c\|}{\|(1-\alpha_3)c\|}\cdot\frac{\|(1-\alpha_2)b\|}{\|-\alpha_2 b\|}=1 \qquad (18)$$

From equation (15) and (16) obtained :

$$\frac{\|\beta c\|}{\|(1-\beta)c\|}=\frac{\|-\alpha_3 c\|}{\|(1-\alpha_3)c\|} \qquad (19)$$

$\alpha_3 = \beta$ or $f = g$

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