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FRACTIONAL INTEGRAL OPERATORS IN GENERALIZED MORREY SPACES DEFINED ON METRIC MEASURE SPACES

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ABSTRACT. We derive some necessary and sufficient conditions for the boundedness of fractional integral operators in generalized Morrey spaces defined on metric measure spaces.

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1. INTRODUCTION

In the present paper we consider the boundedness of the fractional integral operators on metric measure spaces (X, ρ, μ). By this we mean that (X, ρ) is a metric space and μ is a Borel measure. By generalizing the underlying measures, we seek for a better understanding of the fractional integral operators. It seems that Morrey spaces can describe the boundedness property of fractional integral operators very precisely. The most fundamental result of this field is due to Adams [1]. Nowadays there are series of papers that describe the boundedness property of fractional integral operators by means of (generalized) Morrey spaces (see for example, [5, 4, 7, 10, 15, 17]). The boundedness of fractional integral operators defined on nonhomogeneous spaces on $\mathbb{R}^n$ was established in [8] and the same problem on general nonhomogeneous spaces was investigated in [9]. A remarkable progress on function spaces on metric measure spaces was made a decade ago, starting from the papers [11, 18, 19].

To describe our setting, we need some notations. Denote by $\mathcal{B}(X)$ the set of all open balls in X. Throughout the present paper we postulate the following conditions on ϕ: Here and below we denote by $B(a, r)$ the open ball centered at a and of radius $r > 0$. For a ball $B := B(a, r)$, we sometimes write $\phi(a, r) := \phi(B)$. In what follows the letter C will be used to denote constants that may change from one occurrence to another one.

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